Comparative analysis of Large $N_c$ QCD and Quark model approaches to baryons

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Introduction

- Description of baryons
  - Large $N_c$ QCD: $N_c \to \infty$, model-independent
  - Group theory

- Quark Model: model-dependent
  - Hamiltonian dynamics

- Compatibility of both approaches

- Light and heavy baryons
Light baryons


Large $N_c$ expansion (I)

- When $N_c \to \infty$, exact $SU(2N_f)$ symmetry
  - Baryons: $N_c$ quarks
- Large but finite $N_c$
  - $SU(2N_f)$ broken, $1/N_c$ expansion
- Mass formula $M = \sum_i c_i \hat{O}_i$
  - Some operators
    \begin{align*}
    \hat{O}_1 &= N_c \mathbf{1} \\
    \hat{O}_2 &= \frac{1}{N_c} \ell^i S^i \\
    \hat{O}_4 &= \frac{1}{N_c} S^i S^i
    \end{align*}
  - $1/N_c^2$ neglected
  - $c_i$ to be fitted. Contain the QCD dynamics.
Large $N_c$ expansion (II)

- Excited baryons
  - Labelled by an integer $K$, quantum of excitation
  - Harmonic oscillator picture
  - $K = 0$ for ground state baryons
  - $P = (-1)^K$
  - $c_i = c_i(K)$

- Ground state baryons ($N$ and $\Delta$)

$$M = c_1 N_c 1 + c_4 \frac{S^2}{N_c} + O(N_c^{-3})$$
Quark model for baryons (I)

- Dominant order: $H = \sum_i \sqrt{\vec{p}_i^2 + m_i^2} + a|\vec{x}_i - \vec{x}_Y|$
  - Spinless Salpeter Hamiltonian
  - Y-junction as long-range potential

**Lattice QCD**


\[ a \approx 0.16 - 0.2 \text{ GeV}^2 \]
Quark model for baryons (II)

- **Light quarks**
  \[ H = \sum_i \sqrt{\vec{p}_i^2} + a|\vec{x}_i - \vec{R}| \]
  - Toricelli point \( \approx \) Center of mass


- **How to get analytical relations?**
  - Auxiliary field technique
    \[ H \rightarrow H(\mu_j, \nu_j) = \sum_j \frac{\vec{p}_j^2}{2\mu_j} + \frac{a^2(\vec{x}_j - \vec{R})^2}{2\nu_j} + \frac{\mu_j}{2} + \frac{\nu_j}{2} \]
  - Elimination
    \[ \delta_{\mu_k} H(\mu_j, \nu_j) = 0, \quad \mu_k = \sqrt{\vec{p}_k^2} \]
    \[ \delta_{\nu_k} H(\mu_j, \nu_j) = 0, \quad \nu_k = a|\vec{x}_k - \vec{R}| \]

- If seen as numbers... Just a harmonic oscillator
Mass formula (I)

- **Y-junction**
  \[
  M_0 = 6\mu_0 = \sqrt{2\pi a(K + 3)}
  \]
  \[
  K = 2(n_1 + n_2) + (\ell_1 + \ell_2)
  \]

- **Short distances: One gluon exchange**

\[
V_{ij}(r_{ij}) = -\frac{2}{3} \frac{\alpha_s}{r_{ij}} + \text{corrections}
\]

- \(\alpha_s \approx 0.2 - 0.4\) remains small once confinement is separated

- In perturbation,
  \[
  \Delta M_{oge} = -\frac{2\alpha_s}{3} \sum_{i<j} \frac{1}{|\vec{x}_i - \vec{x}_j|}
  \approx -\frac{\pi \alpha_s a}{3\sqrt{3}\mu_0}
  \]
Mass formula (II)


\[
\Delta M_{qse} = -\frac{fa}{\pi} \sum_i \frac{\eta(m_i/\delta)}{2\mu_i} \quad f \in [3, 4], \quad \delta \approx 1 \text{ GeV}
\]

\[
\mu_i = \langle \sqrt{\vec{p}_i^2 + m_i^2} \rangle
\]

- **Light quarks**

\[
\Delta M_{qse} = -\frac{fa}{4\mu_0}
\]

- **Squared mass**

\[
M^2 \approx 2\pi a(K + 3) - \frac{4}{\sqrt{3}} \alpha_s - \frac{12}{(2+\sqrt{3})} fa
\]

Excitation number
First comparison

- Spin-independent part
  - Large $N_c$: $M^2 = c_1^2 N_c^2$
  - Quark model: $M^2 = \beta K + \gamma$

  Do we have $c_1^2 = (\beta K + \gamma) / N_c^2$ ?

![Graph showing the comparison between Large $N_c$ and Quark model results.](image)

OK for
- $a = 0.163 \text{ GeV}^2$
- $\alpha_s = 0.4$
- $f = 4.0$

Standard values
Spin-dependent terms

- Corrections in $1/\mu_0^2$ Yu. A. Simonov, hep-ph/9911237

\[ c_2 = \frac{c_2^0}{K+3}, \quad c_4 = \frac{c_4^0}{K+3} \]

- Expected:

\[ c_2^0 \approx 208 \text{ MeV} \]
\[ c_4^0 \approx 1062 \text{ MeV} \]

Small spin-orbit term
Large $N_c$ and strangeness

- $SU(2N_f)$ symmetry with three flavors ($u$, $d$, $s$)
  - Mass formula $M = \sum_i c_i \hat{O}_i + \sum_j d_j \hat{B}_j$
  - $SU(3)$ breaking

- Strange quarks contribution $n_s \Delta M_s = \sum_j d_j \hat{B}_j$

- Classification number $K$ assumed
Quark model with strangeness

- Analytic results at order $m_s^2$
- Mass formula $M = M_q + n_s \Theta \frac{m_s^2}{\mu_0}$
  \[ \Theta = \Theta(K, \ldots) \]
- $K$ is still a good quantum number

OK with $m_s = 243$ MeV
Charm and bottom baryons

C. Semay, F. Buisseret, and Fl. Stancu,
Experimental data

- In 2007-2008: New heavy baryons

<table>
<thead>
<tr>
<th>Nonstrange</th>
<th>Nonstrange</th>
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</thead>
<tbody>
<tr>
<td>$\Lambda_c = 2286.46 \pm 0.14$ MeV,</td>
<td>$\Lambda_b = 5620.2 \pm 1.6$ MeV,</td>
</tr>
<tr>
<td>$\Sigma_c = 2453.56 \pm 0.16$ MeV,</td>
<td>$\Sigma_b = 5811.5 \pm 1.7$ MeV,</td>
</tr>
<tr>
<td>$\Sigma_c^* = 2518.0 \pm 0.8$ MeV,</td>
<td>$\Sigma_b^* = 5832.7 \pm 1.8$ MeV,</td>
</tr>
<tr>
<td>$\Xi_c = 2469.5 \pm 0.3$ MeV,</td>
<td>$\Xi_b = 5792.9 \pm 3.0$ MeV.</td>
</tr>
<tr>
<td>$\Xi'_c = 2576.9 \pm 2.1$ MeV,</td>
<td>$n_s = 1$</td>
</tr>
<tr>
<td>$\Xi_c^* = 2646.4 \pm 0.9$ MeV,</td>
<td></td>
</tr>
<tr>
<td>$\Omega_c = 2697.5 \pm 2.6$ MeV,</td>
<td>$\Omega_b = 6165 \pm 23$ MeV</td>
</tr>
<tr>
<td>$\Omega_c^* = 2768.3 \pm 3.0$ MeV.</td>
<td>$n_s = 2$</td>
</tr>
</tbody>
</table>

One $c$ quark | One $b$ quark
Large $N_c$ and heavy quarks

- Heavy baryon
  - $N_c - 1$ light quarks, $1/N_c$ expansion
  - One heavy quark: $1/m_Q$ expansion
- Mass formula

$$M = m_Q + \Lambda_{qq} + \lambda_q + \lambda_Q$$

\[
\begin{aligned}
\Lambda_{qq} &= c_0 N_c + \frac{c_2}{N_c} \vec{J}_{qq}^2 \\
\lambda_q &= \frac{c_0'}{2m_Q} + \frac{c_2'}{2N_c^2 m_Q} \vec{J}_{qq}^2 \\
m_Q \text{ and } \lambda_Q &= 2 \frac{c_2''}{N_c m_Q} \vec{J}_{qq} \cdot \vec{J}_Q
\end{aligned}
\]
Quark model

- Mass formula with Y-junction
  - Auxiliary fields + $1/ m_Q$ expansion
  
  $$M_{qqQ} = m_Q + 4\mu_1 + \frac{\pi a}{12m_Q} G(K_1, K_2),$$
  
  $$\mu_1 = \sqrt{\frac{\pi a (K_1+K_2+3)}{12}},$$
  
  $$G(K_1, K_2) = \sqrt{2K_2 + 3} \left( \sqrt{2(K_1 + K_2 + 3)} - \sqrt{2K_2 + 3} \right)$$

- Minimal mass for $K_2 = 0, K_1 = K$
- Heavy quark – diquark picture for excited states
- Explanation of $K$ introduced in Large $N_c$ QCD
Back to Regge trajectories

- Heavy baryons
  \[(M - m_Q)^2 \approx \frac{4\pi a}{3} K \approx 1.3\pi a K\]

- Smaller slope than light baryons
  \[M^2 \approx 2\pi a K\]

- Mesons
  - Light \(q\bar{q}\) \(M^2 \approx 2\pi a K\)
  - Heavy \(Q\bar{q}\) \((M - m_Q)^2 \approx \pi a K\)
Additional terms

- OGE
  - $\alpha_s(qq) \neq \alpha_s(Qq)$
  - Simple choice $\alpha_s(Qq) = 0.7 \alpha_s(qq)$
    

- QSE for heavy quark  $\Delta M_{qse} \propto m_Q^{-3} \approx 0$

- Strangeness
  - Power expansion in $m_s^2$
    
    $\Delta M_s = n_s \Theta(K) \frac{m_s^2}{\mu_1}$
Comparison (I)

\[ M = m_Q + c_0 N_c + \frac{c_2}{N_c} J_{qq}^2 + \frac{c_0'}{2m_Q} + \frac{c_2'}{2N_c^2 m_Q} J_{qq}^2 + \frac{2c_2''}{N_c m_Q} \vec{J}_{qq} \cdot \vec{J}_Q \]

\[ M_{qqQ} = m_Q + 4\mu_1 + \cdots + \frac{a}{2m_Q} G(K, K_2 = 0) + \cdots \]

- Matching between the coefficients
  - Spin effects neglected
- Quark model parameters fixed from light baryons
- Heavy quark masses fitted on \( \Lambda_c, \ \Lambda_b \) (\( J_{qq}^2 = 0 \))
Comparison (II)

- $K = 0$

<table>
<thead>
<tr>
<th></th>
<th>Large $N_c$ (MeV)</th>
<th>Quark Model (MeV)</th>
<th>$\delta$ (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_c$</td>
<td>1315</td>
<td>1252</td>
<td>4.7</td>
</tr>
<tr>
<td>$m_b$</td>
<td>4642</td>
<td>4612</td>
<td>0.6</td>
</tr>
<tr>
<td>$c_0$</td>
<td>324</td>
<td>333</td>
<td>2.7</td>
</tr>
<tr>
<td>$c'_0$</td>
<td>96</td>
<td>91</td>
<td>5.2</td>
</tr>
<tr>
<td>$\Delta M_s$</td>
<td>206</td>
<td>170</td>
<td>17.5</td>
</tr>
</tbody>
</table>

- Satisfactory agreement
Conclusion
Summary

- Compatibility between Large $N_c$ mass formula and quark model for light and heavy baryons
  - Support for the quark model assumptions
  - Physical interpretation of the coefficients in Large $N_c$ mass formula
- Dynamical origin of the classification number $K$ understood from quark model
  - Light baryons: total excitation number
  - Heavy baryons: heavy quark-light diquark picture
Outlook

- Future predictions in the heavy baryon sector
  - $K = 1$ → 5 coefficients in the Large $N_c$ formula
    - Can be fitted on experiment BUT…
  - Quark model parameters fitted on ground state heavy baryons
    - Prediction of mass formula coefficients for excited baryons ($K = 1$)

- Masses of excited baryons from a combined Large $N_c$ - Quark model approach, without fit.