Baby-steps beyond rainbow-ladder

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Excited QCD09 – Zakopane, 9/02/09

Motivation

Desire:
- Poincaré covariant description of mesons
- formulated in the continuum
- description in terms of fundamental quantities of QCD

Natural framework:
- Bethe-Salpeter equations
- Schwinger-Dyson equations

Studied in detail for many years
- Rarely extended beyond simplest truncations
- Ad-hoc ‘improvements’ used.
- Severe approximations (e.g. M-N)

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Baby-steps beyond rainbow-ladder

2 / 32
Motivation

Rainbow-Ladder:

- successful description of light-mesons subject to an apposite phenomenological ansatz for the interaction.
- *e.g.* Maris-Tandy model.
  
  [ P. Maris, P. C. Tandy, PRC 60 (1999) 055214 ]

Lacking in many regards:

- no unquenching effects – pion cloud
  

- no $\eta/\eta'$ splitting – $U_A(1)$ anomaly

- admits $\bar{3}_c$ coloured diquark bound-states – but useful in studying Baryons
  
Moreover:
- Describes only pure $q\bar{q}$-states:
  - No flavour mixing
  - No decay channels
  - No exotics

Rainbow-Ladder provides ONLY
- $\gamma_\mu \otimes \gamma_\mu$ couplings
  - Simplicity of interaction means higher spin states of mesons are poorly represented.
  - No variety in attraction/repulsion.
Motivation

Goal:
- Consistent Green’s function approach
- Ghost/Gluon solutions of DSE
- Quark-Gluon vertex beyond $\gamma^\mu$
- BSE kernel satisfying axWTI

“Break the ladder:”
- Unquenching effects
- Leading Yang-Mills corrections
1. Introduction
   - Bethe-Salpeter equations
   - Schwinger-Dyson equations
   - Rainbow-Ladder

2. Quark-gluon vertex
   - Basic structure

3. Beyond rainbow-ladder: Unquenching effects
   - Modelling the pion-cloud

   - Gluonic Corrections

5. Outlook/Conclusions
Outline

1 Introduction
   - Bethe-Salpeter equations
   - Schwinger-Dyson equations
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4 Beyond rainbow-ladder: Yang-Mills sector
   - Gluonic Corrections

5 Outlook/Conclusions
Bethe-Salpeter equations

Bound states:
- poles in $n \geq 3$-point colour singlet Green’s functions

\[
\Gamma_H(p, P) = r_H \frac{\Gamma_h(p, P)}{P^2 + m_H^2} + \text{regular terms}
\]

$\Gamma_h(p, P)$ solves homogeneous Bethe-Salpeter Equation:

Required inputs
- Quark propagator
- Gluon propagator
- Quark-Gluon vertex
- Scattering kernel $K$
## Schwinger-Dyson equations

Basic objects are the propagators of the theory.

### Quark

\[
\langle \bar{\psi}^a \psi^b \rangle \equiv S_F^{ab}(p) = \delta^{ab} \frac{i \phi + M(p^2)}{p^2 + M^2(p^2)} Z_f(p^2)
\]

### Gluon†

\[
\langle A^a_\mu A^b_\nu \rangle \equiv D_{\mu\nu}(p) = \delta^{ab} \left( \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \frac{Z(p^2)}{p^2}
\]

### Ghost

\[
\langle \bar{c}^a c^b \rangle \equiv D_G^{ab}(p) = -\delta^{ab} \frac{G(p^2)}{p^2}
\]

Each satisfy a SDE in terms of higher–Green’s fns.  

(† in Landau gauge)
Basic objects are the propagators of the theory.

### Quark

\[ \frac{-1}{\cdots} = \frac{-1}{\cdots} + \frac{-1}{\cdots} \]

### Gluon (truncated)

\[ \frac{-1}{\cdots} = \frac{-1}{\cdots} - \frac{1}{2} + \frac{-1}{\cdots} \]

### Ghost

\[ \frac{-1}{\cdots} = \frac{-1}{\cdots} + \frac{-1}{\cdots} \]

Each satisfy a SDE in terms of higher–Green’s fns. († in Landau gauge)
Symmetries help constrain system

**Axial-vector WTI**

\[
P_\mu \Gamma_{\frac{a}{5}\mu}^a(k; P) = S^{-1}(k_+) \frac{1}{2} \chi_f^a i \gamma_5 + \frac{1}{2} \chi_f^a i \gamma_5 S^{-1}(k_-) - M_\zeta i \Gamma_{\frac{a}{5}}^a(k; P) - i \Gamma_{\frac{a}{5}}^a(k; P) M_\zeta.
\]

- Symmetry preserving truncation in DSE and BSE
  → preserve Goldstone character of the pion

**BSE**

\[
\Gamma^{(\mu)}_{tu}(p; P) = \lambda(P^2) \int \frac{d^4 k}{(2\pi)^4} K_{tu;rs}(p, k; P) \left[ S(k_+) \Gamma^{(\mu)}(k; P) S(k_-) \right]_{sr}
\]

Solve by introducing an eigenvalue \( \lambda(P^2) \)
Bethe-Salpeter equations
Rainbow-Ladder truncation

Replace quark-gluon vertex by tree-level form.

Quark-gluon vertex

\[ \Gamma_{\nu}^{qg}(p_1, p_2, p_3) = \gamma_{\nu} \frac{Z_2}{\tilde{Z}_3} \Gamma_{YM}(p_3^2) \]

BSE Kernel constructed by considering AVWTI:

Quark scattering kernel

\[ K_{tu;sr}(q, p; P) = g^2 \frac{Z(k^2) \Gamma_{YM}(k^2) Z_{1F}}{k^2} \left( \delta_{\mu\nu} - \frac{k_{\mu}k_{\nu}}{k^2} \right) \left[ \frac{\lambda^a}{2} \gamma_{\mu} \right]_{ts} \left[ \frac{\lambda^a}{2} \gamma_{\nu} \right]_{ru} \]

BSE obtained from quark SDE by the substitution:

\[ \gamma^\mu S(k) \gamma^\nu \longrightarrow \gamma^\mu S(k^-) \Gamma_{M}^{(\rho)}(k; P) S(k^+) \gamma^\nu \]
Bethe-Salpeter equations

Rainbow-Ladder truncation

Pictorially

-1

= 

-1

YM

Satisfies AV-WTI

Reproduces:
- masses of light pseudoscalar, vectors
- leptonic decay constants
- electromagnetic form factors, pion charge radius.
Moving beyond Rainbow-Ladder:
- akin to looking for a pot of gold at the end of a *rainbow*

Technically very challenging:
- Coupled integral equations
- Must preserve symmetries
- Computationally involved:
  - Calculate input Green’s functions
  - Solve normalisation conditions.

Want:
- Unquenching (quark-loops)
- Yang-Mills corrections
Be more humble and ask for some pi at the end of our rainbow.

- Arises from unquenching (pion-cloud)
- Hadronic contribution (decay widths?)
- Additional tensor structure → beyond the rainbow

Challenging, but tractable within further simplifying approximations.
Also mandatory to think about additional contributions from Yang-Mills sector.

- Use of *ab initio* quantities to determine:
  - gluon propagator
  - quark-gluon interaction

**model determined dynamically**

- non vector-vector couplings

**richer pattern of chiral symmetry breaking exhibited by meson masses**
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1 Introduction
   • Bethe-Salpeter equations
   • Schwinger-Dyson equations
   • Rainbow-Ladder

2 Quark-gluon vertex
   • Basic structure

3 Beyond rainbow-ladder: Unquenching effects
   • Modelling the pion-cloud

4 Beyond rainbow-ladder: Yang-Mills sector
   • Gluonic Corrections

5 Outlook/Conclusions
**Quark-Gluon vertex**

**Quark diagram** Hadronic contributions


**Ghost diagram** Infrared leading

- [R. Alkofer, C. Fischer, F. Llanes-Estrada, MPL A 23, 1105 (2008)]

For all scales vanishing symmetrically, exhibit power law solutions

$$\Gamma^{n,m,l} \sim \left( \frac{p^2}{\Lambda_{QCD}^2} \right)^{(n-m)\kappa - l/2}$$
Quark-Gluon vertex

Quark-Gluon vertex

= + + + + ~

Quark-Gluon vertex – leading skeleton expansion

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Vector and scalar dressing functions

\[ \Gamma^\mu(p_1, p_2) = \sum_{k=1}^{12} \lambda_k(p_1, p_2)L^\mu(p_1, p_2) \]


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5. Outlook/Conclusions
First steps

1. Subsume all Yang-Mills corrections into a vertex dressing
   - form inspired by quark-gluon vertex calculations
   - scales left free (but constrained) for parameter fitting

2. Gluon propagator obtained from SDE solutions

3. Focus on and quantify hadronic effects.

For investigatory purposes we simplify the truncation further:

- Gives idea of necessary difficulty
- Allows techniques to be refined:
  - Solving quark in the complex plane (Euclidean space)
  - Normalisation condition for non-trivial Kernel.
Beyond rainbow-ladder: Unquenching effects

Hadronic Quenching Effects (Pion Cloud)
Modelling the YM part

Yang-Mills part of quark-gluon interaction $\Gamma_\mu$:
- $Z(k^2)\Gamma_{YM} \sim \alpha(k^2)$: for large momenta
- $\Gamma_{YM} \sim (k^2)^{-1/2-\kappa}$: IR soft-singularity in gluon momentum.


- For consistency with axWTI, use $\Gamma_\mu \sim \Gamma_{YM}\gamma_\mu$

Soft-Divergence

$$\Gamma_{YM}(k^2) = \left(\frac{k^2}{k^2 + d_2}\right)^{-1/2-\kappa} \times \left[ \frac{d_1}{d_2 + k^2} + \frac{k^2 d_3}{d_2^2 + (k^2 - d_2)^2} + \frac{k^2}{\Lambda^2_{QCD} + k^2} \right] \times \left[ \frac{4\pi}{\beta_0 \alpha_\mu} \left( \frac{1}{\log\left(\frac{k^2}{\Lambda^2_{QCD}}\right)} - \frac{\Lambda^2_{QCD}}{k^2 - \Lambda^2_{QCD}} \right)^{-2\delta} \right]$$
Yang-Mills part of quark-gluon interaction $\Gamma_\mu$:
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**Soft-Divergence**

![Graph showing soft-divergence comparison between Our Model and Maris-Tandy](graph.png)
Beyond rainbow-ladder: Unquenching effects

Modelling the pion-cloud

Quark-Gluon vertex - Hadronic unquenching

**Truncation**

\[
\begin{align*}
\text{Vertex} &= \text{Vertex} + \text{Vertex} + \text{Vertex} + \text{Vertex} + \text{Vertex} \\
\text{Vertex} &= \text{Vertex} + \text{Vertex} + \text{Vertex} + \text{Vertex} + (\ldots)
\end{align*}
\]

\[
\begin{align*}
\text{Vertex} &= \text{Vertex} - 1 - 1 \\
\text{Vertex} &= \text{Vertex} - 1 - 1 \\
\text{Vertex} &= \text{Vertex} - 1 - 1
\end{align*}
\]
AxWTI satisfied in $\chi$-limit:

$$P_\mu \Gamma^{a\mu}_5(k; P) = S^{-1}(k_+) \frac{1}{2} \lambda^a_i i \gamma_5 + \frac{1}{2} \lambda^a_i i \gamma_5 S^{-1}(k_-)$$

$$- M_\zeta i \Gamma^{a}_5(k; P) - i \Gamma^{a}_6(k; P) M_\zeta .$$

Generalised GMOR relation well-satisfied:

$$f_\pi m^2_\pi = r_\pi \left( m_u(\mu^2) + m_d(\mu^2) \right) .$$
Pion-cloud effects in light mesons

Simple off-shell prescription

\[ \Gamma_{\pi}^{j}(p; P) = \tau^{j} \gamma_{5} \frac{B_{\chi}(p^{2})}{f_{\pi}} \]
Beyond rainbow-ladder: Unquenching effects

Modelling the pion-cloud

Pion-cloud effects in light mesons

Normalisation

\[
\delta^{ij} = 2 \frac{\partial}{\partial P^2} \text{tr} \int \frac{d^4 k}{(2\pi)^4} \left[ 3 \left( \bar{\Gamma}^i_{\pi}(k, -Q) S(k + P/2) \Gamma^j_{\pi}(k, Q) S(k - P/2) \right) + \int \frac{d^4 q}{(2\pi)^4} \left[ \bar{\chi}^i_{\pi} \right]_{sr} (q, -Q) K_{tu;rs}^\text{pion} (q, k; P) \left[ \chi^j_{\pi} \right]_{ut} (k, Q) \right],
\]

Canonical condition:

Demand residue of bound-state in inhomogeneous Bethe-Salpeter equation is equal to unity.
Normalisation

\[ \delta^{ij} = 2 \frac{\partial}{\partial P^2} \text{tr} \int \frac{d^4 k}{(2\pi)^4} \left[ 3 \left( \overline{\Gamma}^i_\pi(k, -Q) S(k + P/2) \Gamma^j_\pi(k, Q) S(k - P/2) \right) + \int \frac{d^4 q}{(2\pi)^4} [\overline{\chi}^i_\pi]_{sr}(q, -Q) K^{\text{pion}}_{tu;rs}(q, k; P) [\chi^j_\pi]_{ut}(k, Q) \right], \]

\[ \delta^{ij} = \frac{\partial}{\partial P^2} \left[ \right] \]
## Results

### Spectrum of light mesons

<table>
<thead>
<tr>
<th></th>
<th>Maris-Tandy w/o pi</th>
<th>Maris-Tandy incl. pi</th>
<th>Our Model w/o pi</th>
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<tbody>
<tr>
<td>$M_\pi$</td>
<td>140</td>
<td>138$^\dagger$</td>
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</tr>
<tr>
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<td>159</td>
<td>162</td>
<td>156</td>
</tr>
<tr>
<td>$M_{a_1}$</td>
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<td>873</td>
<td>1230</td>
</tr>
<tr>
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<tr>
<td>$M_\eta$</td>
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<td>493</td>
<td>497</td>
<td>548</td>
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<tr>
<td>$M_{\eta'}$</td>
<td></td>
<td></td>
<td>949</td>
<td>963</td>
<td>948</td>
</tr>
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*Yang-Mills part of vertex too simple.*
Vector vs. Pseudoscalar mass

Mass plots

- Quenched, $N_f=0$
- Unquenched, $N_f=2$

$M_{\rho}^p$ [MeV]

$(M_\pi)^2$ [MeV$^2$]
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5. Outlook/Conclusions
Gluonic corrections

Quark-Gluon vertex

Consider the approximated system:
Gluonic corrections

Challenges

Solving vertex SDE for real Euclidean Momenta
- Nowadays ROUTINE.
  
  *all basic QCD vertices tackled to date within some approximation*

Bound-states in Euclidean space $\rightarrow P^2 = -M^2$.
- Need both quark propagator and Quark-Gluon vertex for $\mathbb{C}$-momenta
- Only a (very) technical difficulty – surmounted.
Gluonic corrections
Bethe-Salpeter equation

We \textit{really} calculate these two-loop diagrams

No Munczek-Nemirovsky

Axial-Vector WTI preserving truncation.
Gluonic corrections
Simple exploratory model

- Replace three-gluon vertex with tree-level form

\[ \Gamma^{(0)}_{\mu\nu\rho}^{abc}(p, k, q) = g f^{abc} \left( \delta_{\mu\nu} (p - q)_\rho + \delta_{\nu\rho} (q - k)_\mu + \delta_{\rho\mu} (k - p)_\nu \right) \]

- Replace internal quark-gluon vertices with \( \gamma_\mu \).
- Replace gluon with some integrated strength

\[ Z(p^2) = \frac{g^2}{4\pi} \frac{\pi D}{\omega^2} p^4 \exp \left( -\frac{p^2}{\omega^2} \right) \]

NOT representative of what we expect in nature:

- think of effective gluon interaction compensating for lack of internally dressed vertices.
- no ultraviolet support - renormalisation trivial.
Gluonic corrections

Simple exploratory model

- Replace three-gluon vertex with tree-level form

\[
\Gamma_{\mu\nu\rho}^{(0) abc}(p, k, q) = g f^{abc} \left( \delta_{\mu\nu} (p - q)_{\rho} + \delta_{\nu\rho} (q - k)_{\mu} + \delta_{\rho\mu} (k - p)_{\nu} \right)
\]

- Replace internal quark-gluon vertices with \(\gamma_\mu\).
- Replace gluon with some integrated strength
Gluonic corrections
Simple exploratory model – Results

Parameter Set

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Outlook

- Unquenching: Improve pion off-shell prescription
- Inputs: Employ solutions from SDE solutions
- Results: Meson spectrum, EM form factors

Seven down - two to go. But we must still solve . . .
Outlook

Normalisation

$$\delta_{ij} = \frac{\partial}{\partial P^2} \left[ \begin{array}{c}
\text{Diagram 1} \\
\text{Diagram 2} \\
\text{Diagram 3} \\
\text{Diagram 4} \\
\text{Diagram 5} \\
\text{Diagram 6} \\
\text{Diagram 7} \\
\text{Diagram 8} \\
\text{Diagram 9} \\
\end{array} \right]$$

Need in order to determine leptonic decay constants
Conclusions

Summary

Quark-Gluon vertex **critical** object. Contains
- Hadronic unquenching effects
- Yang-Mills corrections

**Demonstrated that effects from the pion-cloud:**
- are generally attractive
- generate effects of right size
- can be successfully modelled in a simple model

**Presented progress on state-of-the-art calculations:**
- incorporation of leading corrections to vertex
- full two-loop calculations - no kinematic restrictions.