# Diffusion coefficient matrix for multiple conserved charges: a Kubo approach

Sourav Dey

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Collaboration: Dr. Amaresh Jaiswal, Dr. Hiranmaya Mishra

# Motivation:

Annalen der Physik und Chemie.94:59 (1855)

- Non-relativistical diffusion: Fick's Law $\vec{J}_a = -K_{aa}\overrightarrow{\nabla} n_a(t,\vec{x})$

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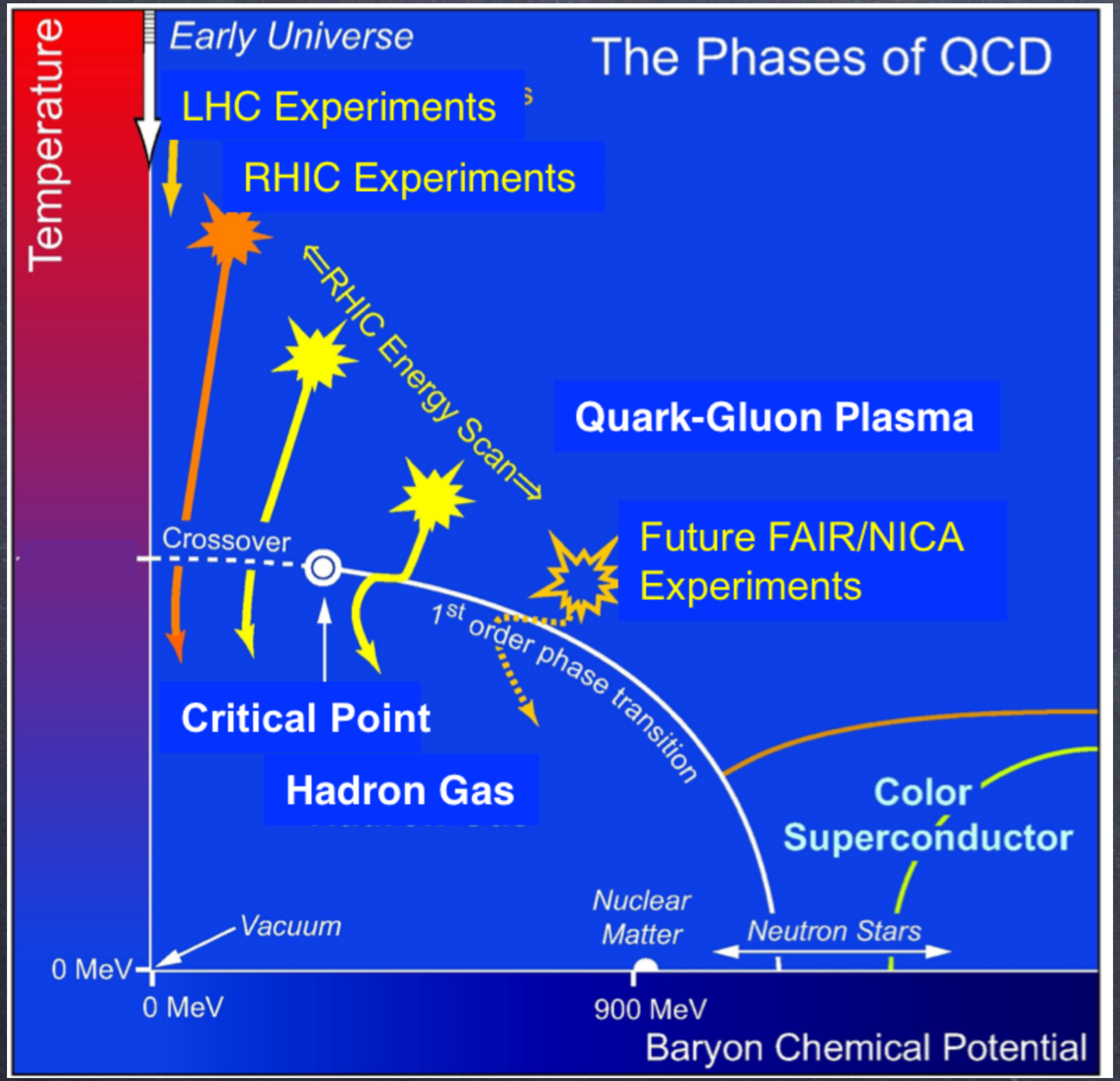


$$\begin{pmatrix} J_B^i \\ J_Q^i \\ J_S^i \end{pmatrix} = \begin{pmatrix} K_{BB} & K_{BQ} & K_{BS} \\ K_{QB} & K_{QQ} & K_{QS} \\ K_{SB} & K_{SQ} & K_{SS} \end{pmatrix} \begin{pmatrix} \nabla^i\left(\mu_B/T\right) \\ \nabla^i\left(\mu_Q/T\right) \\ \nabla^i\left(\mu_S/T\right) \end{pmatrix}$$

Ref:Nayak,Tapan K.J.Phys,:Conf.Ser.1602(2020)012003

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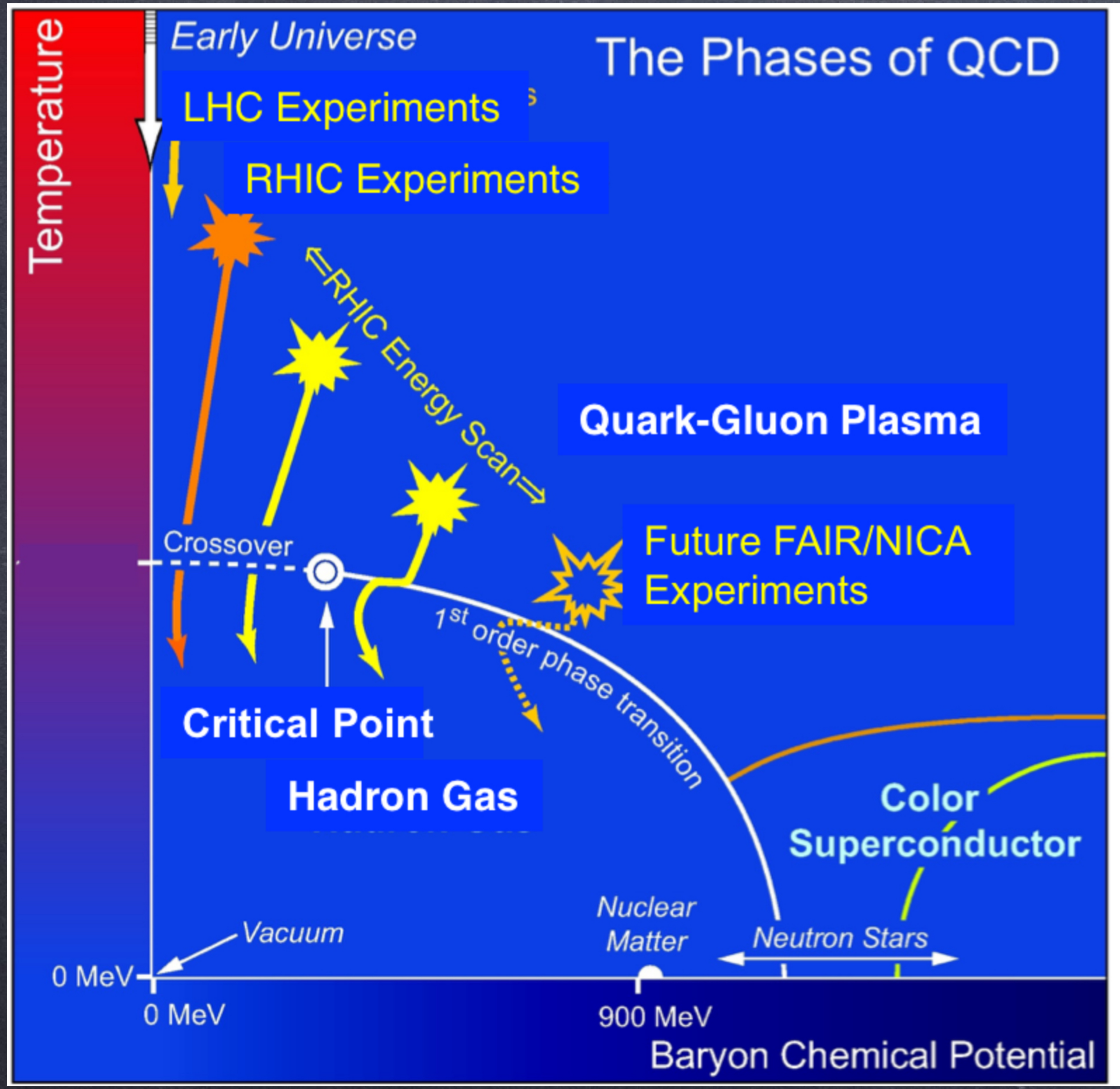


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M.Greif, J.A.Fotakis, G.S.Denicol and C.Greiner, Phys.Rev.Lett(2018)242301

A.Das, H.Mishra, R.Mahapatra, Phys.Rev.D 106 (2022) 1, 014013

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# Zubarev's NESO method

D.N.Zubarev et al,Theoritical and Mathematical Physics 40(1979) 821-831; A.Hosoya et al,Annals of Physics 154(1984)

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    - NESO, which characterizes the non-equilibrium state of system

$$\hat{\varrho}(t) = \frac{1}{\mathcal{Q}} \exp\left[ - \int d^3\mathbf{x}\, \hat{\mathcal{Z}}(\mathbf{x}, t) \right]$$

$$\mathcal{Q} = \mathbf{Tr}\left( exp\left[ - \int d^3\mathbf{x} \hat{\mathcal{Z}}(t, \mathbf{x}) \right] \right)$$

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$$\hat{\mathscr{Z}}(\mathbf{x},t) = \varepsilon \int_{-\infty}^{t} dt_1\, e^{\varepsilon(t_1 - t)} \left[\beta^{\nu}(\mathbf{x},t_1)\,\hat{T}_{0\nu}(\mathbf{x},t_1) - \sum_a \alpha_a(\mathbf{x},t_1)\,\hat{J}^0_a(\mathbf{x},t_1)\right]$$

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$$\partial_\mu \hat{T}^{\mu\nu} = 0. \qquad \partial_\mu \hat{J}^\mu_a = 0$$

Where,

$$\beta^\mu(t, \mathbf{x}) = \beta(t, \mathbf{x})\ u^\mu(t, \mathbf{x})$$

$$\alpha_a(t, \mathbf{x}) = \beta(t, \mathbf{x})\ \mu_a(t, \mathbf{x})$$

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$$\hat{\mathscr{A}} = \int d^3\mathbf{x} \left[ \beta^{\nu}(\mathbf{x}, t) \hat{T}_{0\nu}(\mathbf{x}, t) - \sum_a \alpha_a(\mathbf{x}, t) \hat{J}^0_a(\mathbf{x}, t) \right]$$

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$$\hat{\varrho}(t) = \frac{1}{\mathscr{Q}} \, exp\left[ \hat{\mathscr{A}} - \hat{\mathscr{B}} \right] \longrightarrow \langle \hat{\mathscr{O}}(t,\mathbf{x}) \rangle = Tr\left( \hat{\varrho}(t) \hat{\mathscr{O}}(t,\mathbf{x}) \right)$$

$$\hat{\mathscr{B}} = \int d^3\mathbf{x} \int_{-\infty}^{t} dt_1 e^{\varepsilon(t_1 - t)}\, \hat{\mathscr{C}}(\mathbf{x},t_1)$$

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- The master formula for the whole of linear response theory comes from the first-order time-dependent perturbation theory in Quantum Mechanics,
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  - Adding a time-dependent $\delta\hat{H}(t)$ leads to a change: $\hat{\mathcal{O}}(t, \mathrm{x}) + \delta\hat{\mathcal{O}}(t, \mathrm{x})$

$$\delta\langle\hat{\mathcal{O}}(t, \mathbf{x})\rangle = -i\int_{-\infty}^{t} dt^{'}\langle[\hat{\mathcal{O}}(t, \mathbf{x}), \delta\hat{H}(t^{'}, \mathbf{x})]\rangle$$

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Kubo-Mori-Bogoliubov (KMB) inner product

$$\delta\left\langle\hat{\mathcal{O}}^{\mu_1\mu_2\cdots\mu_n}(\mathbf{x}, t)\right\rangle = \int d^3\mathbf{x}_1\int_{-\infty}^{t} dt_1\, e^{\varepsilon(t_1 - t)}\left(\hat{\mathcal{O}}^{\mu_1\mu_2\cdots\mu_n}(\mathbf{x}, t), \hat{\mathscr{C}}\left(\mathbf{x}_1, t_1\right)\right)$$

- Casting $\hat{\mathscr{C}}(t, x)$ / $\delta \hat{H}(t)$ in terms of Linear source/force term:

$$\delta \hat{H}(t) = -\int d^3\mathbf{x}\, \hat{\mathscr{O}}(t, \mathbf{x})\, \hat{\mathscr{F}}(t, \mathbf{x})$$

$$\delta\langle \hat{\mathscr{O}}(t, \mathbf{x})\rangle = -i \int_{-\infty}^{t} dt'\, d^3\mathbf{x}' \langle [\hat{\mathscr{O}}(t, \mathbf{x}), \hat{\mathscr{O}}(t', \mathbf{x})]\rangle \hat{\mathscr{F}}(t', \mathbf{x})$$

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↓

$$\mathscr{R}(t - t', \mathbf{x} - \mathbf{x}')$$

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→ $$\hat{T}^{\mu\nu} = \hat{\epsilon} u^\mu u^\nu - \hat{P} \Delta^{\mu\nu} + \hat{\mathscr{Q}}^\mu u^\nu + \hat{\mathscr{Q}}^\nu u^\mu + \hat{\mathscr{T}}^{\mu\nu}$$

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$$\langle \hat{n}_a(t, \mathbf{x}) \rangle = n_a(t, \mathbf{x})$$

- Thermodynamical + fluid dynamical relations needs to use:
  - With a proper definition of the Temperature and chemical potential

$$S(t) = \int d^3\mathrm{x}\, s(t, \mathrm{x}) = -\, ln(\hat{\varrho}(t))$$

  - Extensivity + First law of thermodynamics

$$T(t, \mathrm{x})s(t, \mathrm{x}) = \epsilon(t, \mathrm{x}) + P(t, \mathrm{x}) - \sum_a \mu_a(t, \mathrm{x})\; n_a(t, \mathrm{x})$$

$$T(t, \mathrm{x})Ds(t, \mathrm{x}) = D\epsilon(t, \mathrm{x}) - \sum_a \mu_a(t, \mathrm{x})\; Dn_a(t, \mathrm{x})$$ Where, $D = u^\mu \partial_\mu$

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$$TD\epsilon = -\left(\epsilon + P\right)\,\theta\;, \quad Du_\alpha = \frac{1}{\epsilon + P}\nabla_\alpha P\;, \quad Dn_a = -\, n_a\;\theta$$

Where, $\nabla_\mu = \Delta_\mu^\nu \partial_\nu\;, \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$

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- Substituting these equations
  - One can write $\hat{\mathscr{C}}(t, \mathrm{x})$ at first order in gradient

$$\hat{\mathscr{C}} = -\beta\theta\hat{P}^* + \beta\hat{\mathscr{T}}^{\mu\nu}\sigma_{\mu\nu} - \sum_a \hat{\mathscr{J}}^{\mu}_a \nabla_\mu \alpha_a$$

$$\hat{P}^* = \hat{P} - \gamma\hat{\epsilon} - \sum_a \delta_a \hat{n}_a$$

$$\sigma_{\mu\nu} \equiv \frac{1}{2}\left[\Delta^{\alpha}_{\mu}\Delta^{\beta}_{\nu} + \Delta^{\alpha}_{\nu}\Delta^{\beta}_{\mu} - (2/3)\Delta^{\alpha\beta}\Delta_{\mu\nu}\right]\nabla_\alpha u_\beta$$

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A.Harutyunyan, A.Sedrakian, D. Rischke, Annals Phys. 438(2022) 168755

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$$\left\langle \hat{\mathscr{J}}^{\mu}_{a}(\mathbf{x}, t) \right\rangle = -\sum_b \int d^4x_1 \left( \hat{\mathscr{J}}^{\mu}_{a}(\mathbf{x}, t), \hat{\mathscr{J}}^{\nu}_{b}\left(\mathbf{x}_1, t_1\right) \right) \nabla_{\nu}\alpha_b$$

- Dissipative correction to hydro variables at first order :

$$\left\langle \hat{\mathcal{J}}_a^{\mu}(\mathbf{x},t) \right\rangle = \int d^4x_1 \left( \hat{\mathcal{J}}_a^{\mu}(\mathbf{x},t), \hat{\mathcal{C}}\left(\mathbf{x}_1,t_1\right) \right)$$

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$$\downarrow$$

$$\mathcal{K}_{ab}^{\mu\nu}$$

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$$\downarrow \quad u^\mu \hat{\mathcal{Q}}_\mu = u^\mu \hat{\mathcal{N}}_{a\,\mu} = 0$$

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$$\Big\downarrow \quad u^{\mu}\hat{\mathcal{Q}}_{\mu} = u^{\mu}\hat{\mathcal{N}}_{a\,\mu} = 0$$

$$u_{\mu}\mathcal{K}^{\mu\nu}_{ab} = 0$$

- The KMB inner product must satisfy the Onsager reciprocal relations.

$$\mathcal{K}^{\mu\nu}_{ab} = \mathcal{K}^{\nu\mu}_{ba}$$

- These two properties helps to decompose $\mathscr{K}^{\mu\nu}_{ab}$ :

$$u_\mu \mathscr{K}^{\mu\nu}_{ab} = 0 \longrightarrow \mathscr{K}^{\mu\nu}_{ab} = \Delta^{\mu\nu}\kappa_{ab}$$

$$\mathscr{K}^{\mu\nu}_{ab} = \mathscr{K}^{\nu\mu}_{ba} \longrightarrow \kappa_{ab} = \kappa_{ba}$$

- Which leads to the expression for diffusion matrix as :

$$\kappa_{ab} = \frac{1}{3}\Delta_{\mu\nu}\mathscr{K}^{\mu\nu}_{ab} = -\frac{1}{3}\int d^4\mathbf{x}_1 \left( \hat{\mathscr{J}}^{\mu}_{a}(\mathbf{x},t), \hat{\mathscr{J}}_{b\,\mu}\left(\mathbf{x}_1, t_1\right)\right)$$

- Diffusion in particle/charge basis :

L.Gavassino,M.Shokri,P.R.D 108 (2023)

  - Each particle ('a') carries multiple charges ('A'):

$$n_A = \sum_a q_{aA}\, n_a \quad \text{such that} \quad \sum_A \mu_A\, n_A = \sum_a \mu_a\, n_a$$

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  - Accordingly, the diffusion current in the conserved charge basis :

$$\hat{\mathcal{J}}^\mu_A = \sum_a q_{aA}\, \hat{\mathcal{J}}^\mu_a = \hat{\mathcal{N}}^\mu_A - \frac{n_A}{h}\hat{Q}^\mu$$

$$\kappa_{AB} = -\frac{1}{3}\int d^4\mathbf{x}_1 \left( \hat{\mathcal{J}}^\mu_A(\mathbf{x}, t), \hat{\mathcal{J}}_{B\,\mu}\left(\mathbf{x}_1, t_1\right)\right) = \sum_{a,b} q_{aA}\, q_{bB}\, \kappa_{ab}$$

- The computation of the dissipative diffusion coefficients boils down to the evaluation of the KMB product :

  X.Huang, A.Sedrakian, D.Rischke, Ann. Phys. (NY) 326, 3075(2011)

  - KMB for two Hermitian operator can be represented as :

$$\left(\hat{\mathcal{O}}_a(\mathbf{x},t),\hat{\mathcal{O}}_b\left(\mathbf{x}_1,t_1\right)\right)=-\frac{1}{\beta}\int_{-\infty}^{t_1}dt'G^R_{\hat{\mathcal{O}}_a\hat{\mathcal{O}}_b}\left(\mathbf{x}-\mathbf{x}_1,t-t'\right)$$

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Where,

$$G^R_{\hat{\mathcal{O}}_a\hat{\mathcal{O}}_b}\left(\mathbf{x}-\mathbf{x}_1, t-t'\right) = -i\theta(t-t')\left\langle\left[\hat{\mathcal{O}}_a(\mathbf{x}, t), \hat{\mathcal{O}}_b\left(\mathbf{x}_1, t'\right)\right]\right\rangle_l$$

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  - Relation between response function and retarded Green's function :

$$I[\hat{\mathcal{O}}_a\hat{\mathcal{O}}_b]=\lim_{\epsilon\to 0}\int d^3\mathbf{x}_1\int_{-\infty}^{t}dt'\,e^{-\epsilon(t-t')}\left(\hat{\mathcal{O}}_a(\mathbf{x},t),\hat{\mathcal{O}}_b\left(\mathbf{x}_1,t'\right)\right)$$

$$=-T\lim_{\omega\to 0}\lim_{k\to 0}\partial_\omega Im\left(G^R_{\hat{\mathcal{O}}_a\hat{\mathcal{O}}_b}(k,\omega)\right)$$

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  - For the diffusion current - current response function :

$$\kappa_{ab} = \frac{T}{3}\frac{\partial}{\partial\omega}\mathrm{Im}\left(G^R_{\hat{\mathcal{J}}^\mu_a, \hat{\mathcal{J}}_{\mu b}}\right)(\mathbf{0}, \omega)\Big|_{\omega\to 0} .$$

$$\kappa_{AB} = \frac{T}{3}\frac{\partial}{\partial\omega}\mathrm{Im}\left(G^R_{\hat{\mathcal{J}}^\mu_A, \hat{\mathcal{J}}_{\mu B}}\right)(\mathbf{0}, \omega)\Big|_{\omega\to 0} = \sum_{ab} q_{aA} q_{bB}\, \kappa_{ab}$$

- Some Useful connection between Green's Functions :

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- Spectral density/ function :

$$\rho_{\hat{\mathcal{O}}_a\hat{\mathcal{O}}_b}(\omega, \mathbf{k}) = \frac{1}{2}\int d^4x\, e^{iK\cdot X}\left\langle\left[\hat{\mathcal{O}}_a(t, \mathbf{x}), \hat{\mathcal{O}}_b^\dagger(0,\mathbf{0})\right]\right\rangle$$

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$$G^R_{\hat{\mathcal{O}}_a\hat{\mathcal{O}}_b}(\omega,\mathbf{k}) = -\,i\int_0^\infty dt\int d^3\mathbf{x}\, e^{i\omega t - i\mathbf{k}\cdot\mathbf{x}}\left\langle\theta(t)\left[\hat{\mathcal{O}}_a(\mathbf{x},t),\hat{\mathcal{O}}_b^\dagger\,(0,\mathbf{0})\right]\right\rangle_l$$

$$= \int_{-\infty}^{\infty}\frac{dp^0}{\pi}\frac{\rho_{\hat{\mathcal{O}}_a\hat{\mathcal{O}}_b}(P)}{p^0-\omega-i0^+}$$

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Sokhotski-Plemeij theorem

$$\mathrm{Im}\left(G^R_{\hat{\mathcal{O}}_a\hat{\mathcal{O}}_b}(\omega, \mathbf{k})\right) = \rho_{\hat{\mathcal{O}}_a\hat{\mathcal{O}}_b}(\omega, \mathbf{k}) \longleftarrow \frac{1}{\Delta \pm i0^+} = P\left(\frac{1}{\Delta}\right) \mp i\pi\delta(\Delta)$$

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$$I\left[\hat{\mathcal{O}}_a\hat{\mathcal{O}}_b\right] = -T\lim_{\omega\to 0}\lim_{k\to 0}\partial_\omega Im\left(G^R_{\hat{\mathcal{O}}_a\hat{\mathcal{O}}_b}(k,\omega)\right)$$

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$$\kappa_{AB} = \sum_{ab} q_{aA}q_{bB}\,\kappa_{ab}$$

- A toy model calculation with two interacting charged scalar fields :
  - Lagrangian:

$$\mathscr{L}(x) = \partial\phi^\dagger\partial\phi + \partial\xi^\dagger\partial\xi - V(\phi,\xi)$$

Where, $V(\phi,\xi) = m_\phi^2\,\phi^\dagger\phi + m_\xi^2\,\xi^\dagger\xi + \frac{g}{2}\phi^\dagger\phi\,\xi^\dagger\xi + \frac{\lambda_1(\phi^\dagger\phi)^2}{4} + \frac{\lambda_2(\xi^\dagger\xi)^2}{4}$

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$$\begin{pmatrix}\mu_\phi \\ \mu_\xi\end{pmatrix} = \begin{pmatrix} q_{\phi 1} & q_{\phi 2} \\ q_{\xi 1} & q_{\xi 2}\end{pmatrix}\begin{pmatrix}\mu_1 \\ \mu_2\end{pmatrix}$$

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$$Z(T,\mu_\phi,\mu_\xi) = C\int \mathscr{D}\phi\mathscr{D}\xi\, e^{-S_E} = Z(T,\mu_1,\mu_2)$$

- Finite temperature and chemical potential :
  - Fields in Euclidean space :

$$\phi_a(\tau, \mathbf{x}) = \sum\!\!\!\!\!\!\int_P e^{i(p_n + i\mu_a)\tau - i\mathbf{p}\cdot\mathbf{x}} \text{ and } \phi_a^\dagger(\tau, \mathbf{x}) = \sum\!\!\!\!\!\!\int_P e^{-i(p_n + i\mu_a)\tau - i\mathbf{p}\cdot\mathbf{x}}$$

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  - The two-point Eucledian correlation function can be written as :

$$G^E_{a,b}(X) = \left\langle \phi_a(X)\, \phi_b^\dagger(0) \right\rangle_E$$

$$= \delta_{ab} \sum\!\!\!\!\!\!\int_P \frac{e^{i(p_n + i\mu_a)\tau - i\mathbf{p}\cdot\mathbf{x}}}{(p_n + i\mu_a)^2 + \mathbf{p}^2 + m_a^2}$$

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For free propagator,

$$\rho_a^f(k_0, \mathbf{k}) = \frac{k_0}{|k_0|} \pi \delta(k_0^2 - E_{a\,\mathbf{k}}^2)$$

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S.Jeon, Phys.Rev. D52 (1995) 3591-3642

With interaction, RSF :

$$\rho_a(k_0, \mathbf{k}) = \frac{\mathrm{Im}(\Sigma_a(k_0, \mathbf{k}))}{\left(k_0^2 - \mathbf{k}^2 - m_a^2 - \mathrm{Re}(\Sigma_a(k_0, \mathbf{k}))\right)^2 + |\,\mathrm{Im}(\Sigma_a(k_0, \mathbf{k}))\,|^2}$$

- Finite temperature and chemical potential :
  - Fields in Euclidean space :

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$$\rho_a(k_0, \mathbf{k}) \equiv \frac{1}{2i} \left[ G^E_{a,a}(i(k_n - i\mu) \to k^0 + i0^+, \mathbf{k}) - G^E_{a,a}(i(k_n - i\mu) \to k^0 - i0^+, \mathbf{k}) \right]$$

- Field theory with finite temperature and chemical potential :

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- The two-point Eucledian correlation function can be written as :

$$G^E_{\hat{\mathcal{O}}_a,\hat{\mathcal{O}}_b}(X) = \left\langle \hat{\mathcal{O}}_a(X)\,\hat{\mathcal{O}}_b(0) \right\rangle_E$$

$$= \delta_{ab} \int_{-\infty}^{\infty} \frac{dk^0}{\pi} \frac{\rho_a(k^0, \mathbf{k})}{k^0 - ik_n}$$

$$\rho_{\hat{\mathcal{O}}_a,\hat{\mathcal{O}}_b}(k_0, \mathbf{k}) \equiv \frac{1}{2i}\left[ G^E_{\hat{\mathcal{O}}_a,\hat{\mathcal{O}}_b}(ik_n \to k^0 + i0^+, \mathbf{k}) - G^E_{\hat{\mathcal{O}}_a,\hat{\mathcal{O}}_b}(ik_n \to k^0 - i0^+, \mathbf{k}) \right]$$

- One needs to perform Wick rotation $(t \to -i\tau)$ :

$$\hat{Q}^E_i = \hat{T}_{\tau i} = i \sum_a \left( \partial_\tau \phi_a^\dagger \partial_i \phi_a + \partial_i \phi_a^\dagger \partial_\tau \phi_a \right) , \quad \hat{\mathcal{N}}^E{}_{a\,i} = -\, i \left( \partial_i \phi_a^\dagger \phi_a - \phi_a^\dagger \partial_i \phi_a \right)$$

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- The two-point Euclidean correlation function can be written as :

$$G^E_{\hat{Q}_i, \hat{Q}_j}(X) = \left\langle \hat{Q}_i(X)\, \hat{Q}_j(0) \right\rangle_E \; , \qquad G^E_{\hat{\mathcal{N}}_i, \hat{\mathcal{N}}_j}(X) = \left\langle \hat{\mathcal{N}}_i(X)\, \hat{\mathcal{N}}_j(0) \right\rangle_E$$

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$$G^E_{\hat{\mathcal{J}}_a \hat{\mathcal{J}}_b}(X) = \Delta^{ij} \left\langle \hat{\mathcal{J}}_{a\,i}(X) \hat{\mathcal{J}}_{b\,j}(0) \right\rangle_l = -\left[ \left\langle \hat{\mathcal{N}}_{a\,i}(X) \hat{\mathcal{N}}_{b\,i}(0) \right\rangle_l - \frac{n_b}{h} \left\langle \hat{\mathcal{N}}_{a\,i}(X) \hat{\mathcal{Q}}_i(0) \right\rangle_l - \frac{n_a}{h} \left\langle \hat{\mathcal{Q}}_i(X) \hat{\mathcal{N}}_{b\,j}(0) \right\rangle_l + \frac{n_a n_b}{h^2} \left\langle \hat{\mathcal{Q}}_i(X) \hat{\mathcal{Q}}_j(0) \right\rangle_l \right]$$

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$$\rho_{\hat{\mathcal{J}}_a \hat{\mathcal{J}}_b}(k_0, \mathbf{k}) \equiv \frac{1}{2i} \left[ G^E_{\hat{\mathcal{J}}_a \hat{\mathcal{J}}_b}(ik_n \to k^0 + i0^+, \mathbf{k}) - G^E_{\hat{\mathcal{J}}_a \hat{\mathcal{J}}_b}(ik_n \to k^0 - i0^+, \mathbf{k}) \right]$$

- Spectral function of diffusion currents and transport coefficients:

$$\rho_{\mathcal{J}_a\mathcal{J}_b}(\omega, \mathbf{l}) = \delta_{ab}\rho_{\mathcal{N}_a\mathcal{N}_a}(\omega, \mathbf{l}) - \frac{n_b}{h}\rho_{\mathcal{N}_a\mathcal{Q}}(\omega, \mathbf{l}) - \frac{n_a}{h}\rho_{\mathcal{N}_b\mathcal{Q}}(\omega, \mathbf{l}) + \frac{n_a n_b}{h^2}\rho_{\mathcal{Q}\mathcal{Q}}(\omega, \mathbf{l}),$$

$$\rho_{\hat{\mathcal{J}}_a\hat{\mathcal{J}}_b}(\omega, \mathbf{0}) = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\int \frac{d\omega'}{\pi}\sum_c \mathbf{p}^2\Bigg(\Bigg[4\delta_{ab}\delta_{ca} - \frac{2}{h}\big(n_a\delta_{cb} + n_b\delta_{ca}\big)(2\omega' + \omega) + \frac{n_a n_b}{h^2}(2\omega' + \omega)^2\Bigg]$$
$$\times\, \rho_c(\omega', \mathbf{p})\, \rho_c(\omega' + \omega, \mathbf{p})\Big[f_c(\omega' + \omega) - f_c(\omega')\Big]\Bigg)$$

Where,

$$f_a(\omega) = \frac{1}{e^{\beta(\omega - \mu_a)} - 1}$$

- Spectral function of diffusion currents and transport coefficients:

$$\rho_{\mathcal{J}_a\mathcal{J}_b}(\omega,\mathbf{l}) = \delta_{ab}\rho_{\mathcal{N}_a\mathcal{N}_a}(\omega,\mathbf{l}) - \frac{n_b}{h}\rho_{\mathcal{N}_a\mathcal{Q}}(\omega,\mathbf{l}) - \frac{n_a}{h}\rho_{\mathcal{N}_b\mathcal{Q}}(\omega,\mathbf{l}) + \frac{n_a n_b}{h^2}\rho_{\mathcal{Q}\mathcal{Q}}(\omega,\mathbf{l}),$$

$$\rho_{\hat{\mathcal{J}}_a\hat{\mathcal{J}}_b}(\omega,\mathbf{0}) = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\int\frac{d\omega'}{\pi}\sum_c \mathbf{p}^2\Bigg(\left[4\delta_{ab}\delta_{ca} - \frac{2}{h}\left(n_a\delta_{cb} + n_b\delta_{ca}\right)(2\omega'+\omega) + \frac{n_a n_b}{h^2}(2\omega'+\omega)^2\right]$$
$$\times\,\rho_c(\omega',\mathbf{p})\,\rho_c(\omega'+\omega,\mathbf{p})\left[f_c(\omega'+\omega) - f_c(\omega')\right]\Bigg)$$

Where,

$$f_a(\omega) = \frac{1}{e^{\beta(\omega-\mu_a)} - 1}$$

$$\kappa_{ab} = \frac{T}{3}\lim_{\omega\to 0}\frac{\rho_{\hat{\mathcal{J}}_a\hat{\mathcal{J}}_b}(\omega,\mathbf{0})}{\omega}$$

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$$\kappa_{ab} = \frac{T}{3}\lim_{\omega\to 0}\frac{\rho_{\hat{\mathcal{J}}_a\hat{\mathcal{J}}_b}(\omega,\mathbf{0})}{\omega} = \frac{4}{3}\int\frac{d^3\mathbf{p}}{(2\pi)^3}\int\frac{d\omega'}{\pi}\sum_c \mathbf{p}^2\Bigg(\left[\delta_{ab}\delta_{ca} - \frac{\omega'}{h}\big(n_a\delta_{cb} + n_b\delta_{ca}\big) + n_a n_b\left(\frac{\omega'}{h}\right)^2\right] \times \rho_c(\omega',\mathbf{p})^2 f_c(\omega')\Big[1 + f_c(\omega')\Big]\Bigg)$$

- Quasiparticle approximation with finite thermal width :

$$\rho_a(k_0, \mathbf{k}) = \frac{\mathrm{Im}(\Sigma_a(k_0, \mathbf{k}))}{\left(k_0^2 - \mathbf{k}^2 - m_a^2 - \mathrm{Re}(\Sigma_a(k_0, \mathbf{k}))\right)^2 + |\,\mathrm{Im}(\Sigma_a(k_0, \mathbf{k}))\,|^2}$$

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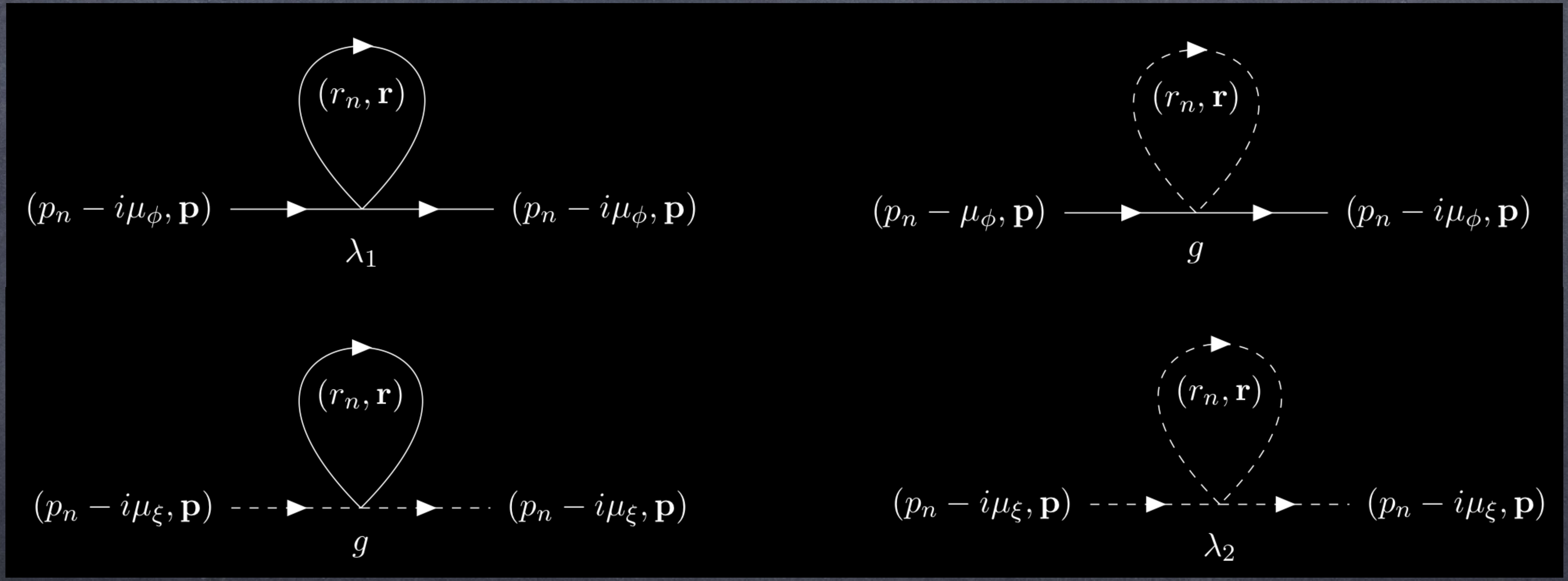
$$\mathbf{Re}(\Sigma_a(k_0,\mathbf{k})) \sim \lambda$$



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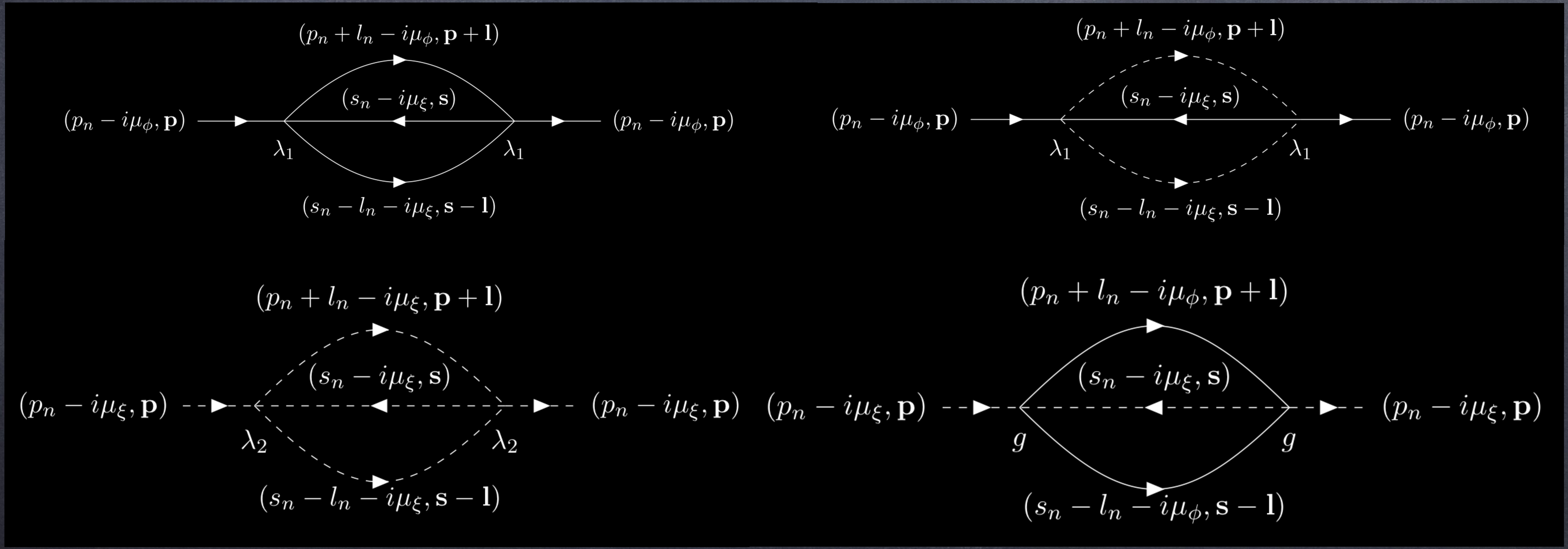

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$$\rho_a(\omega,\mathbf{p}) \approx \frac{1}{2i}\left[\frac{1}{\left(\omega-i\Gamma_{\mathbf{p}a}\right)^2-E_{\mathbf{p}a}^2}-\frac{1}{\left(\omega+i\Gamma_{\mathbf{p}a}\right)^2-E_{\mathbf{p}a}^2}\right]\left[1+\mathcal{O}\left(\lambda^2\right)\right]$$

Where, $$\Gamma_{a\,\mathbf{k}} = \frac{\mathrm{Im}(\Sigma_a(k_0,\mathbf{k}))}{2E_{a\,\mathbf{k}}}$$

- After taking pinching pole approximation, dominating contribution $\sim \dfrac{1}{\Gamma_{\mathbf{p}a}}$ :

$$\kappa_{\phi\phi} = \frac{1}{3}\int \frac{d^3\mathbf{p}}{(2\pi)^3}\mathbf{p}^2\Bigg(\frac{1}{2\Gamma_{\phi\,\mathbf{p}}E^2_{\phi\,\mathbf{p}}}\Bigg[\Big(1-\frac{n_\phi E_{\phi\,\mathbf{p}}}{h}\Big)^2 f_\phi(E_{\phi\,\mathbf{p}})\Big(1+f_\phi(E_{\phi\,\mathbf{p}})\Big)+\Big(1+\frac{n_\phi E_{\phi\,\mathbf{p}}}{h}\Big)^2 \times \bar{f}_\phi(E_{\phi\,\mathbf{p}})\Big(1+\bar{f}_\phi(E_{\phi\,\mathbf{p}})\Big)\Bigg]+\frac{1}{2\Gamma_{\xi\,\mathbf{p}}E^2_{\xi\,\mathbf{p}}}\Big(\frac{n_\xi E_{\xi\,\mathbf{p}}}{h}\Big)^2 \times\Bigg[f_\xi(E_{\xi\,\mathbf{p}})\Big(1+f_\xi(E_{\xi\,\mathbf{p}})\Big)+\bar{f}_\xi(E_{\xi\,\mathbf{p}})\Big(1+\bar{f}_\xi(E_{\xi\,\mathbf{p}})\Big)\Bigg]\Bigg)$$

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$$\left.+\frac{n_\xi E_{\phi\,\mathbf{p}}}{h}\left(\frac{n_\phi E_{\phi\,\mathbf{p}}}{h}+1\right)\bar{f}_\phi(E_{\phi\,\mathbf{p}})\Big(1+\bar{f}_\phi(E_{\phi\,\mathbf{p}})\Big)\right]+\frac{1}{2\Gamma_{\xi\,\mathbf{p}}E^2_{\xi\,\mathbf{p}}}\left[\frac{n_\phi E_{\xi\,\mathbf{p}}}{h}\left(\frac{n_\xi E_{\xi\,\mathbf{p}}}{h}-1\right)\right.$$

$$\left.\left.\times f_\xi(E_{\xi\,\mathbf{p}})\Big(1+f_\xi(E_{\xi\,\mathbf{p}})\Big)+\frac{n_\phi E_{\xi\,\mathbf{p}}}{h}\left(\frac{n_\phi E_{\xi\,\mathbf{p}}}{h}+1\right)\bar{f}_\xi(E_{\xi\,\mathbf{p}})\Big(1+\bar{f}_\xi(E_{\xi\,\mathbf{p}})\Big)\right]\right)$$

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$$\left. + \frac{1}{2\Gamma_{a\mathbf{p}} E_{a\mathbf{p}}^2} \left[ \left( q_{aA} + \frac{n_A E_{a\mathbf{p}}}{h} \right) \left( q_{aB} + \frac{n_B E_{a\mathbf{p}}}{h} \right) \bar{f}_a(E_{a\mathbf{p}}) \Big( 1 + \bar{f}_a(E_{a\mathbf{p}}) \Big) \right] \right)$$

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- Relaxation time :

$$\tau_a = \frac{1}{2\Gamma_{a\mathbf{p}}}$$

A.Das, H.Mishra, R.Mahapatra, Phys.Rev.D 106 (2022) 1, 014013

# Future Works

- Such expressions are rather general and can be used for estimating the diffusion coefficients using nonperturbative methods like lattice QCD or using any effective models of strong interactions like Nambu--Jona-Lasinio (NJL) model, quasi particle models , or sigma models coupled to quarks.

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- In the context of neutron stars, it will be interesting to investigate the effects of diffusion on the oscillation of neutron stars as a result of an internal instability or an external perturbation.

T.K.jha,H.Mishra,V.Sreekanth, Phys.Rev.C82:025803,2010; K.Y. Kraav, M.E. Gusakov and E.M. Kantor, arXiv:2105.08121 [astro-ph.HE]; L. Gavassino, Class. Quantum Grav. 40 165008 (2023).

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A.K. Panda, P. Bagchi, H. Mishra and V. Roy,
arXiv:2404.08431 [nucl-th]

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Thank you