Hadrons in hot and dense matter III

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Outline

1. Transport theory and hydrodynamics
   - phase-space distribution
   - relativistic Boltzmann equation
   - the Boltzmann H theorem
   - hydrodynamics

2. Transport simulations (UrQMD and GiBUU as examples)
   - GiBUU
   - UrQMD
   - Dalitz decays of hadron resonances
   - Baryon-resonance model at SIS energies

3. Dileptons in pp, pA, and AA collisions at SIS energies

4. Quiz
Transport theory and hydrodynamics
Phase-space distribution

- classical many-body system of relativistic particles
  - all particles are on their mass shell: \( E = E_p := \sqrt{\vec{p}^2 + m^2} \)

- Boltzmann equation \([dvv80, CK02, Hee15]\):
  - dynamical equation for phase-space distribution function \( f(t, \vec{x}, \vec{p}) \)

- relativistic covariance of phase-space distribution
  - \( f(t, \vec{x}, \vec{p}) \) defined as Lorentz scalar quantity
  - particle number \( N \): \( dN = d^3 \vec{x} \ d^3 \vec{p} \ f(t, \vec{x}, \vec{p}) \)
  - particle-number four-vector current \((N^\mu) = (n, \vec{N})\)

\[
N^\mu = \int_{\mathbb{R}^3} d^3 \vec{p} \frac{p^\mu}{E_p} f(t, \vec{x}, \vec{p})
\]

- flow-velocity of fluid cell (“Eckart frame”)

\[
\vec{v}_{Eck}(x) = \frac{\vec{N}(x)}{N^0(x)}, \quad u^\mu_{Eck} = \frac{N^\mu}{\sqrt{N_\mu N^\mu}} = \frac{N^\mu}{n_0}
\]

- \( n_0 \): particle density in local fluid (Eckart) restframe
Relativistic Boltzmann equation

- particles moving along trajectories \((\vec{x}(t), \vec{p}(t))\)
- for infinitesimal time step \(dt\)

\[
dN(t + dt) = f(t + dt, \vec{x} + dt \vec{v}, \vec{p} + dt \vec{F}) d^6 \xi(t + dt), \quad d^6 \xi = d^3 \vec{x} d^3 \vec{p}
\]

- Jacobian for phase-space volume

\[
d^6 \xi(t + dt) = d^6 \xi(t) \det \left( \frac{\partial (\vec{x} + dt \vec{v}, \vec{p} + dt \vec{F})}{\partial (\vec{x}, \vec{p})} \right) = d^6 \xi(t) (1 + dt \vec{\nabla}_p \cdot \vec{F}) + \mathcal{O}(dt^2)
\]

- total change of \(dN\)

\[
dN(t + dt) - dN(t) = d^6 \xi(t) dt \left[ \frac{\partial f}{\partial t} + \frac{\vec{p}}{E} \cdot \frac{\partial f}{\partial \vec{x}} + \frac{\partial (\vec{F} f)}{\partial \vec{p}} \right]
\]
Relativistic Boltzmann equation

- covariance: \( d\tau = dt \sqrt{1 - \vec{v}^2} \) proper time, \( \vec{v} = \vec{p}/E_p, \sqrt{1 - \vec{v}^2} = m/E_p \)

\[
dt \left[ \frac{\partial f}{\partial t} + \frac{\vec{p}}{E} \cdot \frac{\partial f}{\partial \vec{x}} \right] = d\tau \frac{p^\mu}{m} \frac{\partial f}{\partial x^\mu} \Rightarrow \text{covariant!}
\]

- covariant equation of motion for point particle

\[
\frac{dp^\mu}{d\tau} = K^\mu, \quad p_\mu p^\mu = m^2 = \text{const} \Rightarrow
\]

\[
k^0 = \frac{\vec{p}}{E_p} \cdot \vec{K} \Rightarrow \frac{d\vec{p}}{dt} = \vec{F} = \vec{K} \frac{m}{E_p}
\]

\[
\frac{E_p}{m} \vec{\nabla}_p (\vec{F} f) = \frac{\partial}{\partial p^\mu} (K^\mu f) \Rightarrow \text{covariant!}
\]

\[
dN(t + dt) - dN(t) = dt \left[ \frac{\partial f}{\partial t} + \frac{\vec{p}}{E} \cdot \frac{\partial f}{\partial \vec{x}} + \frac{\partial (\vec{F} f)}{\partial \vec{p}} \right] = d\tau \left[ \frac{p^\mu}{m} \frac{\partial f}{\partial x^\mu} + \frac{\partial (K^\mu f)}{\partial p^\mu} \right]
\]
Relativistic Boltzmann equation

- change of particle number due to collisions
- short-range interactions: collisions at one point (local) in space
- invariant cross section

\[
dN_{\text{coll}}(p_1', p_2' \leftarrow p_1, p_2) = d^4 x \frac{d^3 \vec{p}_1}{E_1} \frac{d^3 \vec{p}_2}{E_2} \frac{d^3 \vec{p}_1'}{E_1'} \frac{d^3 \vec{p}_2'}{E_2'} f_1 f_2 W(p_1', p_2' \leftarrow p_1, p_2),
\]

\[
d\sigma = \frac{W(p_1', p_2' \leftarrow p_1, p_2) d^4 x \frac{d^3 \vec{p}_1}{E_1} \frac{d^3 \vec{p}_2}{m} \frac{d^3 \vec{p}_1'}{E_1'} \frac{d^3 \vec{p}_2'}{E_2'}}{d^4 x d^3 \vec{p}_1 v_{\text{rel}} f_1 d^3 \vec{p}_2 f_2},
\]

\[
d\sigma = \frac{d^3 \vec{p}_1'}{E_1'} \frac{d^3 \vec{p}_2'}{E_2'} W(p_1', p_2' \leftarrow p_1, p_2) \frac{1}{I}, \quad I = \sqrt{(p_1 \cdot p_2)^2 - m^4}
\]

- important: \(v_{\text{rel}}\) is velocity of particle 1 in rest frame of particle 2
- from relativistic covariance (or unitarity of \(S\)-matrix!) \(\Rightarrow\) detailed-balance relation

\[
W(p_1', p_2' \leftarrow p_1, p_2) = W(p_1, p_2 \leftarrow p_1', p_2')
\]

- Boltzmann equation (manifestly covariant form)

\[
p^\mu \frac{\partial f}{\partial x^\mu} + m \frac{\partial (K^\mu f)}{\partial p^\mu} = \frac{1}{2} \int_{\mathbb{R}^3} \frac{d^3 \vec{p}_2}{E_2} \int_{\mathbb{R}^3} \frac{d^3 \vec{p}_1'}{E_1'} \int_{\mathbb{R}^3} \frac{d^3 \vec{p}_2'}{E_2'} W(p_1', p_2' \leftarrow p, p_2)(f_1' f_2' - f_1 f_2).
\]

- collision integral: “gain minus loss”
Entropy

- input from quantum mechanics: particle in a cubic box (periodic boundary cond.)
  \[ \vec{p} = \frac{2\pi}{L} \vec{n}, \quad \vec{n} \in \mathbb{Z}^3 \]
- \( \Delta^6 \xi_j = L^3 \Delta^3 \vec{p} \) ("microscopically large, macroscopically small")
- contains \( G_j \) single-particle states (\( g \): degeneracy due to spin, isospin, ...)
  \[ G_j = g \frac{\Delta^6 \xi_j}{(2\pi)^3} \]
- statistical weight for \( N_j \) particles in \( \Delta^6 \xi_j \):
  - factor \( 1/N_j! \): indistinguishability of particles
  \[ \Delta \Gamma_j = \frac{1}{N_j!} G_j^{N_j} \]
- entropy a la Boltzmann
  \[ S = \sum_j \ln \Delta \Gamma_j \approx \sum_j [N_j \ln G_j - N_j(\ln N_j - 1)] \]
  \[ = -\int d^3 \vec{x} \, d^3 \vec{p} \, f(x, p) \{\ln[(2\pi)^3 f(x, p)/g] - 1}\]
The Boltzmann H theorem

- $H = \text{greek Eta: Boltzmann’s notation for entropy}$
- covariant description of entropy: entropy four-flow

$$S^\mu(x) = - \int_{\mathbb{R}^3} \frac{d^3 \vec{p}}{E} p^\mu f(x, p) \{ \ln[(2\pi)^3 f(x, p)/g] - 1 \}$$

- Boltzmann equation + symmetries of $W(p_1' p_2' \leftarrow p_1 p_2)$

$$\frac{\partial S^\mu}{\partial x^\mu} := \zeta = + \frac{1}{4} \int_{\mathbb{R}^3} \frac{d^3 \vec{p}}{E} \int_{\mathbb{R}^3} \frac{d^3 \vec{p}_2}{E_2} \int_{\mathbb{R}^3} \frac{d^3 \vec{p}_1'}{E_1'} \int_{\mathbb{R}^3} \frac{d^3 \vec{p}_2'}{E_2'} f f_2$$

$$\times \left[ \frac{f'_1 f'_2}{f f_2} - \ln \left( \frac{f'_1 f'_2}{f f_2} \right) - 1 \right] W(p_1' p_2' \leftarrow p, p_1) \geq 0$$

- (on average) entropy can never decrease with time!
- equilibrium $\iff S$ maximal!
- bracket must vanish $\Rightarrow$ Maxwell-Boltzmann distribution

$$f_{eq}(x, p) = \frac{g}{(2\pi)^3} \exp \left[ -\beta(x) \left( u(x) \cdot p - \mu(x) \right) \right], \quad p^0 = E = \sqrt{m^2 + \vec{p}^2}$$

- $\beta = 1/T$: inverse temperature, $u$: fluid four-velocity, $\mu$: chemical potential
- temperature, chemical potential are Lorentz scalars!
Hydrodynamics

- in the limit of very small mean-free path: system in local thermal equilibrium
- switch to ideal hydrodynamics description
- forget about “particles” ⇒ fluid description
- equations of motion for $\vec{v}(t, \vec{x})$: conservation laws

$$\partial_\mu N^\mu = 0, \quad \partial_\mu T^{\mu \nu} = 0$$

- $N^\mu$: net-baryon number, $T^{\mu \nu}$: energy-momentum tensor
- ideal hydrodynamics

$$N^\mu = n u^\mu, \quad T^{\mu \nu} = (\epsilon + P) u^\mu u^\nu - P \eta^{\mu \nu}$$

$$\partial_\mu N^\mu = 0, \quad \partial_\mu T^{\mu \nu} = 0$$

- $n$: proper net-baryon density, $\epsilon$: proper energy density, $P$: pressure
- 5 equations of motion, 6 unknowns: $\vec{v}$, $n$, $\epsilon$, $P$
- need also equation of state $\epsilon = \epsilon(P)$
- hadron-resonance gas EoS (low energies)
- lQCD based cross-over phase transition (high energies)
Transport simulations
(UrQMD and GiBUU)
Boltzmann-Uehling-Uhlenbeck (BUU) framework for hadronic transport

reaction types: $pA$, $\pi A$, $\gamma A$, $eA$, $\nu A$, $AA$

open-source modular Fortran 95/2003 code

version control via Subversion

publicly available releases: https://gibuu.hepforge.org

Review on hadronic transport (GiBUU): [BGG+12]

all calculations for dileptons: J. Weil
The Boltzmann-Uehling-Uhlenbeck Equation

- time evolution of phase-space distribution functions

\[
[\partial_t + (\hat{\nabla}_p H_i) \cdot \hat{\nabla}_x - (\hat{\nabla}_x H_i) \cdot \hat{\nabla}_p ] f_i(t, \vec{x}, \vec{p}) = I_{\text{coll}}[f_1, \ldots, f_i, \ldots, f_j]
\]

- use Monte-Carlo simulation for test particles

- transition probability \( W \) in collision term used to define stochastic process ("random numbers" on the computer)

- Hamiltonian \( H_i \)
  
  - selfconsistent hadronic mean fields, Coulomb potential, "off-shell potential"

- collision term \( I_{\text{coll}} \)
  
  - two- and three-body decays/collisions
  - multiple coupled-channel problem
  - resonances described with relativistic Breit-Wigner distribution

\[
\mathcal{A}(x, p) = -\frac{1}{\pi} \frac{\text{Im}\Pi}{(p^2 - M^2 - \text{Re}\Pi)^2 + (\text{Im}\Pi)^2}; \quad \text{Im}\Pi = -\sqrt{p^2}\Gamma
\]

- off-shell propagation: test particles with off-shell potential
Ultra-relativistic Molecular Dynamics (UrQMD)

- transport model for hadrons
  - all hadrons (resonances) with masses up to 2.2 GeV included
  - cross sections adapted to experimental data
  - no explicit medium modifications of hadrons implemented

- quantum molecular dynamics
  - hadrons represented by quantum-mechanical Gaussian wave packets

\[ \psi_i(t, \vec{x}) = \left( \frac{2}{\pi L} \right)^{1/4} \exp \left\{ -\frac{2}{L} [\vec{x} - q_i(t)]^2 + i\vec{p}_i(t) \cdot \vec{x} \right\} \]

- \( N \)-body state = product state (no Bose/Fermi symmetrization!)
- classical equations of motion from Lagrangian

\[ L = \sum_i \left[ -\dot{q}_i \cdot \vec{p}_i + \langle T_i \rangle + \frac{1}{2} \sum_{ij} \langle V_{ij}^{(2)} \rangle - \frac{3}{2Lm} \right] \]

- interaction potentials: similar resonance model as in GiBUU
- all calculations for dileptons: S. Endres
Dalitz decays

- Dalitz decay:
  1 particle $\rightarrow$ 3 particles
- $V$: $\omega \rightarrow \pi + \gamma^* \rightarrow \pi + \ell^+ + \ell^-$
- $P, S$: $\pi, \eta \rightarrow \gamma + \gamma^* \rightarrow \gamma + \ell^+ + \ell^-$
- $R$: Baryon resonances
  $\Delta, N^* \rightarrow N + V \rightarrow N + \gamma^* \rightarrow N + \ell^+ + \ell^-$
- vector-meson dominance
Resonance Model

- reactions dominated by resonance scattering: $a b \rightarrow R \rightarrow c d$
- Breit-Wigner cross-section formula

$$\sigma_{ab\rightarrow R\rightarrow cd} = \frac{2s_R + 1}{(2s_a + 1)(2s_b + 1)} \frac{4\pi}{p_{lab}^2} \frac{s\Gamma_{ab\rightarrow R}\Gamma_{R\rightarrow cd}}{(s - m_R^2)^2 + s\Gamma_{tot}^2}$$

- applicable for low-energy nuclear reactions $E_{kin} \lesssim 1.1$ GeV
- example: $\sigma_{\pi^- p \rightarrow \pi^- p}$ [Teis (PhD thesis 1996), data: Baldini et al, Landolt-Börnstein 12 (1987)]
GiBUU: Resonance Model

- further cross sections

![Graphs showing cross sections for various reactions](image-url)
GiBUU: Extension to HADES energies

- [WHM12, WM13]
- keep same resonances (parameters from Manley analysis)

| rating | \( M_0 \) [MeV] | \( T_0 \) [MeV] | \(|\mathcal{M}^2|/16\pi \) [mb GeV\(^2\)] | \( NR \) | \( \Delta R \) | \( \pi N \) | \( \eta N \) | \( \pi \Delta \) | \( \rho N \) | \( \sigma N \) | \( \pi N^* (1440) \) | \( \sigma \Delta \) |
|--------|----------------|----------------|--------------------------------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| P_{11}(1440) | **** | 1462 | 391 | 70 | — | 69 | — | 22_P | — | 9 | — | — |
| S_{11}(1535) | *** | 1534 | 151 | 8 | 60 | 51 | 43 | — | 2_S + 1_D | 1 | 2 | — |
| S_{11}(1650) | **** | 1659 | 173 | 4 | 12 | 89 | 3 | 2_D | 3_D | 2 | 1 | — |
| D_{13}(1520) | **** | 1524 | 124 | 4 | 12 | 59 | — | 5_S + 15_D | 21_S | — | — | — |
| D_{15}(1675) | **** | 1676 | 159 | 17 | — | 47 | — | 53_D | — | — | — | — |
| P_{13}(1720) | * | 1717 | 383 | 4 | 12 | 13 | — | — | 87_P | — | — | — |
| F_{15}(1680) | **** | 1684 | 139 | 4 | 12 | 70 | — | 10_P + 1_F | 5_P + 2_F | 12 | — | — |
| P_{33}(1232) | **** | 1232 | 118 | OBE | 210 | 100 | — | — | — | — | — | — |
| S_{31}(1620) | ** | 1672 | 154 | 7 | 21 | 9 | — | 62_D | 25_S + 4_D | — | — | — |
| D_{33}(1700) | * | 1762 | 599 | 7 | 21 | 14 | — | 74_S + 4_D | 8_S | — | — | — |
| P_{31}(1910) | **** | 1882 | 239 | 14 | — | 23 | — | — | — | 67 | 10_P | — |
| P_{33}(1600) | *** | 1706 | 430 | 14 | — | 12 | — | 68_P | — | — | 20 | — |
| F_{35}(1905) | *** | 1881 | 327 | 7 | 21 | 12 | — | 1_P | 87_P | — | — | — |
| F_{37}(1950) | **** | 1945 | 300 | 14 | — | 38 | — | 18_F | — | — | — | 44_F |

- production channels in Teis: \( NN \rightarrow N\Delta, NN \rightarrow NN^*, N\Delta^*, NN \rightarrow \Delta\Delta \)
- extension to \( NN \rightarrow \Delta N^*, \Delta\Delta^*, NN \rightarrow NN\pi, NN \rightarrow NN\rho, NN\omega, NN\pi\omega, NN\phi, NN \rightarrow BYK \ (B = N, \Delta, Y = \Lambda, \Sigma) \)
GiBUU Extension to HADES energies

- good description of total pp, pn (inelastic) cross section

- dilepton sources
  - Dalitz decays: \( \pi^0, \eta \rightarrow \gamma \ell^+\ell^-; \omega \rightarrow \pi^0 \ell^+\ell^-\), \(\Delta \rightarrow N \ell^+\ell^-\)
  - \(\rho, \omega, \phi \rightarrow \ell^+\ell^-\): invariant mass \(\ell^+\ell^-\) spectra ⇒ spectral properties of vector mesons
  - for details, see [WHM12]
### UrQMD: Baryon resonances

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<th>Resonance</th>
<th>Mass</th>
<th>Width</th>
<th>$N\pi$</th>
<th>$N\eta$</th>
<th>$N\omega$</th>
<th>$N\phi$</th>
<th>$N\pi\pi$</th>
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<th>$N_{1440}^*\pi$</th>
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<td>$\Delta_{1900}^+$</td>
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<td>260</td>
<td>0.25</td>
<td></td>
<td></td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td></td>
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<tr>
<td>$\Delta_{1905}^+$</td>
<td>1.880</td>
<td>350</td>
<td>0.18</td>
<td></td>
<td></td>
<td>0.80</td>
<td>0.02</td>
<td></td>
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<tr>
<td>$\Delta_{1010}^+$</td>
<td>1.900</td>
<td>250</td>
<td>0.30</td>
<td></td>
<td></td>
<td>0.10</td>
<td>0.35</td>
<td>0.25</td>
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<tr>
<td>$\Delta_{1920}^+$</td>
<td>1.920</td>
<td>200</td>
<td>0.27</td>
<td></td>
<td></td>
<td>0.40</td>
<td>0.30</td>
<td>0.03</td>
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<tr>
<td>$\Delta_{1930}^+$</td>
<td>1.970</td>
<td>350</td>
<td>0.15</td>
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<td></td>
<td>0.22</td>
<td>0.20</td>
<td>0.28</td>
<td>0.15</td>
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<tr>
<td>$\Delta_{1950}^+$</td>
<td>1.990</td>
<td>350</td>
<td>0.38</td>
<td></td>
<td></td>
<td>0.08</td>
<td>0.20</td>
<td>0.18</td>
<td>0.12</td>
<td>0.04</td>
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</table>
UrQMD: Baryon resonances

![Graphs showing p+p → p+p'(1440), p+p → p+p'(1520), p+p → p+p'(1680), and p+p → Δ^+, Δ^0 interactions with data and UrQMD predictions.](image-url)
Dileptons in pp, pA, and AA collisions at SIS energies
GiBUU: $p \ p$ at HADES ($E_{\text{kin}} = 1.25 \text{ GeV}$)

$p + p$ at 1.25 GeV

![Graph showing differential cross section $d\sigma/dm_{ee}$ vs. dilepton mass $m_{ee}$ with data and GiBUU total for various decay processes: $\pi^0 \rightarrow e^+e^-\gamma$, $\Delta \rightarrow N e^+e^-$, and $\rho \rightarrow e^+e^-$.](image-url)
triggered on forward protons → quasifree np scattering
model uncertainties:
- $\rho$ production through $D_{13}(1525)$ (isospin symmetric?)
- $S_{11}(1535)$ [enhanced in np; (from $\eta$ production)]
- d-wave function treatable as quasiclassical “distribution”?
- bremsstrahlung contributions
GiBUU: p p at HADES ($E_{\text{kin}} = 3.5$ GeV)

$p + p$ at 3.5 GeV

Data
GiBUU total
\(\rho \rightarrow e^+e^-\)
\(\omega \rightarrow e^+e^-\)
\(\phi \rightarrow e^+e^-\)
\(\omega^0 \rightarrow \pi^0 e^+e^-\)
\(\pi^0 \rightarrow e^+e^-\gamma\)
\(\eta \rightarrow e^+e^-\gamma\)
\(\Delta \rightarrow N e^+e^-\)

$d\sigma / dm_{ee}$ [$\mu b$/GeV]

Dilepton mass $m_{ee}$ [GeV]
GiBUU: $p \ p$ at HADES ($E_{\text{kin}} = 3.5$ GeV)

**Transverse Momentum $p_T$ [GeV]**

- Data
- GiBUU total
- $\rho \rightarrow e^+e^-$
- $\omega \rightarrow e^+e^-$
- $\phi \rightarrow e^+e^-$
- $\omega^0 \rightarrow \pi^0 e^+e^-$
- $\omega^0 \rightarrow \pi^+ e^-$
- $\eta \rightarrow e^+e^-\gamma$
- $\Delta \rightarrow Ne^-e^-$

**Rapidity $y$**

- $m < 150$ MeV
- $150$ MeV $< m < 470$ MeV
- $470$ MeV $< m < 700$ MeV
- $m > 700$ MeV
GiBUU: “$\rho$ meson” in $pp$

- production through hadron resonances
  \[ NN \rightarrow NR \rightarrow NN \rho, \quad NN \rightarrow N\Delta \rightarrow NN\pi\rho \]

- “$\rho$”-line shape “modified” already in elementary hadronic reactions
- due to production mechanism via resonances
GiBUU: Comparison to old DLS data (pp)

- HADES data consistent with DLS data
- checked by comparing HADES data within DLS acceptance

![Graph showing data comparison](image-url)
GiBUU: Comparison to old DLS data (pd)

- HADES data consistent with DLS data
- checked by comparing HADES data within DLS acceptance
GiBUU: Newest development: $\Delta(1232)$ in VMD model

- so far: $\Delta$-Dalitz decay treated separately from other resonances
- now: treating $\Delta$ as all other resonances via VMD model

$p + p$ at 1.25 GeV

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Hadrons in hot and dense matter III
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GiBUU: Newest development: $\Delta(1232)$ in VMD model

- so far: $\Delta$-Dalitz decay treated separately from other resonances
- now: treating $\Delta$ as all other resonances via VMD model

$d + p$ at 1.25 GeV

**Figure:**

- **dilepton mass $m_{ee}$ (GeV)**
- **differential cross section $d \sigma/dm_{ee}$ [µb/GeV]**

Legend:
- data
- GiBUU total
- $\pi^0 \rightarrow e^+e^-\gamma$
- $\eta \rightarrow e^+e^-\gamma$
- $\Delta$ QED
- $\Delta$ VMD
- $N^*$ VMD
- Brems. OBE

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Hadrons in hot and dense matter III

July 17-21, 2017
GiBUU: Newest development: $\Delta(1232)$ in VMD model

- so far: $\Delta$-Dalitz decay treated separately from other resonances
- now: treating $\Delta$ as all other resonances via VMD model

$p + p$ at 2.2 GeV

Data

$\omega \rightarrow e^+ e^-$
$\omega_\pi \rightarrow e^+ e^-$
$\eta \rightarrow e^+ e^-$
$\Delta_{QED}$
$\Delta_{VMD}$
$N^*_{VMD}$
$\Delta^*_{VMD}$
Brems. OBE

Hendrik van Hees (GU Frankfurt/FIAS)
so far: $\Delta$-Dalitz decay treated separately from other resonances
now: treating $\Delta$ as all other resonances via VMD model

$p + p$ at 3.5 GeV

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UrQMD: p p at HADES ($E_{\text{kin}} = 2.2 \text{ GeV and 3.5 GeV}$)

**p+p @ 3.5 GeV**

HADES Acceptance

$\theta_{\text{ee}} > 9^\circ$, $0.08 < p_{e} < 2.0 \text{ GeV/c}$

**p+p @ 2.2 GeV**

HADES Acceptance

$\theta_{\text{ee}} > 9^\circ$, $0.1 < p_{e} < 1.3 \text{ GeV/c}$
GiBUU: medium effects built in transport model
- binding effects, Fermi smearing, Pauli blocking
- final-state interactions
- production from secondary collisions
- sensitivity to additional in-medium modifications of vector mesons?
GiBUU: p Nb at HADES (3.5 GeV)

- with vacuum spectral functions:

![Graph showing dilepton mass distribution](image)

- p + Nb at 3.5 GeV
- preliminary data
- GiBUU total
- \( \rho \rightarrow e^+e^- \)
- \( \omega \rightarrow e^+e^- \)
- \( \phi \rightarrow e^+e^- \)
- \( \omega^0 \rightarrow \pi^0 e^+e^- \)
- \( \pi^0 \rightarrow e^+e^-\gamma \)
- \( \eta \rightarrow e^+e^-\gamma \)
- \( \Delta \rightarrow N e^+e^- \)
with medium modified spectral functions:

no definite hint for medium modifications in p Nb
GiBUU: p Nb at HADES (3.5 GeV)

- medium effects built in transport model
  - binding effects, Fermi smearing, Pauli blocking
  - final-state interactions
  - production from secondary collisions
- sensitivity on medium effects of vector-meson spectral functions?

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GiBUU: Ar KCl at HADES

GiBUU (vacuum SF)

$\frac{1}{N_{\pi,0}} \frac{dN}{dm_{ee}}$

dilepton mass $m_{ee}$ [GeV]

- Data
- Total
- $\rho \rightarrow e^+e^-$
- $\omega \rightarrow e^+e^-$
- $\phi \rightarrow e^+e^-$
- $\omega^0 \pi^0 e^+e^-$
- $\pi^+ \rightarrow e^+e^+$
- $\eta \rightarrow e^+e^+\gamma$
- $\Delta \rightarrow Ne^+e^-$
- NN Brems.
- $D_{13}(1520)$
- $S_{31}(1620)$
- $\pi\pi$

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Hadrons in hot and dense matter III
July 17-21, 2017
VM in-medium effects

d\sigma/dm_{ee} [mb/GeV]

dilepton mass m_{ee} [GeV]
UrQMD: pp and CC at HADES (lowest energies)

HADES Acceptance
$\theta_{ee} > 9^\circ$, $0.05 < p_e < 1.8 \text{ GeV/c}$

$\pi_0$
$\eta$
$\Delta$
$\rho$
$\omega_{Dalitz}$
$\omega_{direct}$
**Sum**

$p+p @ 1.25 \text{ GeV}$

$C+C @ 1 \text{ AGeV}$

HADES Acceptance
$\theta_{ee} > 9^\circ$, $0.0 < p_e < 2.0 \text{ GeV/c}$

$\pi_0$
$\eta$
$\Delta$
$\rho$
$\omega_{Dalitz}$
$\omega_{direct}$
**Sum**
http://dx.doi.org/10.1016/j.physrep.2011.12.001

http://dx.doi.org/10.1007/978-3-0348-8165-4


http://fias.uni-frankfurt.de/~hees/publ/kolkata.pdf
Bibliography II


Quiz
1. What's the difference between the simulation algorithms used in GiBUU (test-particle Monte Carlo simulation) and in UrQMD (quantum molecular dynamics simulation)?

2. Which is the most important empirical input we need for transport models in low-energy heavy-ion collisions?

3. Why are the $\rho$-meson properties in the particle-data booklet defined solely through reactions like $e^+ + e^- \rightarrow \pi + \pi$ and not with $p + p \rightarrow$ hadrons?

4. What's the fundamental difficulty in making use of (quantitative) many-body-QFT calculations of medium-modified spectral functions?

5. How can one solve this approximately and what are the caveats?