$$N = \int dx\ \psi^*(x)\,\psi = \int dx \left(\int \frac{dk}{2\pi}\,\tilde{\psi}^*(k)\,e^{-ikx}\right) \cdot \left(\int \frac{dk}{2\pi}\,\tilde{\psi}(k)\,e^{ikx}\right)$$

$$= \int \frac{dk}{2\pi} \int \frac{dk'}{2\pi}\,\tilde{\psi}^*(k)\,\tilde{\psi}(k') \int dx\ e^{i(k-k')x}$$

Analogie:

$$\left(\sum_{i=1}^{N} a_i\right) \cdot \left(\sum_{i=1}^{N} b_i\right)$$

$$= (a_1 + a_2 + \dots) \cdot (b_1 + b_2 + \dots)$$

$$= a_1 b_1 + a_1 b_2 + \dots\ a_2 b_1 + a_2 b_2 + \dots$$

$$= \sum_{i,j=1}^{N} a_i b_j$$