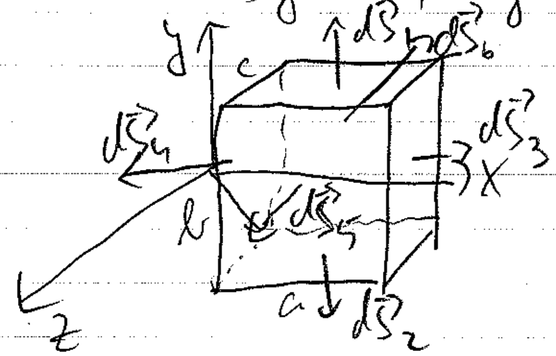
Problems Chpt. 13

(1) $\vec{E} = E_y(t,x)\,\vec{i}_y$



Only $d\vec{S}_1$ and $d\vec{S}_2$ are contributing since $\vec{E} = E_y \vec{i}_y$

S_1: $\vec{r} = x\,\vec{i}_x + (y_0+b)\,\vec{i}_y + z\,\vec{i}_z$; $d\vec{S} = dx\,dy\,\vec{i}_y$

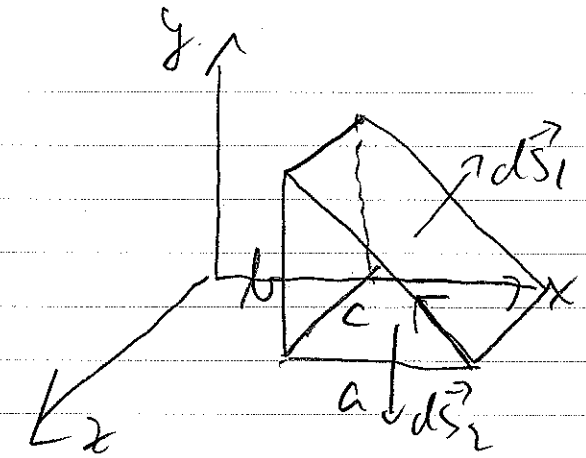
$$\Rightarrow \int_{S_1} d\vec{S}\cdot\vec{E} = \int_{x_0}^{x_0+a} dx \int_{z_0}^{z_0+c} dz\, E_y(t,x)$$

$$= c\int_{x_0}^{x_0+a} dx\, E_y(t,x)$$

S_2: $\vec{r} = x\,\vec{i}_x + y_0\,\vec{i}_y + z\,\vec{i}_z$; $d\vec{S} = -dx\,dy\,\vec{i}_z$

$$\int_{S_2} d\vec{S}\,\vec{E} = -c\int_{x_0}^{x_0+a} dx\, E_y(t,x)$$

$$\Rightarrow \oint_{\partial V} d\vec{S}\cdot\vec{E} = 0$$



②

Again only S_1 and S_2 contribute because the other $d\vec{S} \perp \vec{E}$.

For S_2 we get as in the previous example

$$\int_{S_2} d\vec{S} \cdot \vec{E} = -c \int_{x_0}^{x_0+a} dx \, E_y(t, x)$$

$$S_1 : \vec{r} = \xi \frac{-a\vec{i}_x + b\vec{i}_y}{\sqrt{a^2+b^2}} + z\vec{i}_z + (x_0+a)\vec{i}_x$$

$$d\vec{S} = \pm d\xi \, dz \frac{-a\vec{i}_x + b\vec{i}_y}{\sqrt{a^2+b^2}} \times \vec{i}_z$$

$$= \pm d\xi \, dz \frac{a\vec{i}_y + b\vec{i}_x}{\sqrt{a^2+b^2}}$$

must point out the volume $\Rightarrow$

$$d\vec{S} = + d\xi \, dz \frac{a\vec{i}_y + b\vec{i}_x}{\sqrt{a^2+b^2}}$$

$$\int_{S_1} d\vec{S}\cdot\vec{E} = \int_0^{\sqrt{a^2+b^2}} d\xi \int_{z_0}^{z_0+c} dz\; E_y\left(t_1, x_0+a-\frac{a\xi}{\sqrt{a^2+b^2}}\right)$$

$$= c\int_0^{\sqrt{a^2+b^2}} d\xi\; E_y\left(t_1, x_0+a-\frac{a\xi}{\sqrt{a^2+b^2}}\right)\frac{a}{\sqrt{a^2+b^2}}$$

Substitution

$$x = x_0+a-\frac{a\xi}{\sqrt{a^2+b^2}}$$

$$dx = -\frac{a}{\sqrt{a^2+b^2}}\,d\xi$$

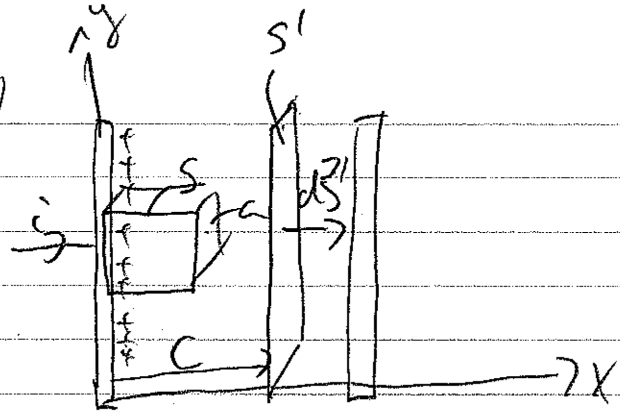
$$x(\xi=0) = x_0+a$$
$$x(\xi=\sqrt{a^2+b^2}) = x_0$$

$$\Rightarrow \int_{S_1} d\vec{S}\cdot\vec{E} = c\,\frac{a}{\sqrt{a^2+b^2}}\int_{x_0}^{x_0+a} dx\,\frac{\sqrt{a^2+b^2}}{a}\; E_y(t_1, x)\cdot\frac{a}{\sqrt{a^2+b^2}}$$

$$= c\int_{x_0}^{x_0+a} dx\; E_y(t_1, x) = -\int_{S_2} d\vec{S}\cdot\vec{E}$$

$$\Rightarrow \oint_{\partial V} d\vec{S}\cdot\vec{E} = 0$$

(2)



Gaussian surface with a gate

$$\vec{E} = E_x \vec{i}_x .$$

$$\oint_S d\vec{S}\,\vec{E} = a E_x = \frac{aQ}{A\varepsilon_0}$$

$$\Rightarrow E_x = \frac{Q}{A\varepsilon_0}$$

$$\Rightarrow Q = \varepsilon_0 A E_x$$

$$\Rightarrow i = \frac{dQ}{dt} = \varepsilon_0 A \frac{dE_x}{dt} = -\varepsilon_0 A \omega E_0 \sin(\omega t)$$

$$\Phi_{\vec{E}} = \int_{S'} d\vec{S}'\,\vec{E} = A E_0 \cos(\omega t)$$

$$i_D = \varepsilon_0 \frac{d\Phi_{\vec{E}}}{dt} = -A\varepsilon_0 E_0 \omega \sin(\omega t) = i$$

(3) $$V = \int_C d\vec{r}\cdot\vec{E} = E_x d = \frac{Qd}{A\varepsilon_0}$$

$$C = \frac{Q}{V} = \frac{A\varepsilon_0}{d}$$

$$i_D = \frac{dQ}{dt} = C\frac{dV}{dt}$$ (here we used the previous problem)

Exercises Chapt. 13

(1a)

$$E_y = A\cos[a(x-bt)]$$

$$\frac{1}{c^2}\frac{\partial^2 E_y}{\partial t^2} = \frac{\partial^2 E_y}{\partial x^2}$$

$$\Rightarrow \frac{1}{c^2}(-a^2b^2)A\cos[a(x-bt)] = -a^2 \; A\cos[ax-bt]$$

$$\Rightarrow \quad \frac{b^2}{c^2} = 1 \Rightarrow b = \pm c$$

(b) $k = a \Rightarrow \lambda = \frac{2\pi}{k} = \frac{2\pi}{a}$

(c) $\omega = ab \Rightarrow T = \frac{2\pi}{\omega} = \frac{2\pi}{ab}$

(2) $\omega = 2\pi f$; $\lambda = cT = 2\pi\frac{c}{\omega} = \frac{c}{f}$

$c = 3\cdot 10^8 \frac{m}{s}$

Name	f (Hz)	λ (m)
UV	10^{16}	$3\cdot 10^{-8}$
IR	10^{13}	$3\cdot 10^{-5}$
microwaves	10^{10}	$3\cdot 10^{-2}$
X ray	10^{19}	$3\cdot 10^{-11}$
vis. light	10^{15}	$3\cdot 10^{-7}$

(3) see lecture : $B_0 = \frac{1}{c} E_0$