Bound states in gauge theory I

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Standard Model @ LHC
& Paul Hoyer Fest
literature


outline

• motivation & introduction
• bound state equation
  • Poincaré invariance
  • solutions
  • spectrum
• conclusions

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motivation

• understanding of
  • bound state dynamics
  • bound state spectra
  • qcd & field theory

• wanted:
  hadronic basis for scattering amplitudes
observations

• QCD bound states
  ∞ # of constituents vs. few valence quarks
observations

• QCD bound states
  ∞ # of constituents vs. few valence quarks

• bound state spectra
  e.g. charmonium and positronium qualitatively similar
Similarity of atomic and hadronic spectra

\[ V(r) = c r^{-\alpha} + \text{DD} \]

from one of Paul’s talks
observations

- QCD bound states
  \( \propto \) # of constituents vs. few valence quarks

- bound state spectra
  e.g. charmonium and positronium qualitatively similar

- \( \alpha_s \) might freeze in already for moderate virtualities
Figure 3: 95% CL contours for jet shape means (dashed) and some distributions (solid). The curves for the $T$, $C$ and old $B_T$ and $B_W$ distributions are taken from [8]. The curves for the means are to be taken as purely indicative since we have not accounted for the correlations between systematic errors (which, where available, are added in quadrature to the statistical errors). Additionally for some observables we may not have found all the available data.

As a first step it would be necessary to carry out a resummed PT calculation for the DIS broadening. This has so far not been done.

The situation for the mean broadening measured with respect to the thrust axis is fairly simple though, since (modulo factors of two associated with the definition of the broadening in DIS [23]) it is equivalent to a single hemisphere in $e^+e^-$:

$$\langle B \rangle_{\text{DIS, thrust}} - \langle B \rangle_{\text{(PT) DIS, thrust}} = P(\pi \sqrt{\alpha_{CMW}(\bar{Q})} + \frac{3}{4} - \beta_0 C + \eta_0 + O(\sqrt{\alpha_s}))$$

(5.3)

For the mean broadening defined with respect to the photon ($z$) axis the situation is more complicated because of the dependence on perturbative initial-state radiation. To a first approximation, at moderate $x$, one can view the DIS event as a rotated $e^+e^-$ event where the broadening is measured in the right hemisphere with respect to the axis of the quark in the left hemisphere: i.e. the relevant transverse momentum for determining the rapidity available to the NP correction is $p = |\vec{p}_1 + \vec{p}_2|$. Since this is very similar to $\max(p_1, p_2)$ we have a situation like that for the wide-jet broadening, and the leading power correction is suppressed by factor $\sqrt{2}$ compared to (5.3):

$$\langle B \rangle_{\text{DIS, z}} - \langle B \rangle_{\text{(PT) DIS, z}} = P(\pi \sqrt{2} \sqrt{\alpha_{CMW}(\bar{Q})} + \frac{3}{4} - \beta_0 C + \eta_0 + O(1))$$

(5.4)

Even though we have chosen to include some subleading terms of $O(1)$, it is likely that there are other terms of $O(1)$, arising through the $x$ dependence of the problem.
The coupling is

\[ \alpha_s(M_Z) \approx 0.1 \]

\[ \alpha_0 \approx 0.5 \]


Even though we have chosen to include some subleading terms, there are other terms of importance. For the mean broadening defined with respect to the photon distribution, this is particularly complicated because of the dependence on perturbative initial data. This is discussed further in Sec. VI.

In DIS, it is equivalent to a single hemisphere in the left hemisphere: i.e. the relevant transverse momentum transfer (continuous band). At fixed first approximation to QCD. Color confinement is introduced by the leading power correction is exponential behavior, a fact which allows us to extend the functional dependence of the coupling to large distances. The harmonic oscillator potential, but effectively QCD couplings extracted from different observables as well as lattice results. Details on the hadronic model obtained from the dilaton-modified AdS space are compared to previous calculations.

The results show no sign of a phase transition, cusp, or other nonanalytical behavior, a fact which allows us to extend the functional dependence of the coupling to large distances. The harmonic oscillator potential, but effectively QCD couplings extracted from different observables as well as lattice results. Details on the hadronic model obtained from the dilaton-modified AdS space are compared to previous calculations.

The nonperturbative confining effective theory. The nonperturbative confining effective QCD couplings extracted from different experiments. Figure 1 also displays other couplings from di-effecive coupling from LF holographic mapping for (5.3): $\alpha_s(M_Z) \approx 0.1$ $\alpha_0 \approx 0.5$


<table>
<thead>
<tr>
<th>$\alpha_s(Q)/\pi$</th>
<th>$\alpha_0$</th>
<th>$\alpha_s(M_Z)$</th>
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</thead>
<tbody>
<tr>
<td>Modified AdS</td>
<td>AdS</td>
<td>$\alpha_g/\pi$ (pQCD)</td>
</tr>
<tr>
<td>$\alpha_g/\pi$</td>
<td>$\alpha_g/\pi$ world data</td>
<td>$\alpha_{F3}/\pi$</td>
</tr>
<tr>
<td>$\alpha_t/\pi$ OPAL</td>
<td>$\alpha_g/\pi$ JLab CLAS</td>
<td>$\alpha_g/\pi$ Hall A/CLAS</td>
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observations

• QCD bound states
  ∞ # of constituents vs. few valence quarks

• bound state spectra
  e.g. charmonium and positronium qualitatively similar

• $\alpha_s$ might freeze in already for moderate virtualities

• virtual gluons more costly than anticipated
  $\Leftarrow$ large-angle scattering, Okubo-Zweig-Iizuka rule, ...
large-ear hadron
large-ar

hadron

also
Baller et al.
PRL 60 (1988) 1118

scaling (pp):
Gunion, Brodsky &
Blankenbecler
PRD 6 (1972) 2652

\[ \frac{d\sigma}{dt} \propto \frac{1}{s^2 u^4 t^4} \]

\[ \frac{d\sigma}{dt} \propto \frac{1}{t^8} \]

Landshoff
PRD 10 (1974) 1024

White et al.
PRD 49 (1994) 58
observations

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  \( \infty \# \) of constituents vs. few valence quarks

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  $\Leftarrow$ large-angle scattering, Okubo-Zweig-Iizuka rule, ...

  $\Rightarrow$ try valence fermions
  + low-order instantaneous interactions
not less!

\[ |e^-(x_1)e^+(x_2)\rangle \]

\[ \nabla \cdot A = 0 \]

\[ -\nabla^2 A^0(x) = e [\delta(x - x_1) - \delta(x - x_2)] \]

\[ A^0(x) = \frac{e}{4\pi} \left[ \frac{1}{|x - x_1|} - \frac{1}{|x - x_2|} \right] \]

vanishing field is no solution in presence of charges!

\[ \frac{1}{2} [eA^0(x_1) - eA^0(x_2)] = -\alpha/|x_1 - x_2| \]

⇒ in non-relativistic limit: hydrogen Schrödinger problem
The interaction potential is then specified for an interaction of the type

\[ - \nabla^2 A^0(x) = e \left[ \delta(x - x_1) - \delta(x - x_2) \right] \]

\[ A^0(x) = \frac{e}{4\pi} \left[ \frac{1}{|x - x_1|} - \frac{1}{|x - x_2|} \right] \]

\[ A^0_\Lambda(x) = \Lambda^2 \hat{\ell} \cdot x \]

homogeneous solution

\[ \hat{\ell} \parallel x_1 - x_2 \]

\[ V(x_1 - x_2) = \frac{1}{2} [eA^0_\Lambda(x_1) - eA^0_\Lambda(x_2)] = \frac{1}{2} e\Lambda^2 |x_1 - x_2| \]
\[ l + l d \ qed \]

- no transverse gauge bosons
- linear potential
- more tractable
The present paper we discuss cases where the wave functions of relativistic bound states, further complicating the transformation of atomic wave functions in the Coulomb gauge. We derive below the Poincaré algebra for QED starting from first principles, by identifying the linear potential as a fundamental parameter of the relativistic two-fermion system with a relativistic two-fermion interaction. Since there are no transverse photons the interaction is fully given from Gauss' law, which allows to express the equation of motion for the wave function describing a single particle with both positive and negative energy. The possibility to describe relativistic bound states, which have an infinite number of Fock space pairs, is a fundamental feature of QED in Coulomb gauge after integrating out the gauge bosons, in Sect. III we present the corresponding bound state wave function under boosts, and in Sect. V we analyze the behavior of the bound-state wave function depending on the distance between two (or more) particles, such as QED atoms.

The Dirac bound states mentioned above are not translation invariant since they are independent of the choice of wave function. Adding relativistic corrections will increase the number of Fock space pairs, which arise from the non-local fermionic action, which may be found in QED, for the system with a relativistic two-fermion interaction as a property of their bound state equation.

The QED action in 1+1 dimensions is then

\[
S = \int d^2 x \left[ -\frac{1}{2} (\partial_1 A^0) (\partial^1 A^0) + \sum_f \psi_f^\dagger(x) \gamma^0 (i\partial^0 - m_f - e\gamma^0 A^0) \psi_f(x) \right]
\]

\[
- \partial_1^2 A^0(x) = e \sum_f \psi_f^\dagger \psi_f(x)
\]

\[
A^0(x) = -\frac{e}{2} \sum_f \int dy^1 |x^1 - y^1| \psi_f^\dagger \psi_f(x^0, y^1)
\]

Leibbrandt,

*Introduction to noncovariant gauges*

Rev. Mod. Phys. 59 (1987) 1067
eliminate A

\[ S = \int d^2x \left[ -\frac{1}{2} (\partial_1 A^0) (\partial^1 A^0) + \sum_f \psi_f^\dagger(x) \gamma^0 (i\phi - m_f - e\gamma^0 A^0) \psi_f(x) \right] \]

\[ A^0(x) = -\frac{e}{2} \sum_f \int dy^1 |x^1 - y^1| \psi_f^\dagger \psi_f (x^0, y^1) \]

\[ S \equiv S_F + S_V = \sum_f \int d^2x \psi_f^\dagger(x) \gamma^0 (i\phi - m_f) \psi_f(x) + \]

\[ + \frac{e^2}{4} \sum_{f, f'} \int d^2x d^2y \delta(x^0 - y^0) \psi_f^\dagger \psi_f(x) |x^1 - y^1| \psi_{f'}^\dagger \psi_{f'}(y) \]
Poincaré invariance

\[
\begin{align*}
[P^0, P^1] &= 0 \\
[P^0, M^{01}] &= iP^1 \\
[P^1, M^{01}] &= iP^0
\end{align*}
\]
spatial translation

\[ \psi_f(x^0, x^1) \rightarrow \psi_f(x^0, x^1 - \epsilon(x^0) d\ell) \]

\[ \delta S = d\ell \int dx^0 \epsilon'(x^0) P^1 \]

\[ P^1 = -i \sum_f \int dx^1 \psi_f^\dagger(x) \partial_1 \psi_f(x) \]

momentum
spatial translation

\[ \psi_f(x^0, x^1) \rightarrow \psi_f(x^0, x^1 - \epsilon(x^0) d\ell) \]

total momentum conserved in potential interaction

\[ P^1 = -i \sum_f \int dx^1 \, \psi_f^\dagger(x) \partial_1 \psi_f(x) \]

momentum
temporal translation

\[ \psi(x^0, x^1) \rightarrow \psi(x^0 - \epsilon(x^0)dt, x^1) \]

\[ P_F^0 = \sum_f \int dx^1 \psi_f^\dagger(x)(-i\gamma^0\gamma^1\partial_1 + m_f\gamma^0)\psi_f(x), \]

\[ P_V^0 = -\frac{e^2}{4} \sum_{f,f'} \int dx^1 dy^1 \psi_f^\dagger\psi_f(x^0, x^1)|x^1 - y^1|\psi_{f'}^\dagger\psi_{f'}(x^0, y^1) \]

Hamiltonian
temporal translation

\[ \psi(x^0, x^1) \rightarrow \psi(x^0 - \epsilon(x^0)dt, x^1) \]

balance between potential and kinetic energy

\[ P^0_F = \sum_f \int dx^1 \psi_f(x) \frac{1}{2m} (\dot{x}^1)^2 \psi_f(x) \]

\[ P^0_V = -\frac{e^2}{4} \sum_{f,f'} \int dx^1 dy^1 \psi_f^\dagger \psi_f(x, y) (x^1 - y^1) \psi_{f'}^\dagger \psi_{f'}(x^0, y^1) \]

Hamiltonian
Combined with the standard boost transformation we have then
\[ A^0 \rightarrow A^0 + \epsilon(x^0) \gamma^0 \gamma^1 \partial_0 \psi \] \[ \psi(x^0, x^1) \rightarrow [1 + \frac{1}{2} \epsilon(x^0) \gamma^0 \gamma^1 \partial_0 \psi \] \[ x^0 \rightarrow x^0 + \epsilon(x^0) \gamma^0 \gamma^1 \partial_0 \psi \] \[ (A^0, A^1 = 0) \rightarrow (A^0, \epsilon(x^0) \gamma^0 \gamma^1 \partial_0 \psi) \] \[ \psi(x) \rightarrow \exp(-i \epsilon(x^0) \gamma^0 \gamma^1 \partial_0 \psi) \psi(x) \] \[ \partial_1 \theta(x) = eA^0(x) = -\frac{e^2}{2} \int d^2 y \delta(x^0 - y^0) |x^1 - y^1| \psi^\dagger \psi(y) \] \[ \psi(x^0, x^1) \rightarrow [1 + \frac{1}{2} \epsilon(x^0) \gamma^0 \gamma^1 \partial_0 \psi \] \[ x^0 - \epsilon(x^0) x^1 \partial_0 \psi, x^1 - \epsilon(x^0) x^0 \partial_0 \psi \]
boost

\[ x^0 \rightarrow x^0 + d\xi x^1 \quad x^1 \rightarrow x^1 + d\xi x^0 \]

**non-covariant gauge**

\[ \partial_1 \theta(x) = eA^0(x) = -\frac{1}{2} \int d^2 y \, \delta(x^0 - y^0) |x^1 - y^1| \psi^\dagger \psi(y) \]

\[ \psi(x^0, x^1) \rightarrow \left[ 1 + \frac{1}{2} \epsilon(x^0) \gamma^0 \gamma^1 d\xi - i \epsilon(x^0) \theta(x^0, x^1) d\xi \right] \psi(x^0 - \epsilon(x^0)x^1 d\xi, x^1 - \epsilon(x^0)x^0 d\xi) \]

**but:** \( A^1 = 0 \)

must restore gauge condition
Poincaré invariance

\[ M^{01}_F = x^0 P^1 + \sum_f \int dx^1 \psi^\dagger_f(x) \left[ x^1 (i \gamma^0 \gamma^1 \partial_1 - \gamma^0 m_f) + \frac{i}{2} \gamma^0 \gamma^1 \right] \psi_f(x) \]

\[ M^{01}_V = \frac{e^2}{8} \sum_{f,f'} \int dx^1 dy^1 \psi^\dagger_f \psi_f(x^0, x^1) (x^1 + y^1) |x^1 - y^1| \psi^\dagger_{f'} \psi_{f'}(x^0, y^1) \]

as densities

\[ P^0 = \psi \left( - \frac{1}{2} i \gamma^1 \partial_1 + m \right) \bar{\psi} - \frac{e^2}{4} \int dy^1 \psi^\dagger(x^0, x^1) |x^1 - y^1| \psi^\dagger \psi(x^0, y^1) \]

\[ P^1 = \psi \left( - \frac{1}{2} i \gamma^0 \partial_1 \right) \bar{\psi}, \]

\[ M^{01} = x^0 P^1 - x^1 P^0 \]

cross check ✓
\[ M_{F}^{01} = x^{0}P^{1} + \sum_{f} \int dx^{1} \psi_{f}^{\dagger}(x) \left[ x^{1}(i\gamma^{0}\gamma^{1}\partial_{1} - \gamma^{0}m_{f}) + \frac{i}{2}\gamma^{0}\gamma^{1} \right] \psi_{f}(x) \]

\[ M_{V}^{01} = \frac{e^2}{8} \sum_{f,f'} \int dx^{1}dy^{1} \psi_{f}^{\dagger}\psi_{f}(x^{0},x^{1})(x^{1} + y^{1})|x^{1} - y^{1}|\psi_{f}^{\dagger},\psi_{f'}(x^{0},y^{1}) \]

as densities

\[ P^{0} = \bar{\psi}(-\frac{1}{2}i\gamma^{1}\partial_{1} + m)\psi - \frac{e^2}{4} \int dy^{1} \psi^{\dagger}\psi(x^{0},x^{1})|x^{1} - y^{1}|\psi^{\dagger}\psi(x^{0},y^{1}) \]

\[ P^{1} = \bar{\psi}(-\frac{1}{2}i\gamma^{0}\partial_{1})\psi, \]

\[ M^{01} = x^{0}P^{1} - x^{1}P^{0} \]

cross check ✔
Poincaré invariance

\[
\begin{align*}
[P^0, P^1] &= 0 \\
[P^0, M^{01}] &= iP^1 \\
[P^1, M^{01}] &= iP^0
\end{align*}
\]

check using

\[
\left\{ \psi_{f\alpha}(x^0, x^1), \psi^\dagger_{f',\beta}(x^0, y^1) \right\} = \delta(x^1 - y^1)\delta_{ff'}\delta_{\alpha\beta}
\]
Poincaré invariance

\[ [P^0, P^1] = 0 \]

for QCD it closes only in the singlet sector

\[ [P^1, M^{01}] = iP^0 \]

check using

\[ \left\{ \psi_{f \alpha}(x^0, x^1), \psi_{f', \beta}^\dagger(x^0, y^1) \right\} = \delta(x^1 - y^1)\delta_{f f'}\delta_{\alpha \beta} \]
bound state equation

valence quarks only!

\[ |0\rangle_R = N^{-1} \prod_{p^1} d^\dagger(p^1) |0\rangle \]

retarded vacuum

\[ \psi(x) |0\rangle_R = 0 \]

low orders only! \Rightarrow no loops

\[ |E, k\rangle \equiv \int dx_1 dx_2 \exp \left[ \frac{1}{2} ik(x_1 + x_2) \right] \overline{\psi_1}(0, x_1)e^{i\varphi} \Phi(x_1 - x_2) \psi_2(0, x_2) |0\rangle_R \]

\[ \psi_1(x) |0\rangle_R = \psi_2^\dagger(x) |0\rangle_R = 0 \]
bound state equation

\[ P^0 |E, k\rangle = E |E, k\rangle \]

\[ P^0_F = \sum_f \int dx^1 \psi_f^\dagger(x)(-i\gamma^0\gamma^1\partial_1 + m_f\gamma^0)\psi_f(x), \]

\[ P^0_V = -\frac{e^2}{4} \sum_{f,f'} \int dx^1 dy^1 \psi_f^\dagger \psi_f(x^0, x^1)|x^1 - y^1|\psi_f^\dagger \psi_f'(x^0, y^1) \]

\[ |E, k\rangle \equiv \int dx_1 dx_2 \exp \left[ \frac{1}{2}ik(x_1 + x_2) \right]\bar{\psi}_1(0, x_1)e^{i\varphi}\Phi(x_1 - x_2)\psi_2(0, x_2)|0\rangle_R \]
\[ \gamma^0 = \sigma_3, \quad \gamma^1 = i\sigma_2, \quad \gamma^0\gamma^1 = \sigma_1 \]

**bound state equation**

\[ P^0 |E, k\rangle = E |E, k\rangle \]

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\]

\[ i\partial_x \{ \sigma_1, \Phi(x) \} - (\partial_x \varphi) \{ \sigma_1, \Phi(x) \} - \frac{1}{2} k [\sigma_1, \Phi(x)] + m_1 \sigma_3 \Phi(x) - m_2 \Phi(x) \sigma_3 = [E - V(x)] \Phi(x) \]
\[\gamma^0 = \sigma_3, \quad \gamma^1 = i\sigma_2, \quad \gamma^0\gamma^1 = \sigma_1\]

**bound state equation**

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\[P^0_V = -\frac{e^2}{4} \sum \int dx^1 dy^1 \psi_f^\dagger \psi_f(x^0, x^1)|x^1 - y^1|\psi_f^\dagger, \psi_f', (x^0, y^1)\]

\[|E, k\rangle \equiv \int dx_1 dx_2 \exp \left[\frac{1}{2}ik(x_1 + x_2)\right]\bar{\psi}_1(0, x_1)e^{i\varphi}\Phi(x_1 - x_2)\psi_2(0, x_2) |0\rangle_R\]

\[i\partial_x \{\sigma_1, \Phi(x)\} - (\partial_x \varphi) \{\sigma_1, \Phi(x)\} - \frac{1}{2}k[\sigma_1, \Phi(x)] + m_1\sigma_3\Phi(x) - m_2\Phi(x)\sigma_3 = [E - V(x)]\Phi(x)\]
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\[ P^0_V = -\frac{e^2}{4} \sum_{f,f'} \int dx^1 dy^1 \psi_f^\dagger \psi_f(x^0, x^1) |x^1 - y^1\rangle \langle \psi_f^\dagger, \psi_{f'}(x^0, y^1) |0\rangle_R \]

\[ |E, k\rangle \equiv \int dx_1 dx_2 \exp \left[ \frac{1}{2}ik(x_1 + x_2) \right] \bar{\psi}_1(0, x_1)e^{i\varphi}\Phi(x_1 - x_2)\psi_2(0, x_2) |0\rangle_R \]

\[ i\partial_x \{\sigma_1, \Phi(x)\} - (\partial_x \varphi) \{\sigma_1, \Phi(x)\} - \frac{1}{2}k [\sigma_1, \Phi(x)] + m_1\sigma_3\Phi(x) - m_2\Phi(x)\sigma_3 \]

\[ = [E - V(x)] \Phi(x) \]
decomposition

\[ i \partial_x \{ \sigma_1, \Phi(x) \} - (\partial_x \varphi) \{ \sigma_1, \Phi(x) \} - \frac{1}{2} k [\sigma_1, \Phi(x)] + m_1 \sigma_3 \Phi(x) - m_2 \Phi(x) \sigma_3 \]

\[
\Phi(x) \equiv \Phi_0(x) + \sum_{j=1}^{3} \Phi_j(x) \sigma_j
\]

no derivatives on #2 & #3
change of variable

\[ s(x) \equiv \frac{1}{2} \int_0^x du \left[ E - V(u) \right] = \frac{\varepsilon(x)}{2e^2} \left[ 2EV(x) - V(x)^2 \right] \]

\[ M \equiv \sqrt{E^2 - k^2} \]

\[ i\partial_s \Phi_1(s) = \left[ 1 - \frac{(m_1 - m_2)^2}{p^2} \right] \Phi_0(s) \]

\[ i\partial_s \Phi_0(s) = \left[ 1 - \frac{(m_1 + m_2)^2}{p^2} \right] \Phi_1(s) \]

\[ p^2 = (E - V)^2 - k^2 \]

\[ k \text{ independent @ fixed } s \text{ & } M ! \]
Reference frames. According to (3.10) and (3.18) the full wave function corresponding to the infinitesimal boost defined by (A8), is generated by

$$s(x) = \frac{1}{E - V(x)}$$

From its definition (3.3) the coeeficients (3.5), whereas the Poincaré algebra defines the bound state equation (3.13) gives the corresponding term on the right.

$$dx/ds = 2/(E - V(x))$$

$$V \ll E \quad 1/E$$

Cross check: direct boosting ✔

$$i\partial_s \Phi_1(s) = \left[ \frac{1}{p^2} \right] \Phi_0(s)$$

$$i\partial_s \Phi_0(s) = \left[ 1 - \frac{(m_1 + m_2)^2}{p^2} \right] \Phi_1(s)$$
change of variable

boosts:

equations form invariant in terms of s relation between s & x frame dependent

\[
\frac{dx}{ds} = \frac{2}{(E - V(x))}
\]

\(V \ll E\)  \(\frac{1}{E}\)

cross check: direct boosting ✓

works only for the linear potential
solutions

• possible analytically 😊

• selection of spectrum ?
The Dirac Electron in Simple Fields*

By Milton S. Plesset

Sloane Physics Laboratory, Yale University

(Received June 6, 1932)

The relativity wave equations for the Dirac electron are transformed in a simple manner into a symmetric canonical form. This canonical form makes readily possible the investigation of the characteristics of the solutions of these relativity equations for simple potential fields. If the potential is a polynomial of any degree in $x$, a continuous energy spectrum characterizes the solutions. If the potential is a polynomial of any degree in $1/x$, the solutions possess a continuous energy spectrum when the energy is numerically greater than the rest-energy of the electron; values of the energy numerically less than the rest-energy are barred. When the potential is a polynomial of any degree in $r$, all values of the energy are allowed. For potentials which are polynomials in $1/r$ of degree higher than the first, the energy spectrum is again continuous. The quantization arising for the Coulomb potential is an exceptional case.

\[ \varphi(x \to \infty) \sim \exp(\pm ix^2/4) \]

**Oscillations**

- **Dirac** \( \varphi(x) \) \((b=0)\)
- **Schrödinger** \( \rho(x) \)

\[ m = 2.5 \]
\[ \varphi(x \to \infty) \sim \exp(\pm ix^2/4) \]

**Oscillations**

- **Dirac** \( \varphi(x) \) \( (b=0) \)
- **Schrödinger** \( \psi(x) \)

**Dirac equation inclusive:**
- Oscillations \( \leftrightarrow \) presence of pairs (e.g. Klein paradoxon)
- Retarded propagators give inclusive cross sections

\( m = 2.5 \)
solutions \((m_1=m_2)\)

- oscillations as in Dirac

- only 1d subspace of solutions regular @ \(s=0\) opposed to Dirac

- selection of parity even or odd \(\Rightarrow\) spectrum
Regge trajectories

\[ M_n^2 - (2m)^2 \]

\[ n \]

\[ m = 0.1 \]

\[ m = 4.0 \]
solutions \((m_1 \neq m_2)\)

- no solutions regular @ \(s=0\)
- orthogonality relations ✔
- duality normalisation for highly excited states
boosting

Ground state

Excited state

\[ |\Phi_0|^2 + |\Phi_1|^2 \]

\( P = 0 \)

\( M = 5.11 \)

\( M = 3.15 \)

\( P = 5 \)

\( M = 5.11 \)

\( M = 3.15 \)

\[ m_1/e = 1.0 \]

\[ m_2/e = 1.5 \]
summary

• objective: towards a Born term for hadrons
  • valence quarks only
  • no transverse gluons
  • perturbation theory
  • Poincaré invariance
• $1+1$ QED as model
Thank you very much for your attention!

Thanks, Paul!