Tutorial "General Relativity"

Winter term 2016/2017

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Sheet No. 6

will be discussed on January 31, 2017

1. Newtonian limit

Find the relation between the geodesic equation and the Newtonian equation of motion for a particle moving in a static gravitational field, i.e. show that Newtonian gravity can be described by a metric of the form

$$\mathrm{d}s^2 = \left[1 + \frac{2}{c^2}\phi(\vec{x})\right] c^2 \mathrm{d}t^2 - g_{jk}\mathrm{d}x^j \mathrm{d}x^k \tag{1}$$

where $\phi(\vec{x}) = -\frac{GM}{r}$ is the gravitational potential.

Hint: Use the ansatz $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}(\vec{x})$ with $|h_{\mu\nu}| \ll 1$ for a weak static gravitational field in the equations of motion for a freely falling particle and use the non-relativistic approximation of the equations of motion,

$$\frac{\mathrm{d}^2\vec{x}}{\mathrm{d}t^2} = -\vec{\nabla}\phi(\vec{x}) \tag{2}$$

to find the relation between h_{00} and ϕ.

Remark: Since in a static gravitational field $\mathrm{d}s^2$ should be time independent, it cannot change under time reversal, $t \to -t$, and thus g_{0j} $(j \in \{1,2,3\} = 0)$. Note that in this way, we cannot make any further statement about the spatial components of the metric, g_{jk}, in the non-relativistic limit.

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2. Schwarzschild solution with cosmological term

The cosmological constant has been added by Einstein as a fudge factor. At the time it was believed that the universe was static. The Einstein equations however predict a dynamical universe. When the observations of Hubble proved beyond reasonable doubt that the universe was expanding, Einstein threw out the cosmological constant and claimed it to be *"Die größte Eselei meines Lebens"*.

Recent observations seem to indicate that some sort of "vacuum energy" is at work, so that the cosmological constant is coming back to style. The vacuum Einstein equations with a cosmological constant read

$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} + \Lambda g_{\mu\nu} = 0 \tag{3}$$

What is the influence (consequence) of Λ on the Schwarzschild solution outside the star? To this purpose calculate the *modified* line element using the same ansatz as used in the lecture to derive the Schwarzschild metric given as

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 \exp\nu - \mathrm{d}r^2 \exp\lambda - r^2(\mathrm{d}\vartheta^2 + \mathrm{d}\varphi^2 \sin^2\theta) \tag{4}$$

Is the asymptotic limit for $r \to \infty$ still a Minkowski space-time?
