# Baryon Spectrum and Confinement from Superconformal Quantum Mechanics and Holographic QCD


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**STRONG INTERACTIONS IN THE LHC ERA**

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In collaboration with Stan Brodsky (SLAC) and Hans G. Dosch (Heidelberg)

## Introduction

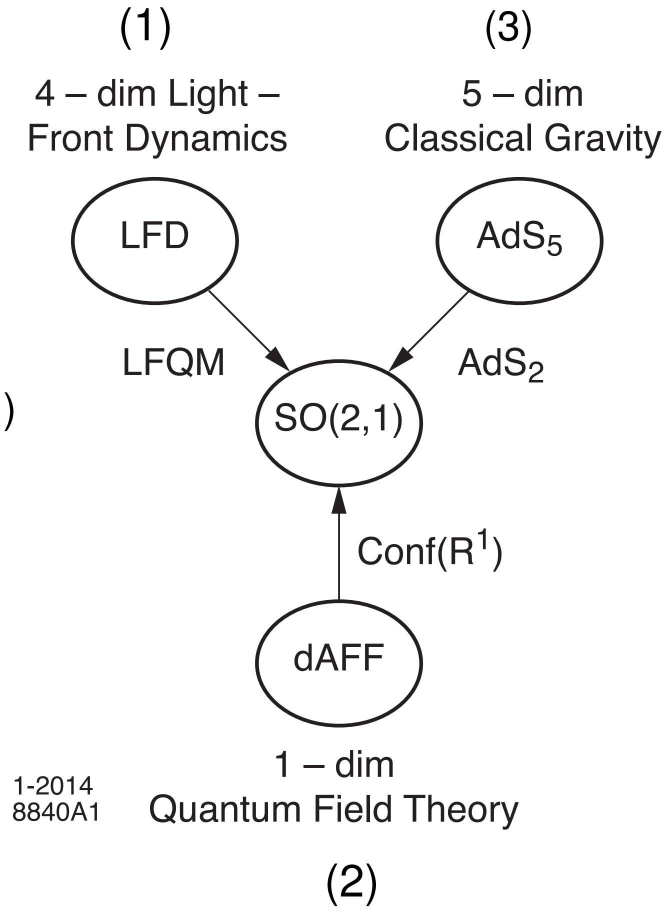
- Classical Lagrangian of QCD is invariant under scale transformations in the limit of massless quarks
- Meson and baryon bound-states have well-defined ground and excited states with well defined properties such as mass and spin
- Fundamental question in hadron physics: understand the mechanism which endows a nominally conformal theory with a mass scale and a spectrum
- Quest for semiclassical equations to describe bound-states in QCD similar to Schrödinger or Dirac Eqs. in atomic physics, corrected for quantum fluctuations
- Convenient frame-independent Hamiltonian framework for treating relativistic bound-states is light-front (LF) quantization
- Remarkable connections of LF Hamiltonian and AdS equations, Conformal and Superconformal QM
- Result is a light-front wave equation which reproduce prominent aspects of hadronic data, such as the the mass pattern observed in the radial and orbital excitations of the light mesons

## Three Steps Towards a Semiclassical Approximation for Bound States in QCD

1. Reduce strongly correlated multi-parton light-front Hamiltonian problem in QCD to effective 1-dim QFT: Complexities from strong interactions hidden in effective potential $U$

2. Construct effective LF confining potential $U$ which captures underlying confinement dynamics, emergence of a mass scale and Regge behavior $\longrightarrow$ Extension of conformal QM to LF !
   Isomorphism of 1-dim conformal group *Conf*$(R^1)$ with $SO(2,1)$:
   One of the generators of $SO(2,1)$ is compact (discrete spectrum)
   Baryons $\longrightarrow$ Extension of superconformal QM to the LF

3. Embedding of LF Hamiltonian in AdS for arbitrary spin:
   Effective LF potential $U$ extended for arbitrary integer and half-integer spin representations

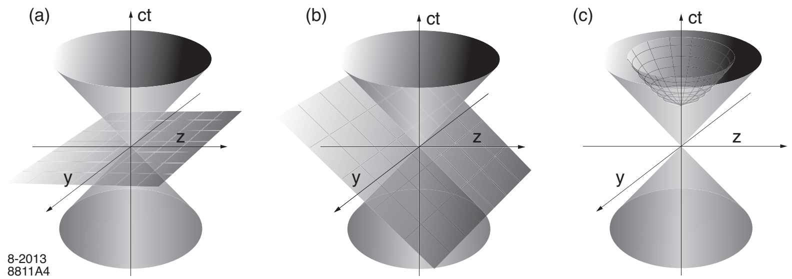



Conformal Quantum Mechanics: de Alfaro, Fubini and Furlan (dAFF) (for II)
Superconformal Quantum Mechanics: Fubini and Rabinovici (for II)
Light-Front Holographic QCD: S. J. Brodsky, GdT and H. G. Dosch (for I and III)

# Dirac Forms of Relativistic Dynamics

[Dirac (1949)]

- Poincaré generators $P^\mu$ and $M^{\mu\nu}$ separated into kinematical and dynamical
- Kinematical generators act along initial hypersurface and contain no interactions
- Dynamical generators are responsible for evolution of the system and depend on the interactions
- Each front has its Hamiltonian and evolve with different time, but results computed in any front should be identical
- *(a) Instant form, (b) Front form, (c) Point Form*




- Hadron with LF 4-momentum $P = (P^+, P^-, \mathbf{P}_\perp)$, $P^\pm = P^0 \pm P^3$, mass-shell relation $P_\mu P^\mu = M^2$ leads to LF Hamiltonian equation

$$P^-|\psi(P)\rangle = \frac{M^2 + \mathbf{P}_\perp^2}{P^+}|\psi(P)\rangle$$

- Construct LF invariant Hamiltonian $H_{LF} = P_\mu P^\mu = P^- P^+ - \mathbf{P}_\perp^2$ $(P^+ \text{ and } \mathbf{P}_\perp \text{ kinematical})$

$$\boxed{H_{LF}|\psi(P)\rangle = M^2|\psi(P)\rangle}$$

# Semiclassical Approximation to QCD in the Light Front (1)

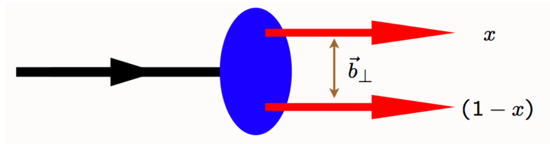
[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- LF eigenvalue equation $P_\mu P^\mu|\phi\rangle = M^2|\phi\rangle$ is a LF wave equation for $\phi$ $\quad (L = |L^z|_{\max})$

$$\Big(\underbrace{-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2}}_{kinetic\ energy\ of\ partons} + \underbrace{U(\zeta)}_{confinement}\Big)\phi(\zeta) = M^2\phi(\zeta)$$

- Invariant variable in transverse impact space is

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2$$




- Critical value $L=0$ corresponds to lowest possible stable solution: ground state of the LF Hamiltonian
- Relativistic and frame-independent LF Schrödinger equation: $U$ is instantaneous in LF time
- A linear potential $V_{\rm eff}$ in the *instant form* of dynamics implies quadratic potential $U_{\rm eff}$ in the *front form* at large distances $\longrightarrow$ Regge trajectories

$$U_{\rm eff} = V_{\rm eff}^2 + 2\sqrt{p^2+m_q^2}\,V_{\rm eff} + 2\,V_{\rm eff}\sqrt{p^2+m_q^2}$$

[A. P. Trawiński, S. D. Glazek, S. J. Brodsky, GdT, H. G. Dosch, PRD **90**, 074017 (2014)]

# Conformal Quantum Mechanics and Light Front Dynamics (2)

[S. J. Brodsky, GdT and H.G. Dosch, PLB **729**, 3 (2014)]

- Incorporate in a 1-dim QFT – as an effective theory, the fundamental conformal symmetry of the 4-dim classical QCD Lagrangian in the limit of massless quarks

- Invariance properties of 1-dim field theory under the full conformal group from dAFF action
[V. de Alfaro, S. Fubini and G. Furlan, Nuovo Cim. A **34**, 569 (1976)]

$$S = \frac{1}{2} \int dt \Big( \dot{x}^2 - \frac{g}{x^2} \Big), \quad g \text{ dimensionless}, \quad [t] = L^2$$

- Absence of dimensional constants implies action is invariant under a large group of transformations, the general conformal group

$$t' = \frac{\alpha t + \beta}{\gamma t + \delta}, \qquad x'(t') = \frac{x(t)}{\gamma t + \delta}, \quad \alpha\delta - \beta\gamma = 1$$

- Using Noether's theorem obtain conserved generators ($p = \dot{x}$)

  a. Hamiltonian: $H = \frac{1}{2} \left( p^2 + \frac{g}{x^2} \right)$

  b. Dilatation: $D = -\frac{1}{4} \left( px + xp \right)$

  c. Special conformal transformations: $K = \frac{1}{2} x^2$

- Using canonical commutation relations $[x, p] = i$ find

$$[H, D] = iH, \quad [H, K] = 2iD, \quad [K, D] = -iK,$$

the algebra of the generators of the conformal group *Conf*$(R^1)$

- Introduce the linear combinations

$$J^{12} = \frac{1}{2}\left(\frac{1}{a}K + aH\right), \;\; J^{01} = \frac{1}{2}\left(\frac{1}{a}K - aH\right), \;\; J^{02} = D,$$

where $a$ has dimension $t$ since $H$ and $K$ have different dimensions

- Generators $J$ have commutation relations

$$[J^{12}, J^{01}] = iJ^{02}, \;\; [J^{12}, J^{02}] = -iJ^{01}, \;\; [J^{01}, J^{02}] = -iJ^{12},$$

the algebra of $SO(2,1)$

- $J^{0i}, \; i = 1, 2$, boost in space direction $i$ and $J^{12}$ rotation in the (1,2) plane

- $J^{12}$ is compact and has thus discrete spectrum with normalizable eigenfunctions

- The relation between the generators of conformal group and generators of $SO(2,1)$ suggests that the scale $a$ may play a fundamental role

- dAFF construct a new generator $G$ as a superposition of the 3 constants of motion

$$G = uH + vD + wK$$

and introduce new time variable $\tau$

$$d\tau = \frac{dt}{u + vt + wt^2}$$

- Find usual quantum mechanical evolution for time $\tau$

$$G|\psi(\tau)\rangle = i\frac{d}{d\tau}|\psi(\tau)\rangle$$

with the new Hamiltonian $G$ in the Schrödinger picture $p \to -i\frac{d}{dx}$

$$G = \frac{1}{2}u\left(-\frac{d^2}{dx^2} + \frac{g}{x^2}\right) + \frac{i}{4}v\left(x\,\frac{d}{dx} + \frac{d}{dx}\,x\right) + \frac{1}{2}wx^2.$$

- Scale appears in the Hamiltonian without affecting the conformal invariance of the action !

- Operator $G$ is compact for $4uw - v^2 > 0$

## Connection to Light-Front Dynamics

- Compare de dAFF Hamiltonian $G$

$$G = \frac{1}{2}u\left(-\frac{d^2}{dx^2} + \frac{g}{x^2}\right) + \frac{i}{4}v\left(x\,\frac{d}{dx} + \frac{d}{dx}\,x\right) + \frac{1}{2}wx^2.$$

with the LF Hamiltonian $H_{LF}$

$$H_{LF} = -\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)$$

and identify dAFF variable $x$ with LF invariant variable $\zeta$

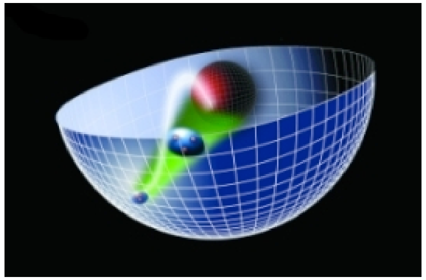
- Choose $u = 2, \;\; v = 0$, find the Casimir operator from LF kinematical constraints $g = L^2 - \frac{1}{4}$
- $w = 2\lambda^2 \sim \frac{1}{a^2}$ fixes the LF potential to harmonic oscillator in the LF plane $\lambda^2\,\zeta^2$

$$U \sim \lambda^2\zeta^2$$

- In contrast to fermionic case one can perform a level shift by adding an arbitrary constant to the confining term in the LF potential $U$

# Higher Integer-Spin Wave Equations in AdS Space (3)

[GdT, H.G. Dosch and S. J. Brodsky, PRD **87**, 075004 (2013)]



- Description of higher spin modes in AdS space (Frondsal, Fradkin, Vasiliev, Metsaev . . . )
- Integer spin-$J$ fields in AdS conveniently described by tensor field $\Phi_{N_1 \dots N_J}$ with effective action

$$S_{eff} = \int d^d x \, dz \sqrt{|g|} \, e^{\varphi(z)} \, g^{N_1 N_1'} \cdots g^{N_J N_J'} \Big( g^{M M'} D_M \Phi^*_{N_1 \dots N_J} \, D_{M'} \Phi_{N_1' \dots N_J'} - \mu^2_{eff}(z) \, \Phi^*_{N_1 \dots N_J} \, \Phi_{N_1' \dots N_J'} \Big)$$

$D_M$ is the covariant derivative which includes affine connection and dilaton $\varphi(z)$ breaks conformality

- Effective mass $\mu_{eff}(z)$ is determined by precise mapping to light-front physics
- Non-trivial geometry of pure AdS encodes the kinematics and the additional deformations of AdS encode the dynamics, including confinement

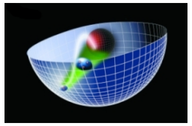
- Physical hadron has plane-wave and polarization indices along $3{+}1$ physical coordinates and a profile wavefunction $\Phi(z)$ along holographic variable $z$

$$\Phi_P(x,z)_{\mu_1\cdots\mu_J} = e^{iP\cdot x}\Phi(z)_{\mu_1\cdots\mu_J}, \qquad \Phi_{z\mu_2\cdots\mu_J} = \cdots = \Phi_{\mu_1\mu_2\cdots z} = 0$$

with four-momentum $P_\mu$ and invariant hadronic mass $P_\mu P^\mu {=} M^2$

- Variation of the action gives AdS wave equation for spin-$J$ field $\Phi(z)_{\nu_1\cdots\nu_J} = \Phi_J(z)\epsilon_{\nu_1\cdots\nu_J}$

$$\boxed{\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}}\partial_z\left(\frac{e^{\varphi}(z)}{z^{d-1-2J}}\partial_z\right) + \left(\frac{mR}{z}\right)^2\right]\Phi_J = M^2\Phi_J}$$



with

$$(m\,R)^2 = (\mu_{eff}(z)R)^2 - Jz\,\varphi'(z) + J(d-J+1)$$

and the kinematical constraints to eliminate the lower spin states $J-1,\, J-2, \cdots$

$$\eta^{\mu\nu}P_\mu\,\epsilon_{\nu\nu_2\cdots\nu_J} = 0, \quad \eta^{\mu\nu}\,\epsilon_{\mu\nu\nu_3\cdots\nu_J} = 0.$$

- Kinematical constrains in the LF imply that $m$ must be a constant

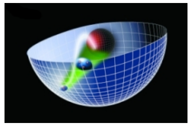
[See also: T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Phys. Rev. D **85**, 076003 (2012)]

## Light-Front Mapping

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

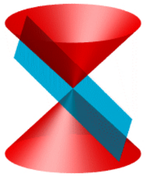
- Upon substitution $\Phi_J(z) \sim z^{(d-1)/2-J} e^{-\varphi(z)/2} \phi_J(z)$ and $z \to \zeta$ in AdS WE

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}}\partial_z\left(\frac{e^{\varphi(z)}}{z^{d-1-2J}}\partial_z\right)+\left(\frac{mR}{z}\right)^2\right]\Phi_J(z)=M^2\Phi_J(z)$$



we find LFWE $(d = 4)$

$$\left(-\frac{d^2}{d\zeta^2}-\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right)\phi_J(\zeta)=M^2\phi_J(\zeta)$$



with
$$U(\zeta)=\tfrac{1}{2}\varphi''(\zeta)+\frac{1}{4}\varphi'(\zeta)^2+\frac{2J-3}{2z}\varphi'(\zeta)$$

and $(mR)^2 = -(2-J)^2 + L^2$

- Unmodified AdS equations correspond to the kinetic energy terms for the partons
- Effective confining potential $U(\zeta)$ corresponds to the IR modification of AdS space
- AdS Breitenlohner-Freedman bound $(mR)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$

## Meson Spectrum

- Dilaton profile in the dual gravity model determined from one-dim QFTh (dAFF)

$$\varphi(z) = \lambda z^2, \qquad \lambda^2 = w/2$$

- Effective potential: $U = \lambda^2 \zeta^2 + 2\lambda(J-1)$

- LFWE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda(J-1)\right)\phi_J(\zeta) = M^2 \phi_J(\zeta)$$

- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z) = 1$

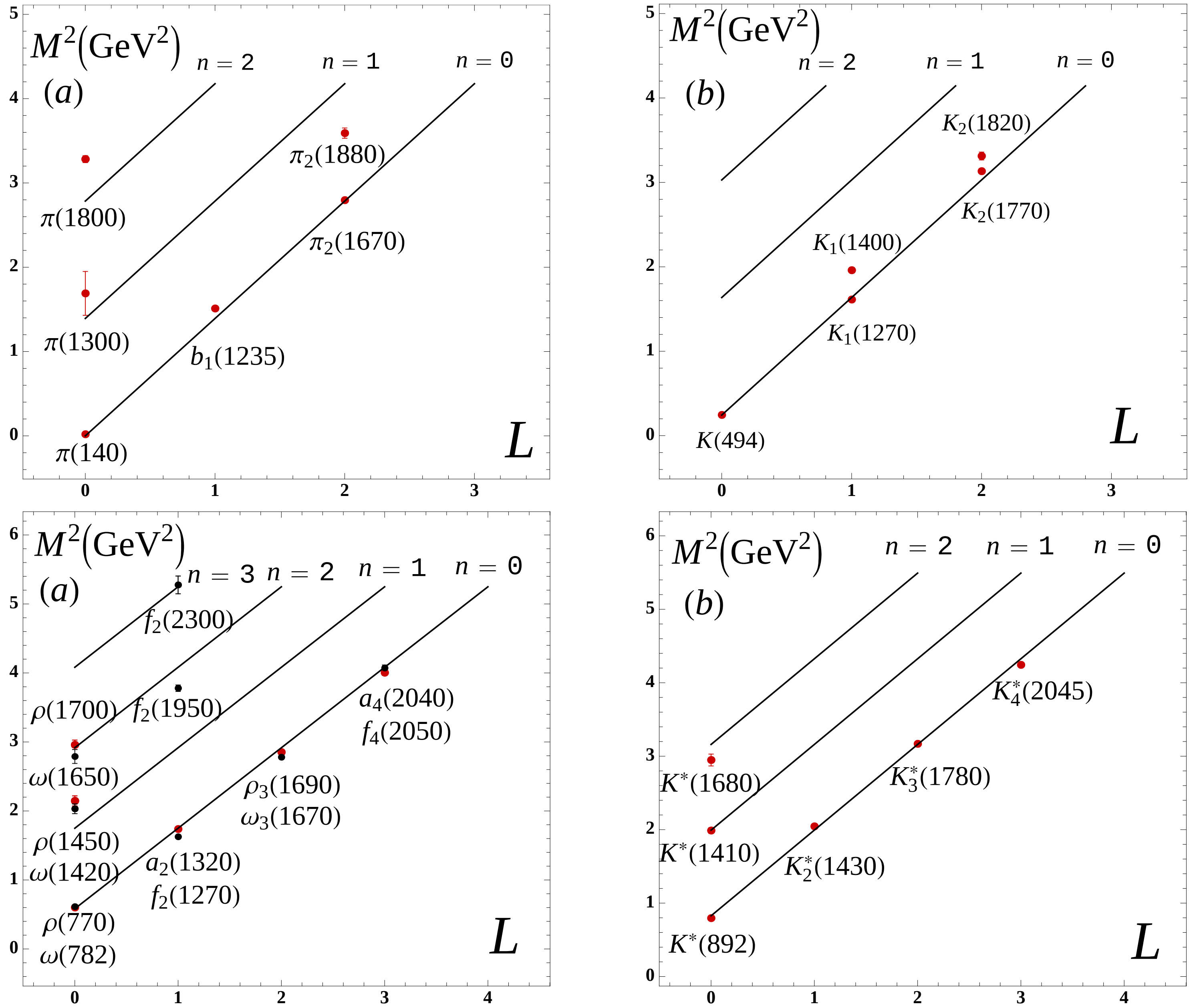
$$\phi_{n,L}(\zeta) = |\lambda|^{(1+L)/2} \sqrt{\frac{2n!}{(n{+}L)!}}\, \zeta^{1/2+L} e^{-|\lambda|\zeta^2/2} L_n^L(|\lambda|\zeta^2)$$

- Eigenvalues for $\lambda > 0$

$$\mathcal{M}^2_{n,J,L} = 4\lambda\left(n + \frac{J+L}{2}\right)$$

- Results are easily extended to light quarks

- $\lambda < 0$ incompatible with LF constituent interpretation




Orbital and radial excitations for $\sqrt{\lambda} = 0.59$ GeV (pseudoscalar) and 0.54 GeV (vector mesons)

## Supersymmetric Quantum Mechanics and Light-Front Dynamics (1)

[E. Witten, NPB **188**, 513 (1981)]

[GdT, H.G. Dosch and S. J. Brodsky, arXiv:1411.XXXX]

- SUSY QM contains two fermionic generators $Q$ and $Q^\dagger$, and a bosonic generator, the Hamiltonian $H$
- Closure under the graded-algebra $sl(1/1)$:

$$\tfrac{1}{2}\{Q, Q^\dagger\} = H$$
$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0, \qquad [Q, H] = [Q^\dagger, H] = 0$$

- Minimal realization of group generators $\quad$ ($W(x)$ is the 'superpotential')

$$Q = \chi \left( \frac{d}{dx} + W(x) \right), \qquad Q^\dagger = \chi^\dagger \left( -\frac{d}{dx} + W(x) \right)$$

- In a $2 \times 2$ Pauli-spin-matrix representation the spinor operators $\chi$ and $\chi^\dagger$ satisfy the relations

$$\{\chi, \chi^\dagger\} = 1 \quad \text{and} \quad [\chi, \chi^\dagger] = \sigma_3$$

- Hamiltonian has identical eigenvalues for up and down components

$$H = \frac{1}{2} \left( -\frac{d^2}{dx^2} + W^2(x) + \sigma_3 \, W'(x) \right)$$

## SUSY Relativistic Light-Front Extension

- In a $4 \times 4$ Dirac-matrix representation the spinor operators $\chi$ and $\chi^\dagger$ satisfy the relations

$$\{\chi, \chi^\dagger\} = 1 \quad \text{and} \quad [\chi, \chi^\dagger] = \gamma_5$$

- Thus the SUSY LF Hamiltonian

$$H = \{Q, Q^\dagger\} = -\frac{d^2}{dx^2} + W^2(x) + \gamma_5 \, W'(x)$$

- Take take 'square root' of the LF Hamiltonian $H_{LF}$: $\quad H_{LF} \, \psi = D_{LF}^2 \psi = M^2 \psi$

- Thus the linear Dirac equation: $\quad (D_{LF} - M) \, \psi = 0$

- In a $2 \times 2$ chiral spinor component representation $\psi_\pm$

$$\begin{aligned} -\frac{d}{d\zeta}\psi_- - \frac{\nu + \frac{1}{2}}{\zeta}\psi_- - u(\zeta)\psi_- &= M\psi_+ \\ \frac{d}{d\zeta}\psi_+ - \frac{\nu + \frac{1}{2}}{\zeta}\psi_+ - u(\zeta)\psi_+ &= M\psi_- \end{aligned}$$

where we have separated kinematic and dynamic effects in the superpotential $W(\zeta)$

$$W(\zeta) = \frac{\nu + 1/2}{\zeta} + u(\zeta)$$

# Higher Half-Integer Spin Wave Equations in AdS Space (3)

[J. Polchinski and M. J. Strassler, JHEP **0305**, 012 (2003)]

[GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[GdT, H.G. Dosch and S. J. Brodsky, PRD **87**, 075004 (2013)]

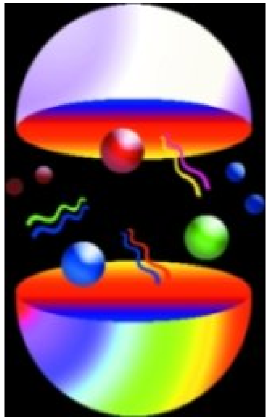


Image credit: N. Evans

- Important similarities between spectra of mesons and baryons: similar slope and spacing of orbital and radial excitations, similar multiplicity
- Holographic embeddings in AdS also explains distinctive features, such as the absence of spin-orbit coupling for baryons
- Extension of holographic ideas to spin-$\frac{1}{2}$ (and higher half-integral $J$) hadrons by considering wave equations for Rarita-Schwinger (RS) spinor fields in AdS space and their mapping to light-front physics
- Half-integer spin $J = T + \frac{1}{2}$ represented by RS spinor $\left[\Psi_{N_1 \cdots N_T}\right]_\alpha$

- Effective AdS action

$$S_{eff} = \frac{1}{2} \int d^d x \, dz \, \sqrt{|g|} \, g^{N_1 \, N_1'} \cdots g^{N_T \, N_T'}$$
$$\left[ \overline{\Psi}_{N_1 \cdots N_T} \Big( i \, \Gamma^A \, e_A^M \, D_M - \mu - V(z) \Big) \Psi_{N_1' \cdots N_T'} + h.c. \right]$$

where covariant derivative $D_M$ includes affine connection and spin connection

- $e_M^A$ is the vielbein and $\Gamma^A$ tangent space Dirac matrices $\left\{ \Gamma^A, \Gamma^B \right\} = \eta^{AB}$

- Variation of the AdS action leads to Dirac equation

$$\begin{aligned} \left[ -\frac{d}{dz} - \frac{\mu R}{z} - \frac{R}{z} V(z) \right] \Psi^-_{\nu_1 \nu_2 \cdots \nu_T} &= M \Psi^+_{\nu_1 \nu_2 \cdots \nu_T} \\ \left[ \frac{d}{dz} - \frac{\mu R}{z} - \frac{R}{z} V(z) \right] \Psi^+_{\nu_1 \nu_2 \cdots \nu_T} &= M \Psi^-_{\nu_1 \nu_2 \cdots \nu_T} \end{aligned}$$

and the Rarita-Schwinger condition in physical space-time

$$\gamma^\nu \Psi_{\nu \nu_2 \, \ldots \, \nu_T} = 0$$

The RS AdS chiral spinor $\Psi^\pm_{\nu_1 \nu_2 \cdots \nu_T} \equiv \Psi^\pm_T$ has polarization indices along physical coordinates

- Compare AdS Dirac equation for spin $J = T + \frac{1}{2}$

$$
\begin{aligned}
-\frac{d}{dz}\Psi_T^- - \frac{\mu R}{z}\Psi_T^- - \frac{R}{z}V(z)\Psi_T^- &= M\Psi_T^+ \\
\frac{d}{dz}\Psi_T^+ - \frac{\mu R}{z}\Psi_T^+ - \frac{R}{z}V(z)\Psi_T^+ &= M\Psi_T^-
\end{aligned}
$$

with LF SUSY Dirac equation

$$
\begin{aligned}
-\frac{d}{d\zeta}\psi_- - \frac{\nu + \frac{1}{2}}{\zeta}\psi_- - u(\zeta)\psi_- &= M\psi_+ \\
\frac{d}{d\zeta}\psi_+ - \frac{\nu + \frac{1}{2}}{\zeta}\psi_+ - u(\zeta)\psi_+ &= M\psi_-
\end{aligned}
$$

- Identifying holographic variable $z$ with invariant LF variable $\zeta$, map AdS into LF Dirac eq. $\Psi_T \to \psi$

- AdS mass is related to parameter $\nu$ by $\mu R = \nu + \frac{1}{2}$ and

$$
u(\zeta) = \frac{R}{\zeta}V(\zeta)
$$

a $J$-independent potential – No spin-orbit coupling along a given trajectory !

# Light-Front Superconformal Quantum Mechanics (2)

S. Fubini and E. Rabinovici, NPB **245**, 17 (1984)

[GdT, H.G. Dosch and S. J. Brodsky, arXiv:1411.XXXX]

- Compelling to examine the properties of supersymmetric algebra and its superconformal extension to describe baryons in complete analogy to the mesons

- Following F&R consider a 1-dim QFT invariant under conformal and supersymmetric transformations

- Conformal superpotential ($f$ is dimensionless)

$$W(x) = \frac{f}{x}$$

- Graded-Lie algebra has in addition to Hamiltonian $H$ and supercharges $Q$ and $Q^\dagger$, a new operator $S$ related to generator of conformal transformations $K$

- Use 1-dim QFT representation of the operators

$$\begin{aligned} Q &= \chi \left(\frac{d}{dx} + \frac{f}{x}\right), \qquad Q^\dagger = \chi^\dagger \left(-\frac{d}{dx} + \frac{f}{x}\right) \\ S &= \chi\, x, \qquad S^\dagger = \chi^\dagger x \end{aligned}$$

- Find enlarged algebra (Superconformal algebra of Haag, Lopuszanski and Sohnius (1974))

$$
\begin{aligned}
\tfrac{1}{2}\{Q, Q^\dagger\} &= H, \qquad \tfrac{1}{2}\{S, S^\dagger\} = K \\
\tfrac{1}{2}\{Q, S^\dagger\} &= \frac{f}{2} + \frac{\sigma_3}{4} + iD \\
\tfrac{1}{2}\{Q^\dagger, S\} &= \frac{f}{2} + \frac{\sigma_3}{4} - iD
\end{aligned}
$$

where the operators

$$
\begin{aligned}
H &= \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{f^2 - \sigma_3 f}{x^2}\right) \\
K &= \tfrac{1}{2}x^2 \\
D &= \frac{i}{4}\left(\frac{d}{dx}x + x\frac{d}{dx}\right)
\end{aligned}
$$

satisfy the conformal algebra

$$
[H, D] = iH, \quad [H, K] = 2iD, \quad [K, D] = -iK
$$

- Define a supercharge $R$, a linear combination of the generators $Q$ and $S$

$$R = \sqrt{u}\, Q + \sqrt{w}\, S$$

and compute the QM evolution operator $G$

$$G = \tfrac{1}{2}\{R, R^\dagger\}$$

We find

$$G = uH + wK + \tfrac{1}{2}\sqrt{uw}\,(2f + \sigma_3)$$

which is a compact operator for $uw > 0$

- Light-front extension of superconformal results follows from

$$x \to \zeta, \quad f \to \nu + \tfrac{1}{2}, \quad \sigma_3 \to \gamma_5, \quad 2G \to H_{LF}$$

- Obtain:

$$H_{LF} = -\frac{d^2}{d\zeta^2} + \frac{\left(\nu + \frac{1}{2}\right)^2}{\zeta^2} - \frac{\nu + \frac{1}{2}}{\zeta^2}\gamma_5 + \lambda^2\zeta^2 + \lambda(2\nu + 1) + \lambda\gamma_5$$

where coefficients $u$ and $w$ are fixed to $u = 2$ and $w = 2\lambda^2$

## Baryon Spectrum

- In $2 \times 2$ block-matrix form

$$H_{LF} = \begin{pmatrix} -\frac{d^2}{d\zeta^2} - \frac{1-4\nu^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda(\nu+1) & 0 \\ 0 & -\frac{d^2}{d\zeta^2} - \frac{1-4(\nu+1)^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda\nu \end{pmatrix}$$

- The light-front eigenvalue equation $H_{LF}|\psi\rangle = M^2|\psi\rangle$ has eigenfunctions

$$\begin{aligned} \psi_+(\zeta) &\sim \zeta^{\frac{1}{2}+\nu} e^{-\lambda\zeta^2/2} L_n^{\nu}(\lambda\zeta^2) \\ \psi_-(\zeta) &\sim \zeta^{\frac{3}{2}+\nu} e^{-\lambda\zeta^2/2} L_n^{\nu+1}(\lambda\zeta^2) \end{aligned}$$

and eigenvalues

$$M^2 = 4\lambda(n + \nu + 1)$$

identical for both plus and minus eigenfunctions

- In contrast with meson spectrum, baryon spectrum does not depend on $J$: no spin-orbit coupling

[See also: T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Phys. Rev. D **85**, 076003 (2012)]

- Lowest possible state $n = 0$ and $\nu = 0$: orbital excitations $\nu = 0, 1, 2 \cdots = L$

$$M^2 = 4\lambda(n + L + 1)$$

- $L$ is the relative LF angular momentum between the active quark and spectator cluster

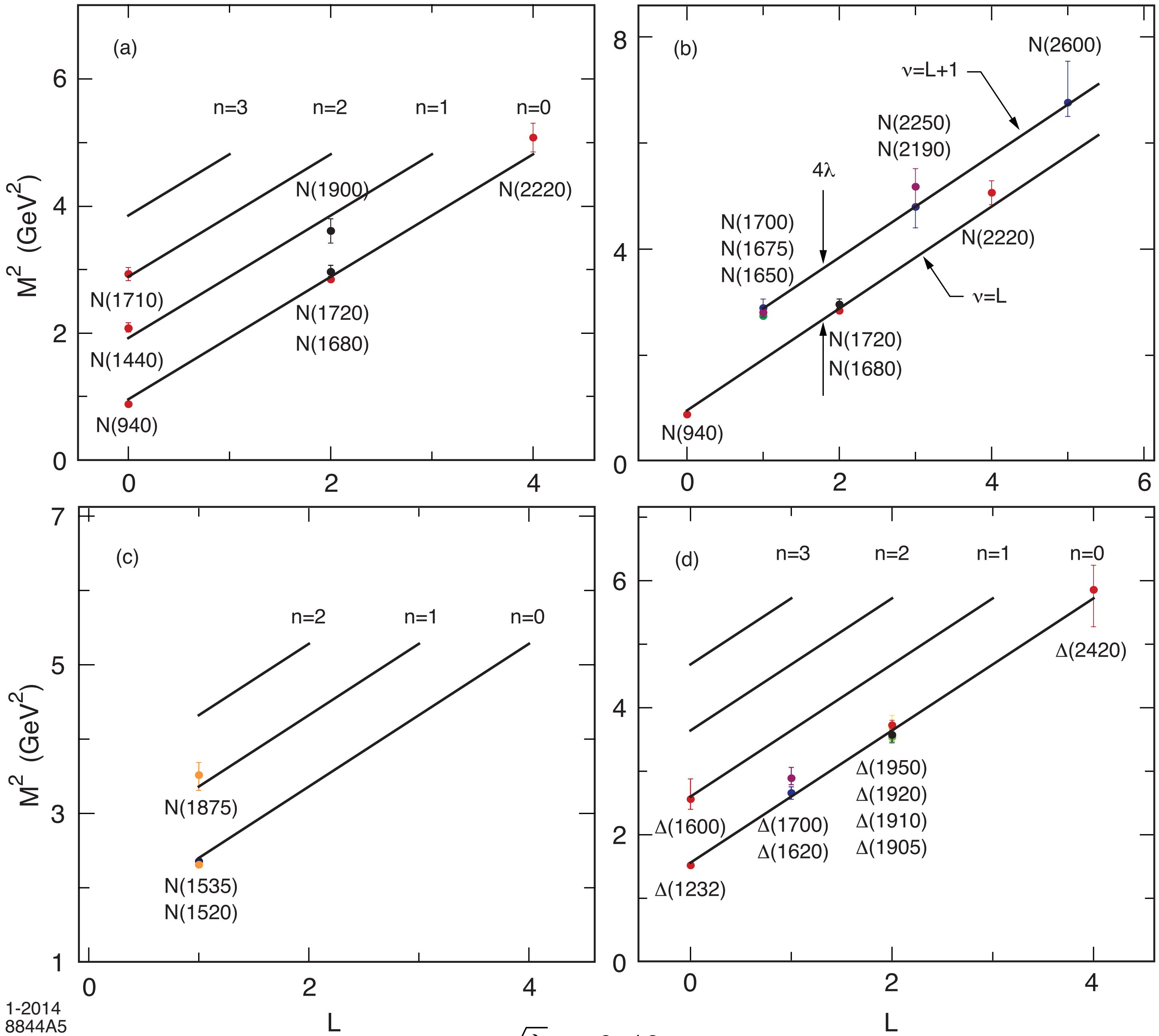
- In general $\nu$ depends on internal spin and parity

  The assignment

| | $S = \frac{1}{2}$ | $S = \frac{3}{2}$ |
|---|---|---|
| P = + | $\nu = L$ | $\nu = L + \frac{1}{2}$ |
| P = - | $\nu = L + \frac{1}{2}$ | $\nu = L + 1$ |

  describes the full light baryon orbital and radial excitation spectrum

| $SU(6)$ | $S$ | $L$ | $n$ | Baryon State |
|---|---|---|---|---|
| **56** | $\frac{1}{2}$ | 0 | 0 | $N\,\frac{1}{2}^{+}(940)$ |
| | $\frac{3}{2}$ | 0 | 0 | $\Delta\,\frac{3}{2}^{+}(1232)$ |
| **56** | $\frac{1}{2}$ | 0 | 1 | $N\,\frac{1}{2}^{+}(1440)$ |
| | $\frac{3}{2}$ | 0 | 1 | $\Delta\,\frac{3}{2}^{+}(1600)$ |
| **70** | $\frac{1}{2}$ | 1 | 0 | $N\,\frac{1}{2}^{-}(1535)$ $N\,\frac{3}{2}^{-}(1520)$ |
| | $\frac{3}{2}$ | 1 | 0 | $N\,\frac{1}{2}^{-}(1650)$ $N\,\frac{3}{2}^{-}(1700)$ $N\,\frac{5}{2}^{-}(1675)$ |
| | $\frac{1}{2}$ | 1 | 0 | $\Delta\,\frac{1}{2}^{-}(1620)$ $\Delta\,\frac{3}{2}^{-}(1700)$ |
| **56** | $\frac{1}{2}$ | 0 | 2 | $N\,\frac{1}{2}^{+}(1710)$ |
| | $\frac{1}{2}$ | 2 | 0 | $N\,\frac{3}{2}^{+}(1720)$ $N\,\frac{5}{2}^{+}(1680)$ |
| | $\frac{3}{2}$ | 2 | 0 | $\Delta\,\frac{1}{2}^{+}(1910)$ $\Delta\,\frac{3}{2}^{+}(1920)$ $\Delta\,\frac{5}{2}^{+}(1905)$ $\Delta\,\frac{7}{2}^{+}(1950)$ |
| 70 | $\frac{3}{2}$ | 1 | 1 | $N\,\frac{1}{2}^{-}$ $N\,\frac{3}{2}^{-}(1875)$ $N\,\frac{5}{2}^{-}$ |
| | $\frac{3}{2}$ | 1 | 1 | $\Delta\,\frac{5}{2}^{-}(1930)$ |
| **56** | $\frac{1}{2}$ | 2 | 1 | $N\,\frac{3}{2}^{+}(1900)$ $N\,\frac{5}{2}^{+}$ |
| **70** | $\frac{1}{2}$ | 3 | 0 | $N\,\frac{5}{2}^{-}$ $N\,\frac{7}{2}^{-}$ |
| | $\frac{3}{2}$ | 3 | 0 | $N\,\frac{3}{2}^{-}$ $N\,\frac{5}{2}^{-}$ $N\,\frac{7}{2}^{-}(2190)$ $N\,\frac{9}{2}^{-}(2250)$ |
| | $\frac{1}{2}$ | 3 | 0 | $\Delta\,\frac{5}{2}^{-}$ $\Delta\,\frac{7}{2}^{-}$ |
| **56** | $\frac{1}{2}$ | 4 | 0 | $N\,\frac{7}{2}^{+}$ $N\,\frac{9}{2}^{+}(2220)$ |
| | $\frac{3}{2}$ | 4 | 0 | $\Delta\,\frac{5}{2}^{+}$ $\Delta\,\frac{7}{2}^{+}$ $\Delta\,\frac{9}{2}^{+}$ $\Delta\,\frac{11}{2}^{+}(2420)$ |
| **70** | $\frac{1}{2}$ | 5 | 0 | $N\,\frac{9}{2}^{-}$ $N\,\frac{11}{2}^{-}$ |
| | $\frac{3}{2}$ | 5 | 0 | $N\,\frac{7}{2}^{-}$ $N\,\frac{9}{2}^{-}$ $N\,\frac{11}{2}^{-}(2600)$ $N\,\frac{13}{2}^{-}$ |




Baryon orbital and radial excitations for $\sqrt{\lambda} = 0.49$ GeV (nucleons) and 0.51 GeV (Deltas)

**Thanks !**

For a review: S. J. Brodsky, G. F. de Téramond, H. G. Dosch and J. Erlich, arXiv:1407.8131 [hep-ph]