Hidden Symmetries and HEFT

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based on arXiv:1610.00150

Bad Honnef, February 2017
Outlook

Probing the Higgs potential
The model
Functional Symmetries and Renormalization
Expanding around the p.c. renormalizable theory:
  the privileged operator $\partial_\mu (\Phi^\dagger \Phi) \partial^\mu (\Phi^\dagger \Phi)$
Applications: anomalous trilinear Higgs coupling
Conclusions
Probing the Higgs potential

The discovery at LHC of a particle compatible with the Standard Model Higgs boson in 2012 has been an unprecedented breakthrough in the study of the electroweak symmetry breaking mechanism.

Parameterization of Beyond-the-Standard-Model (BSM) physics at the LHC is usually carried out within the Effective Field Theory approach.

Recent summary:
Handbook of LHC Higgs cross sections:
4. Deciphering the nature of the Higgs sector
arXiv:1610.07922
Higgs Effective Field Theories

Add operators of higher dimension to the SM Lagrangian compatible with the symmetries of the theory

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_i \frac{c_i^{(5)}}{\Lambda} \mathcal{O}_i^{(5)} + \sum_i \frac{c_i^{(6)}}{\Lambda^2} \mathcal{O}_i^{(6)} + \sum_i \frac{c_i^{(7)}}{\Lambda^3} \mathcal{O}_i^{(7)} + \sum_i \frac{c_i^{(8)}}{\Lambda^4} \mathcal{O}_i^{(8)} + \ldots \]

c are the Wilson coefficients, \( \Lambda \) is some large energy scale
Dimension-six bosonic operators

Table 97: Bosonic $D=6$ operators in the SfLH basis.

<table>
<thead>
<tr>
<th>Bosonic CP-even</th>
<th>Bosonic CP-odd</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_H$</td>
<td>$\frac{1}{2\nu^2} \left[ \partial_\lambda (H^\dagger H) \right]^2$</td>
</tr>
<tr>
<td>$O_T$</td>
<td>$\frac{1}{2\nu^2} \left( H^\dagger D_\mu H \right)^2$</td>
</tr>
<tr>
<td>$O_6$</td>
<td>$-\frac{\Delta}{\nu^3} (H^\dagger H)^3$</td>
</tr>
<tr>
<td>$O_g$</td>
<td>$\frac{g^2}{m_W^2} H^\dagger H G^{\alpha}<em>{\mu\nu} G^\alpha</em>{\mu\nu}$</td>
</tr>
<tr>
<td>$O_\gamma$</td>
<td>$\frac{g^2}{m_W^2} H^\dagger H B_{\mu\nu} B_{\mu\nu}$</td>
</tr>
<tr>
<td>$O_W$</td>
<td>$\frac{ig}{2m_W^2} \left( H^\dagger \sigma_\lambda D_\mu H \right) D_\nu W_{\mu\nu}^i$</td>
</tr>
<tr>
<td>$O_B$</td>
<td>$\frac{ig}{2m_W^2} \left( H^\dagger D_\mu H \right) \partial_\nu B_{\mu\nu}$</td>
</tr>
<tr>
<td>$O_{HW}$</td>
<td>$\frac{ig}{m_W} \left( D_\mu H^\dagger \sigma^i D_\nu H \right) W_{\mu\nu}^i$</td>
</tr>
<tr>
<td>$O_{HB}$</td>
<td>$\frac{ig}{m_W} \left( D_\mu H^\dagger D_\nu H \right) B_{\mu\nu}$</td>
</tr>
<tr>
<td>$O_{2W}$</td>
<td>$\frac{ig}{m_W^2} D_\mu W_{\mu\nu}^i D_\nu W_{\mu\nu}^i$</td>
</tr>
<tr>
<td>$O_{2B}$</td>
<td>$\frac{ig}{m_W^2} \partial_\mu B_{\mu\nu} \partial_\nu B_{\mu\nu}$</td>
</tr>
<tr>
<td>$O_{2G}$</td>
<td>$\frac{ig}{m_W^2} D_\mu G^{\alpha}<em>{\mu\nu} D</em>\nu G^\alpha_{\mu\nu}$</td>
</tr>
<tr>
<td>$O_{3W}$</td>
<td>$\frac{g^3}{m_W^2} \epsilon^{ijk} \tilde{W}<em>{\mu\nu}^{ij} W</em>{\rho\sigma}^{k}$</td>
</tr>
<tr>
<td>$O_{3G}$</td>
<td>$\frac{g^3}{m_W^2} f^{abc} G^{\alpha}<em>{\mu\nu} G^{b}</em>{\nu\rho} G^{c}_{\rho\mu}$</td>
</tr>
</tbody>
</table>

From arXiv:1610.07922
UV properties of the HEFT

Power-counting renormalizability is lost

Physical Unitarity (cancellation of ghost states) guaranteed by BRST symmetry & Slavnov-Taylor identities

Froissart bound usually not respected

In general all possible terms allowed by symmetry must be included in an EFT approach

One-loop anomalous dimensions in the HEFT

However a *tour de force* computation of one-loop anomalous dimensions in the HEFT has revealed surprising cancellations.

R. Alonso, E. Jenkins, A. Manohar, M. Trott

Not all mixings in principle allowed by the symmetries do indeed arise at one loop level
Holomorphy

Holomorphic part of the Lagrangian: built out of
\[ X^+, R, \bar{L} \]

Anti-holomorphic part of the Lagrangian: built out of
\[ X^-, \bar{R}, L \]

The remnant is non-holomorphic

Basic idea: holomorphic operators
do not mix with anti-holomorphic and non-holomorphic operators

True at the one-loop level (up to some breaking proportional to Yukawa couplings)
on the S-matrix elements (not off-shell)
One-loop Anomalous Dimensions for a gauge theory

Table II. Anomalous dimension matrix for dimension six operators in a general quantum field theory. The shaded entries vanish by our non-renormalization theorems, in full agreement with [2]. Here $y^2$ and $y'^2$ label entries that are non-zero due to non-holomorphic Yukawa couplings, $\times$ labels entries that vanish because there are no diagrams [13], and $\times^*$ labels entries that vanish by a combination of counterterm analysis and our non-renormalization theorems.

UV properties of HEFT dimension-six operators from derivative interactions

We restrict ourselves to the Higgs potential sector.
In the first approximation we do not couple the scalars
with the gauge bosons and the fermions

\[ S = \int d^4x \left[ \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma + \frac{1}{2} \partial^\mu \phi_a \partial_\mu \phi_a - \frac{M^2}{2} X_2^2 ight. 
\quad + \left. \frac{1}{v} (X_1 + X_2) \Box \left( \frac{1}{2} \sigma^2 + v \sigma + \frac{1}{2} \phi_a^2 - v X_2 \right) \right]. \]

\[ \Phi = \frac{1}{\sqrt{2}} \left( i \phi_1 + \phi_2 \right) \quad \text{SU(2) doublet} \quad X_2 \quad \text{SU(2) singlet} \]
Equations of motion for the auxiliary field

\[
\frac{\delta S}{\delta X_1} = \frac{1}{v} \Box \left( \frac{1}{2} \sigma^2 + v \sigma + \frac{1}{2} \phi^2 - vX_2 \right)
\]

Going on-shell with the auxiliary field and neglecting zero modes of the Laplacian we obtain

\[
X_2 = \frac{1}{2v} \sigma^2 + \sigma + \frac{1}{2v} \phi^2
\]

\[
= \Phi^\dagger \Phi - \frac{v^2}{2}
\]
On-shell equivalence with the standard formulation

By substituting the e.o.m. solution for the singlet field in the classical action one gets back the ordinary quartic potential

\[
S|_{\text{on-shell}} = \int d^4x \left[ \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma + \frac{1}{2} \partial^\mu \phi_a \partial_\mu \phi_a - \frac{1}{2} \frac{M^2}{v^2} \left( \frac{1}{2} \sigma^2 + v\sigma + \frac{1}{2} \phi_a^2 \right)^2 \right]
\]

with coupling constant

\[
\lambda = \frac{1}{2} \frac{M^2}{v^2}
\]

right sign of the quartic potential needed to ensure stability from the sign of the mass term, in turn fixed by the requirement of the absence of tachyons.
Dangerous interactions for renormalizability

The model contains derivative interactions of the schematic form

\[ \chi \Box \chi^2 \]

i.e. an operator of dimension 5.

Renormalizability?

More symmetries needed
BRST implementation of the on-shell constraint

Off-shell there is one more scalar field $X_1$. What about this field? Physical or unphysical?

BRST symmetry (it does not originate from gauge invariance)

$$sX_1 = vc, \quad sc = 0, \quad s\sigma = s\phi_a = sX_2 = 0,$$

$$s\bar{c} = \frac{1}{2}\sigma^2 + v\sigma + \frac{1}{2}\phi_a^2 - vX_2.$$  

Ghost action

$$S_{\text{ghost}} = -\int d^4x \bar{c} \Box c.$$  

Invariance under the nilpotent BRST symmetry formally associated with a $U(1)_{\text{constr}}$ group

Full tree-level vertex functional

\[ \Gamma^{(0)} = \int d^4 x \left[ \frac{1}{2} \partial^\mu \sigma \partial_\mu \sigma + \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - \frac{M^2}{2} X_2^2 ight. \\
- \bar{c} \Box c + \frac{1}{v} (X_1 + X_2) \Box \left( \frac{1}{2} \sigma^2 + v \sigma + \frac{1}{2} \phi_a^2 - v X_2 \right) \\
+ \bar{c}^* \left( \frac{1}{2} \sigma^2 + v \sigma + \frac{1}{2} \phi_a^2 - v X_2 \right) \].

Antifield

Slavnov-Taylor identity

\[ \int d^4 x \left( v c \frac{\delta \Gamma}{\delta X_1} + \frac{\delta \Gamma}{\delta \bar{c}^*} \frac{\delta \Gamma}{\delta \bar{c}} \right) = 0 \]
Propagators

The quadratic part is diagonalized by \( \sigma = \sigma' + X_1 + X_2 \)

\[
\begin{align*}
\Delta_{\sigma'\sigma'} &= \frac{i}{p^2}, & \Delta_{\phi_a\phi_b} &= \frac{i\delta_{ab}}{p^2}, & \Delta_{\bar{c}c} &= \frac{i}{p^2} \\
\Delta_{X_1X_1} &= -\frac{i}{p^2}, & \Delta_{X_2X_2} &= \frac{i}{p^2 - M^2}.
\end{align*}
\]

The derivative interaction only depends on \( X = X_1 + X_2 \), whose propagator has an improved UV behaviour

\[
\Delta_{XX} = \frac{iM^2}{p^2(p^2 - M^2)}
\]

Thus the derivative interaction is harmless and p.c. renormalizability still holds.
Physical states

They are described by the standard BRST construction of the physical Hilbert space in terms of the asymptotic charge $Q$

$$\mathcal{H}_{phys} = \ker Q / \text{Im } Q$$

The scalar singlet is physical:

$$X_1, \sigma' \notin \mathcal{H}_{phys}$$

$$X_2 \in \mathcal{H}_{phys}$$

$$c = \{Q, X_1\}, \quad [Q, \bar{c}] = \sigma' + X_1$$

The only physical degrees of freedom are the scalar $X_2$ and the $\phi_a$ components of the SU(2) doublet
Functional identities & Renormalization

Ghost and antighost equation
\[
\frac{\delta \Gamma}{\delta \bar{c}} = -\Box c, \quad \frac{\delta \Gamma}{\delta c} = \Box \bar{c}
\]

\(X_1\) equation
\[
\frac{\delta \Gamma}{\delta X_1} = \frac{1}{v} \frac{\delta \Gamma}{\delta \bar{c}^*}
\]

\(X_2\) equation & shift symmetry
\[
\frac{\delta \Gamma}{\delta X_2(x)} = \frac{1}{v} \frac{\delta \Gamma}{\delta \bar{c}^*} + \Box (X_1 + X_2) - M^2 X_2 - v \bar{c}^*
\]

Global SU(2) invariance
\[
\int d^4x \left[ -\frac{1}{2} \alpha_a \phi_a \frac{\delta \Gamma}{\delta \sigma} + \left( \frac{1}{2} \sigma + v \right) \alpha_a + \frac{1}{2} \epsilon_{abc} \phi_b \alpha_c \frac{\delta \Gamma}{\delta \phi_a} \right] = 0.
\]
Stability of the theory

The ghosts remain free fields also at the quantum level.

The $X_1$ and $X_2$ equations fix the amplitudes involving at least one $X_1$ and $X_2$ fields in terms of amplitudes without. The latter have a good UV behaviour.

Global SU(2) invariance is a symmetry of the theory.
**UV indices**

<table>
<thead>
<tr>
<th>Field/ext. source</th>
<th>UV index</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\bar{c}^*$</td>
<td>2</td>
</tr>
<tr>
<td>$\sigma'$</td>
<td>1</td>
</tr>
<tr>
<td>$\phi_a$</td>
<td>1</td>
</tr>
</tbody>
</table>

There is no power-counting with positive UV index on the physical singlet and the auxiliary field $X_1$, however they are controlled by the functional identities in terms of ancestor amplitudes with external legs with positive UV dimensions.
Counterterms for ancestor amplitudes

\[ \mathcal{L}_{ct} = \mathcal{L}_{ct,1} + \mathcal{L}_{ct,2} \]

\[ \mathcal{L}_{ct,1} = -\mathcal{Z}\left(\frac{1}{2}\partial^\mu \sigma \partial_\mu \sigma + \frac{1}{2}\partial^\mu \phi_a \partial_\mu \phi_a \right) - \mathcal{M}\left(\frac{1}{2}\sigma^2 + v\sigma + \frac{1}{2}\phi_a^2 \right) \]

\[ -\mathcal{G}\left(\frac{1}{2}\sigma^2 + v\sigma + \frac{1}{2}\phi_a^2 \right)^2 - \mathcal{R}_1 \bar{c}^* \left(\frac{1}{2}\sigma^2 + v\sigma + \frac{1}{2}\phi_a^2 \right). \]

\[ \mathcal{L}_{ct,2} = -\mathcal{R}_2 \bar{c}^* - \frac{1}{2} \mathcal{R}_3 (\bar{c}^*)^2 \]
SM with the Derivative Representation of the Higgs potential

The coupling to the gauge fields and the fermions happens through the doublet

\[ \partial_\mu \Phi^\dagger \partial^\mu \Phi \rightarrow D_\mu \Phi^\dagger D^\mu \Phi \]

\[ -G_Y \bar{\psi}_L \Phi \psi_R - \tilde{G}_Y \bar{\psi}_L \psi_R \Phi^C + \text{h.c.} \]

The mass term for \( X_2 \), the \((X_1+X_2)\)-dependent sector as well as the ghosts and antighosts associated with the SU(2) constraint are left unchanged.
Custodial Symmetry

Still true in the derivative formulation of the Higgs potential

$$\Omega = (\Phi^C, \Phi)$$

$$\frac{1}{2} \sigma^2 + v \sigma + \frac{1}{2} \phi_a^2 = \frac{1}{2} \text{Tr} \ (\Omega^\dagger \Omega)$$

By requiring $X_1, X_2$ to be invariant under global $\text{SU}(2)_L \times \text{SU}(2)_R$
we recover the usual custodial symmetry

$$\Omega \rightarrow L \Omega R^\dagger$$

in the limit where the hypercharge coupling vanishes.
SU(2) x U(1) x U(1)_{\text{constr}} \text{ BRST symmetry}

The full BRST symmetry of the theory is

\[ s = s_0 + s_1 \]

\( s_0 \) is the SU(2)xU(1) BRST symmetry of the SM
\( s_1 \) is the U(1)_{\text{constr}} BRST symmetry

\[ s_0 \chi = 0 \quad \text{for } \chi \in \{X_2, X_1, \bar{c}, c\} \]
\[ s_1 \chi = 0 \quad \text{on gauge fields, fermions and the } \Phi \text{ doublet} \]

\[ s^2 = 0, \quad s_0^2 = s_1^2 = 0, \quad \{s_0, s_1\} = 0 \]
The $\mathbb{Z}$-model

The $X_2$ equation is not the most general functional symmetry holding true for the vertex functional.

The breaking term on the R.H.S. of the shift symmetry stays linear in the quantum fields even if one adds a kinetic term for the scalar singlet

\[ \int d^4 x \frac{z}{2} \partial^\mu X_2 \partial_\mu X_2 \]

Upon integration over the auxiliary field this is equivalent to the addition of the dimension-six operator

\[ \int d^4 x \frac{z}{v^2} \partial_\mu \Phi^\dagger \Phi \partial^\mu \Phi^\dagger \Phi \]
Modified identities and propagators

The $X_2$ equation changes

$$\frac{\delta \Gamma}{\delta X_2(x)} = \frac{1}{v} \Box \frac{\delta \Gamma}{\delta \bar{c}^*} + \Box (X_1 + (1 - z)X_2) - M^2 X_2 - v\bar{c}^*$$

The $X_2$ propagator becomes

$$\Delta_{x_2x_2} = \frac{i}{(1 + z)p^2 - M^2}$$

and p.c. renormalizability is lost, as a consequence of the worsened UV behaviour of the $X$ field

$$\Delta_{XX} = \frac{i(-zp^2 + M^2)}{p^2[(1 + z)p^2 - M^2]}$$
$z$-dependence of the vertex functional

$$\frac{\partial \Gamma}{\partial z} = \int d^4x \frac{\delta \Gamma}{\delta R(x)}$$

$R(x)$ coupled to $-\frac{1}{2} X_2 \Box X_2$

Thus we can expand the non p.c.-renormalizable theory around $z=0$ in terms of insertion of the kinetic operator for $X_2$ at zero momentum.

**FIG. 1:** Mass insertions (the dots) on a $X_2$ propagator (drawn as a solid line)
z-dependence from tree-level Feynman rules

Rescale the $X_2$ lines by a factor $1/(1+z)$
and redefine the mass through

$$M^2 \rightarrow M^2/(1+z)$$

Side remark:

No power-counting renormalizability,
however no freedom to add finite counterterms
vanishing at $z=0$
(otherwise further sources of $z$-dependence would be added)
Application: anomalous trilinear Higgs coupling

G. Degrassi, P. Giardino, F. Maltoni, D. Pagani
arXiv:1607.04251

\[ V_{H^3} = \lambda_3 v H^3 \]

induced e.g. by an anomalous dim. 6 operator \((\Phi^\dagger \Phi)^3\)

Mixing with \(\partial_\mu (\Phi^\dagger \Phi) \partial^\mu (\Phi^\dagger \Phi)\)?
Gauge invariance?
NLO corrections

According to arXiv:1607.04251 there are two types of corrections:

\[ \Sigma_{NLO} = Z_H \Sigma_{LO}(1 + \kappa \lambda C_1) \]

\( Z_H \) is related to the wave function correction associated with the diagram

\( C_1 \) is a process-dependent coefficient arising from the interference from one-loop virtual EW corrections
Examples of EW vertex corrections

Processes involving massive vector bosons

Contributions to $t\bar{t}H$
The X2-equation in the presence of a cubic interaction

\[ g_6 X_2^3 \]

One needs one more external source \( L \) to define the X2-equation in the presence of a cubic interaction

\[ \int d^4 x \, L X_2^2 \]

The X2-equation becomes valid to all orders in the loop expansion

\[ \frac{\delta \Gamma}{\delta X_2} = \frac{1}{v} \Box \frac{\delta \Gamma}{\delta \bar{c}^*} + 3g_6 \frac{\delta \Gamma}{\delta L} + \Box (X_2 + 1(1 - z)X_2) - M^2 X_2 - v \bar{c}^* + 2LX_2 \]

valid to all orders in the loop expansion
The two-point function

\[
\Gamma_{X_2(-p)X_2(p)}^{(j)} = \frac{1}{v^2} p^4 \Gamma_{\bar{c}^*(-p)\bar{c}^*(p)}^{(j)} - \frac{3g_6}{v} \Gamma_{L(-p)\bar{c}^*(p)}^{(j)} - \frac{3g_6}{v} p^2 \Gamma_{L(p)\bar{c}^*(-p)}^{(j)} + 9g_6^2 \Gamma_{L(-p)L(p)}^{(j)}
\]

\[
\sim_{j=1} \left( \partial_\mu X_2 \partial^\mu X_2 \right)^2
\]

At one loop level (j=1) the amplitudes involving the external sources on the r.h.s. are logarithmically divergent. One generates divergences for monomials involving \(X_2^2\) and up to four derivatives.
Basis dependence

The form of the $X^2$-equation depends on the basis. It simplifies if one uses

$$X = X_1 + X_2, \quad X_2 \sigma$$

$$\frac{\delta \Gamma}{\delta X_2} = 3g_6 \frac{\delta \Gamma}{\delta L} - \Box X - M^2 X_2 - z \Box X_2 - v \tilde{c}^*$$

so that one has for the two-point function

$$\Gamma^{(j)}_{X_2(-p)X_2(p)} = 9g_6^2 \Gamma^{(j)}_{L(-p)L(p)} \sim_{j=1} X_2^2$$

However one needs to consider mixed propagators

$$\Delta_{XX_2}, \Delta_{\sigma X_2}$$

and mixed 1-PI Green’s functions $\Gamma^{(j)}_{\sigma X_2}, \Gamma^{(j)}_{XX_2}$

yielding back the divergences described before
Vertex functions

Anomalous coupling associated with $g_6X_2^3$

SM contribution controlled by the derivative interaction and the sigma-dependent part of the action

At the one-loop level diagrammatic (gauge-invariant) separation of the SM and BSM part in each process

Sample SM and BSM diagrams
UV divergences in vertex functions

Again controlled by the X2-equation

More complicated relations involving external sources insertions

The latter have better UV properties than the field X2

Valid off-shell, to any order in the loop expansion
Outlook

• A particular dimension-six operator has special UV properties in the HEFT extensions of the SM

• The latter properties are transparent in the derivative Higgs interaction formalism (application to the study of anomalous interactions)

• The non-power-counting renormalizable theory is constructed as an expansion around the p.c. renormalizable theory at $z=0$