Towards Reggeon Field Theory in QCD

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Perturbation Theory, DIS, Partons, DGLAP, Hard phenomena: jets

$\nwarrow \; \mathbf{Q^2 \to \infty}$

Perturbative QGP, Quark Gluon Plasma

$\nearrow \; \mathbf{T \to \infty}$

QCD

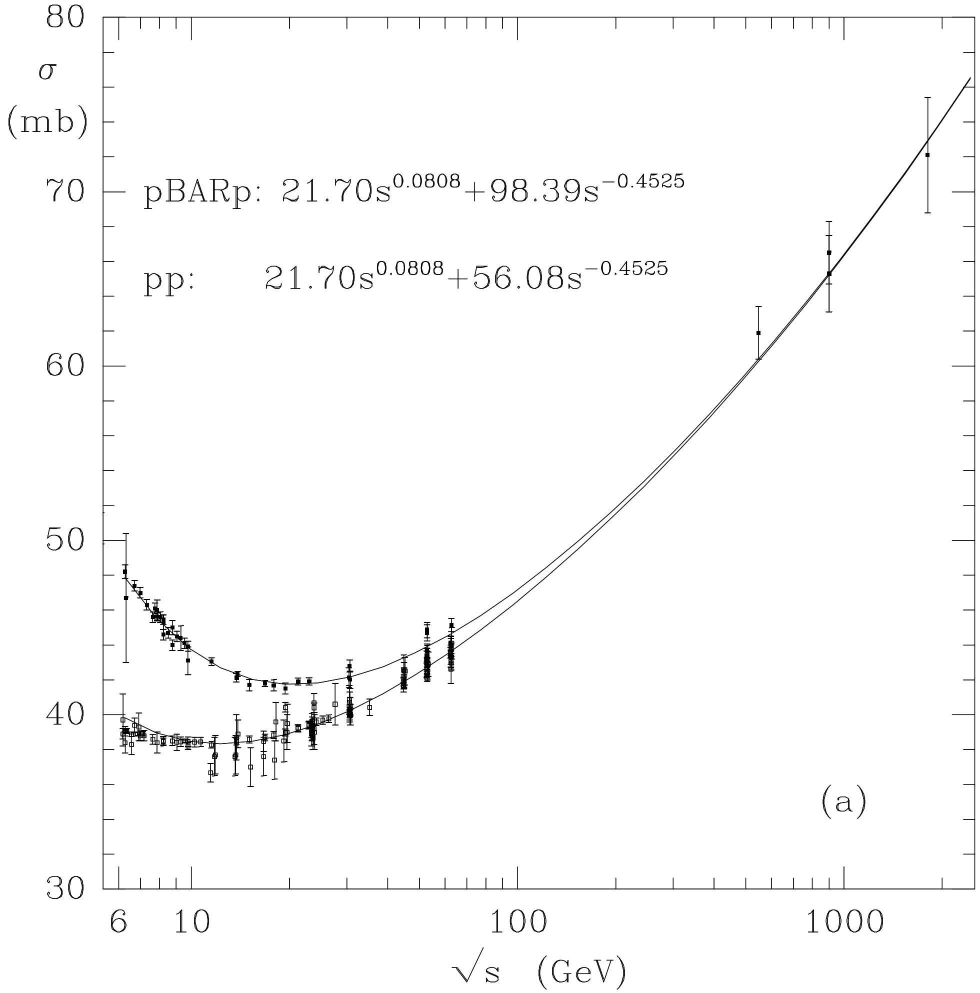
$\swarrow \; \mathbf{m \to 0}$

χ – Lagrangians

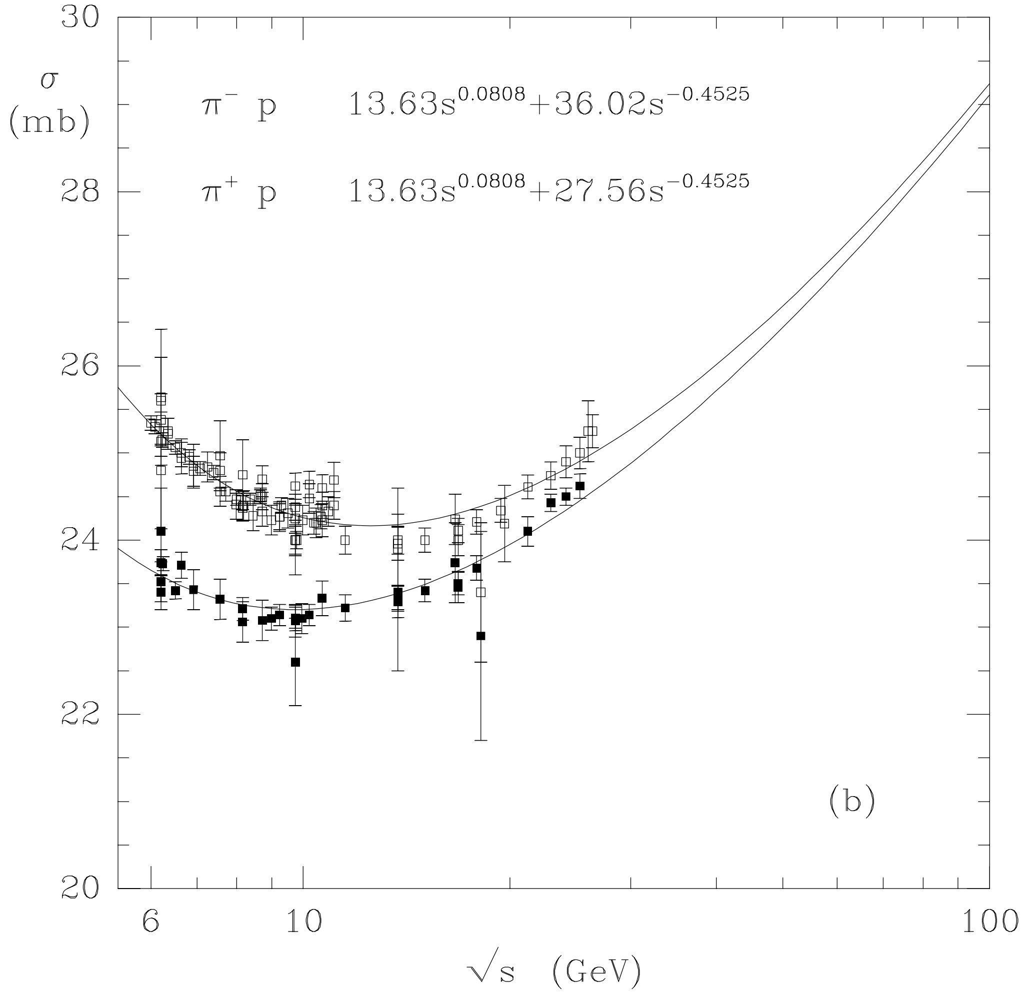
$\searrow \; \mathbf{s \to \infty}$

RFT: All Collider experiments, Cosmic rays, ultra relativistic neutrinos

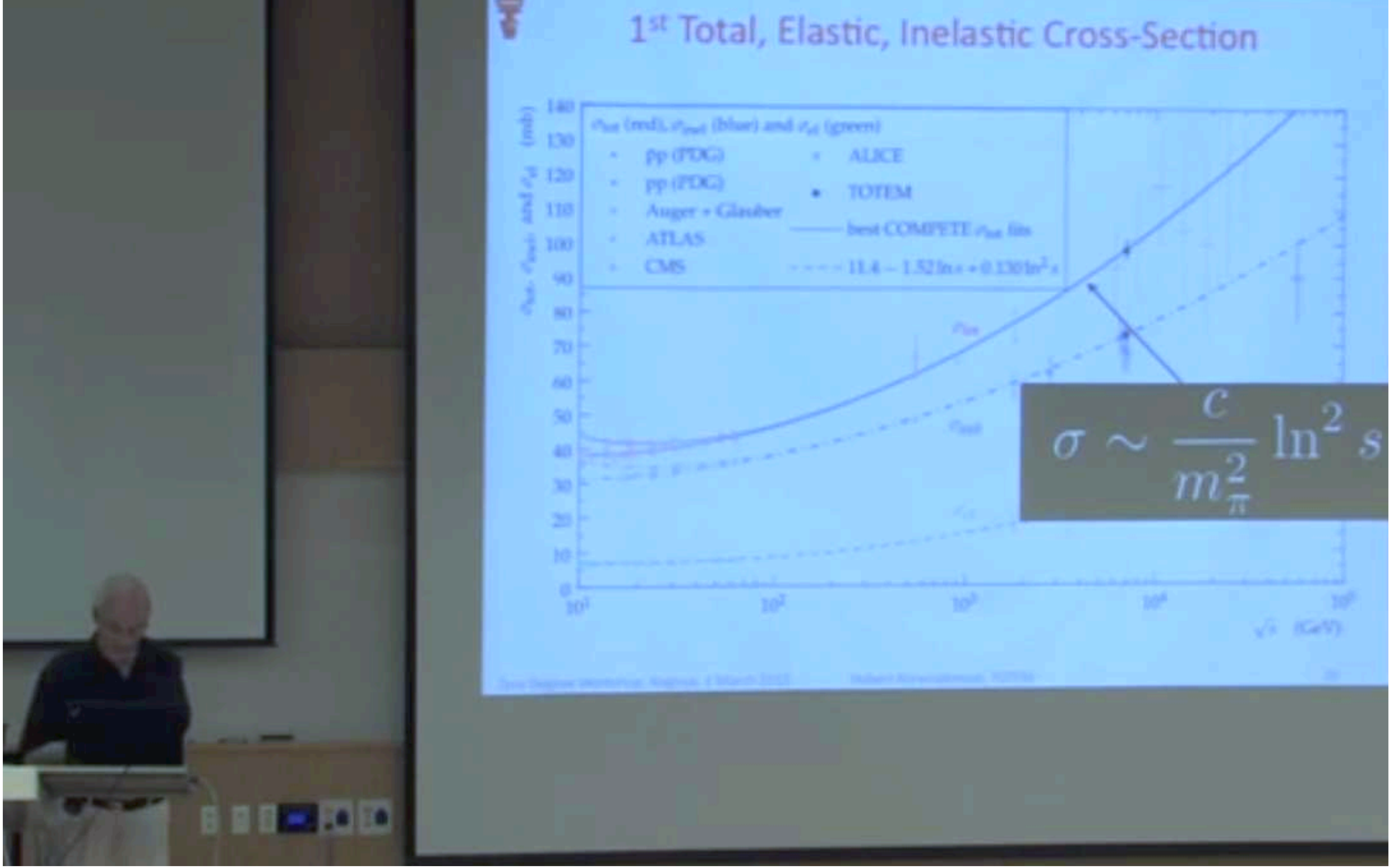
Confinement, String Theory



80
70
60
50
40
30
σ
(mb)
pBARp: 21.70s^0.0808+98.39s^-0.4525
pp: 21.70s^0.0808+56.08s^-0.4525
(a)
6
10
100
1000
√s (GeV)

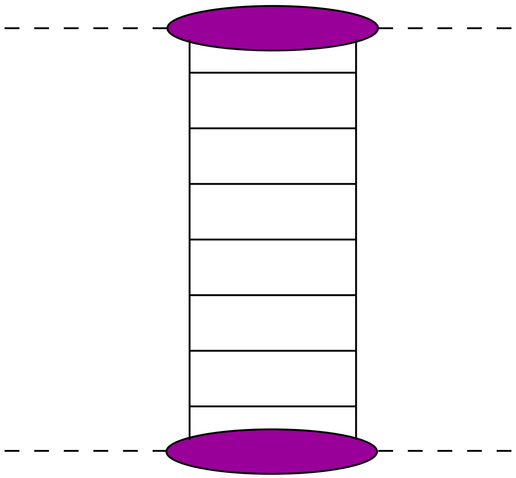


σ
(mb)
π⁻ p   13.63s^0.0808+36.02s^-0.4525
π⁺ p   13.63s^0.0808+27.56s^-0.4525
30
28
26
24
22
20
6
10
100
√s (GeV)
(b)



1st Total, Elastic, Inelastic Cross-Section
p̄p (PDG)
pp (PDG)
Auger + Glauber
ATLAS
CMS
ALICE
TOTEM
best COMPETE fits
11.4 − 1.52 ln s + 0.130 ln² s
$\sigma \sim \frac{c}{m_\pi^2} \ln^2 s$

Pomeron in QCD

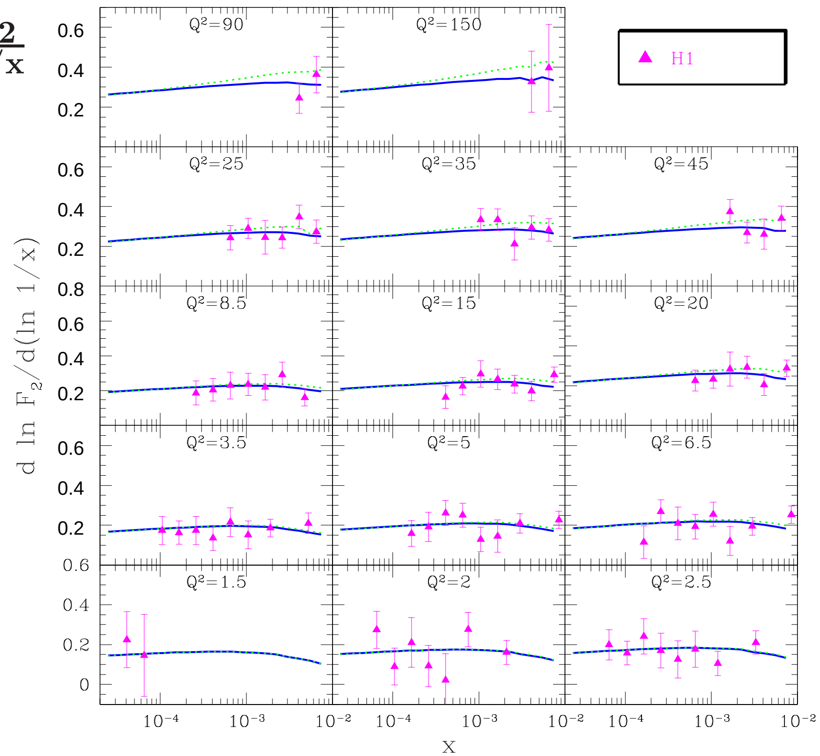


BFKL ladder $(Q^2 \;>>\; \Lambda^2_{QCD})$

$$\sigma \;\sim\; \sum (c\,\alpha_s)^n \ln^n s \sim \; s^{c\,\alpha_s}$$

$c\,\alpha_s \;\simeq\; 0.3 \;-\; 0.5$ (Hard Pomeron)

Seen in data on $\lambda \;=\; \frac{d\;\ln F_2}{d\;\ln 1/x}$



Two important questions:

What is a relation between Soft and Hard Pomerons?

How the scattering theory gets unitarized?

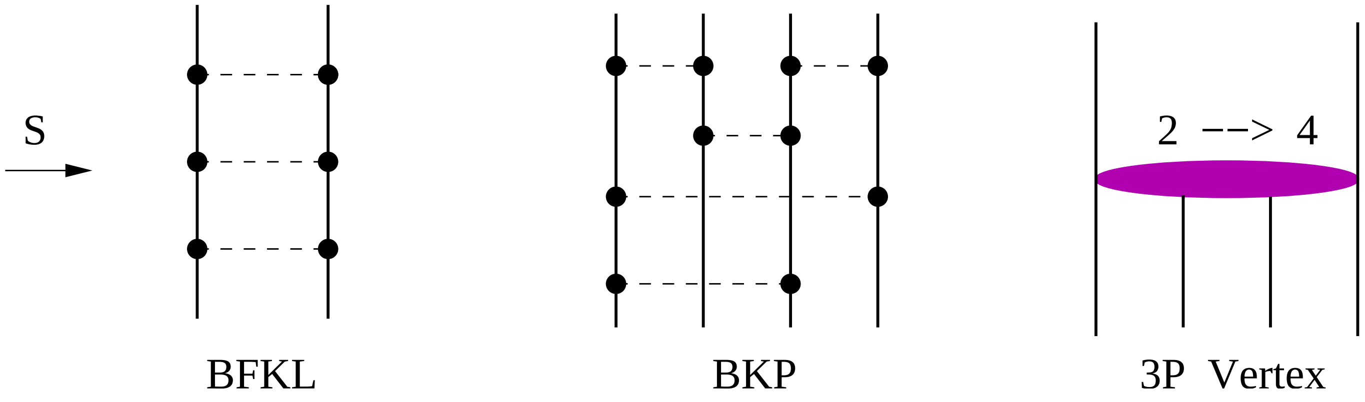
Possible answer: Reggeon Field Theory in QCD

Pomeron interaction vertices in QCD; parton density saturation effects

RFT - History

60' V. Gribov: RFT with supercritical bare Pomeron s^{Δ}, $\Delta > 0$.

70' BFKL (Balitsky, Fadin, Kuraev, Lipatov) ladder - Hard Pomeron, $s^{c\,\alpha_s}$



80' BKP (Bartels, Kwiecinski, Praszalowicz), GLR (L. Gribov, Levin, Ryskin)

90' 3P Vertex (Bartels, Wusthoff, M. Braun), Lipatov's action, Mueller's dipole model,

B-JIMWLK (Balitsky, Jalilian Marian, Iancu, McLerran, Leonidov, Kovner).

since 2005 (A. Kovner and M.L.) JIMWLK+, KLWMIJ, Dense-Dilute Duality (DDD),

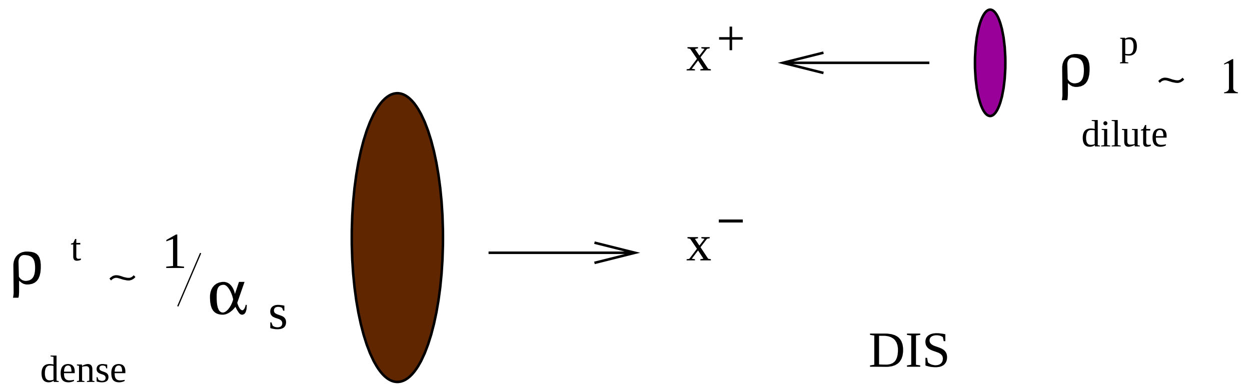
Self-Duality of RFT , Pomeron loops, and much more

Some Major Questions

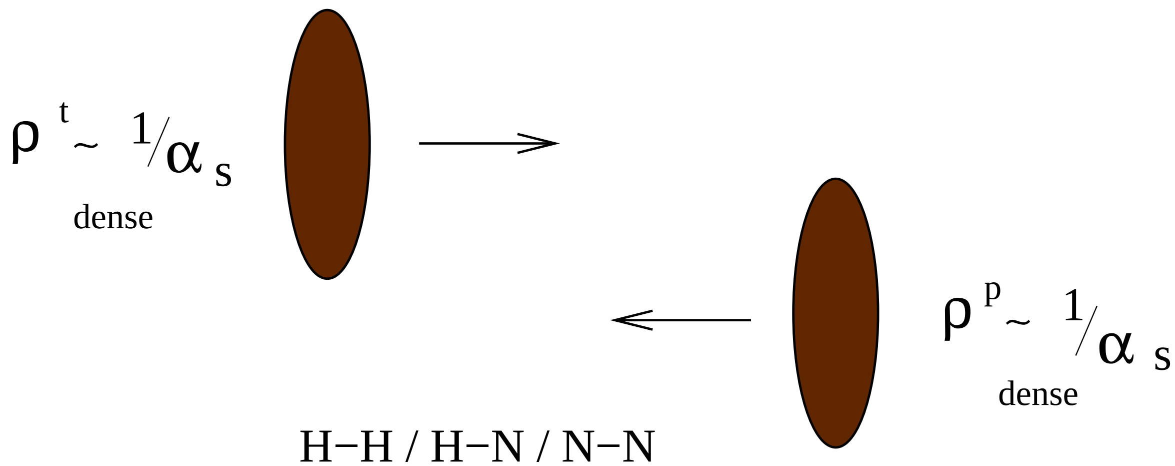
- **How does the unitarity of QCD get manifested in high energy scattering amplitudes?**
- **How do gluon densities grow with energy? Do they saturate? Scales?**
- **What are applicability limits of factorization theorems?**
- **What are final states in collisions of dense objects (jets, multiplicities, correlations)?**
- **How to compute total cross sections?**
- **How to get thermalization in high energy collisions of very dense objects (nuclei)?**

DIS vs. Hadron-Hadron

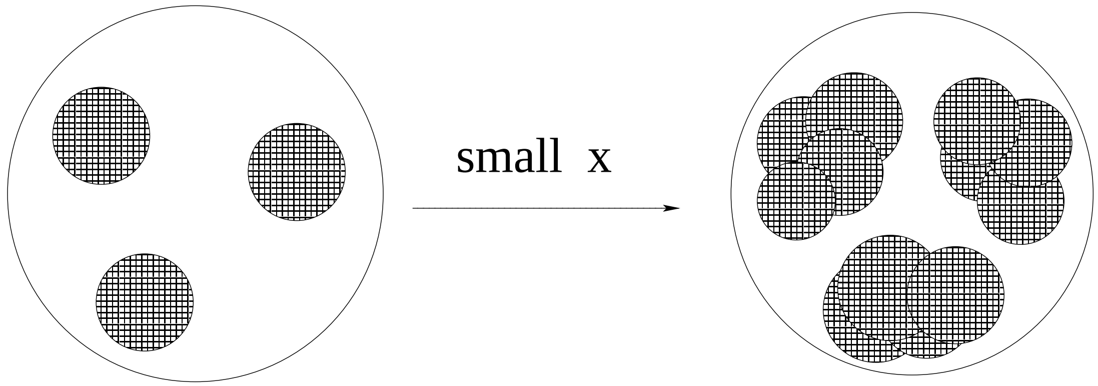
HERA - EIC - LHeC



LHC - RHIC -TeVatron



Dilute regime: $\delta\rho \sim \rho \quad \rightarrow \quad \rho \simeq \mathrm{e}^{\mathrm{c\,Y}}$ **BFKL** $\quad s = \exp[Y] = 1/x$



Evolution is generated by boost. Accelerated (color) charged particles radiate

Fast particles emit softer ones

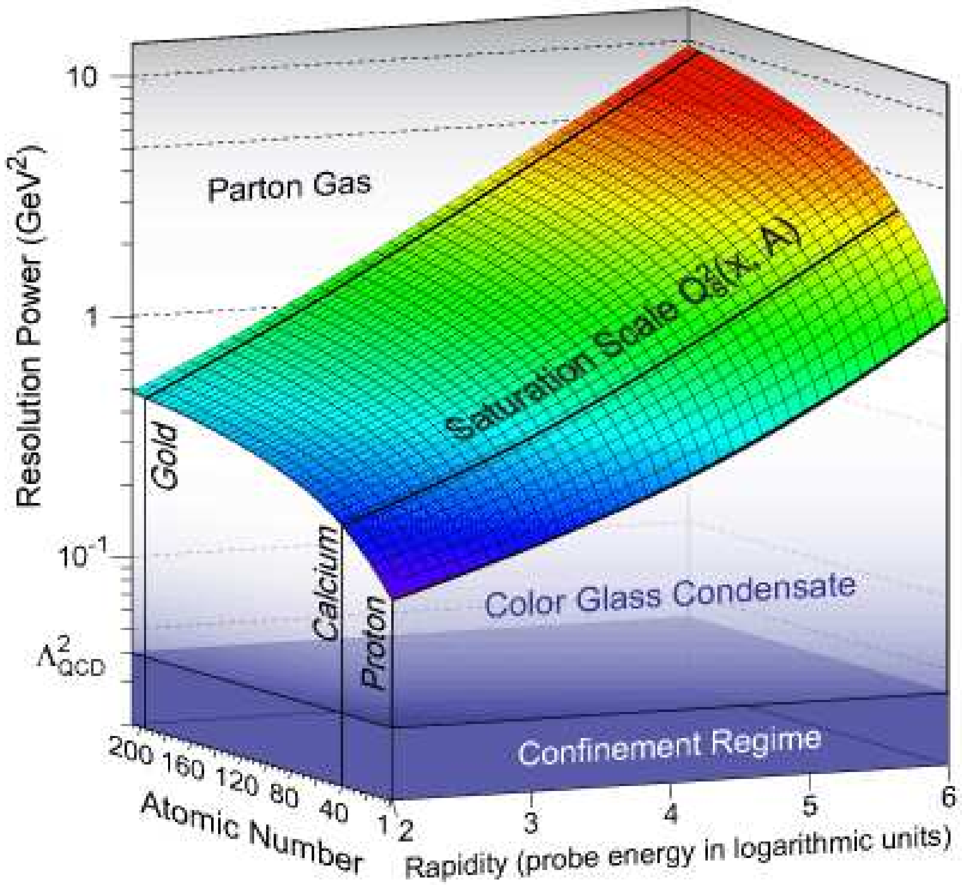
High energy limit = soft gluon emission approximation

Exponential growth of gluon densities leads to unitarity violation.

At high densities the growth should be slowed down due to non-linear effects.

Transition to a non-linear regime is characterized by emergence of a new scale $\mathbf{Q_s}$, known as saturation scale.

$\mathbf{Q_s} \gg \Lambda_{\mathrm{QCD}}$ and perturbative methods are applicable.



Physics is more perturbative.

Classical background fields are strong

Atomic number enhancement

$$\mathbf{Q_s^2 \sim A^{1/3}}$$

motivation for EIC

Saturation occures when the parton density becomes of the order $1/\alpha_s$.
This high density of (color) charges produces strong (non-abelian) classical fields, the Color Glass Condensate.

The situation is somewhat similar to non-linear QED in high intensity lasers.

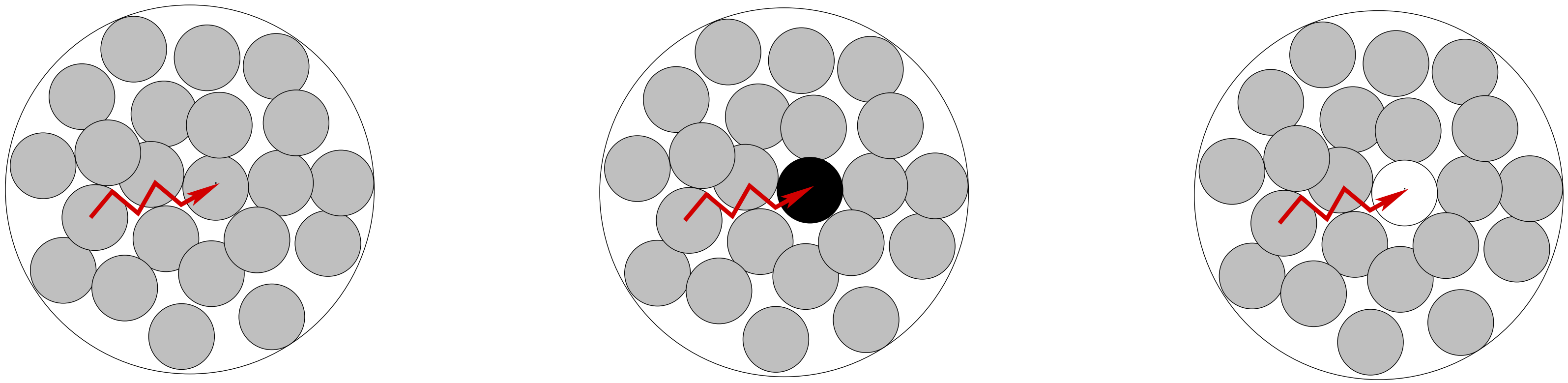
Inside Color Glass Condensate (CGC)

Dense regime:

(1) Hadron is almost black

(2) Emission probability is independent of density

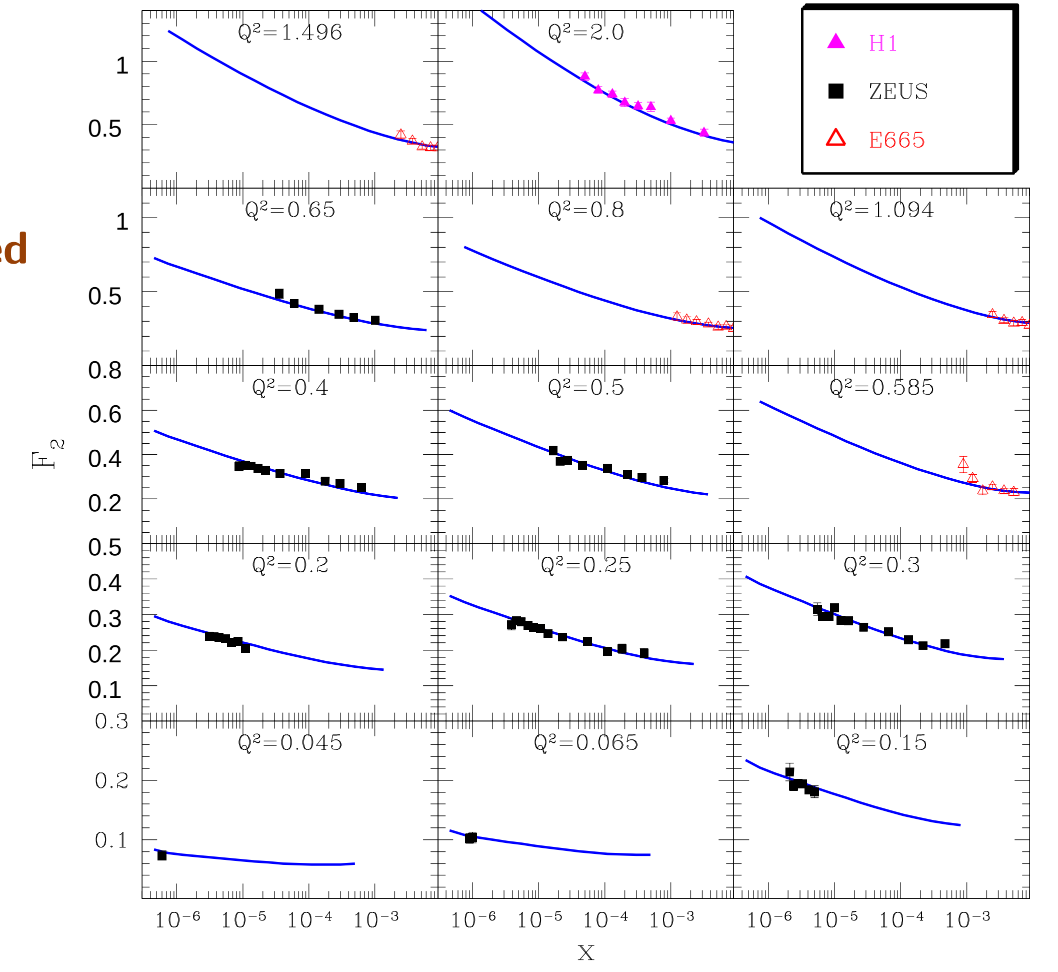
(3) "Bleaching of color"



Random walk $\rho \sim \sqrt{\mathbf{Y}}$

CGC at work

Gotsman, Levin, Lublinsky, Maor (2002)



Most of HERA DIS data are well described by the CGC/Saturation physics.
This is particularly true
in the low x / low Q regime, where
DGLAP fails.

High Energy Scattering

Target $(\rho^t = \rho^-; k^- > \Lambda)$ **Projectile** $(\rho^p = \rho^+; k^+ > \Lambda)$

$$\langle \mathbf{T}| \quad \rightarrow \qquad\qquad \leftarrow \quad |\mathbf{P}\rangle$$

S**-matrix:**

$$\mathbf{S(Y)} = \langle \mathbf{T} \langle \mathbf{P} | \; \hat{\mathbf{S}}(\rho^{\mathrm{t}}, \rho^{\mathrm{p}}) \; |\mathbf{P}\rangle \, \mathbf{T}\rangle$$

or, more generally, any observable $\hat{\mathcal{O}}(\rho^{\mathrm{t}}, \rho^{\mathrm{p}})$

$$\langle \hat{\mathcal{O}} \rangle_{\mathbf{Y}} = \langle \mathbf{T} \langle \mathbf{P} | \; \hat{\mathcal{O}}(\rho^{\mathrm{t}}, \rho^{\mathrm{p}}) \; |\mathbf{P}\rangle \, \mathbf{T}\rangle$$

The question we pose is how these averages change with increase in energy of the process

Projectile averaged operators:

$$\langle\, \mathbf{P}|\ \hat{\mathcal{O}}(\rho^{\mathrm{t}},\ \rho^{\mathrm{p}})\ |\mathbf{P}\rangle \ = \ \int \mathbf{D}\rho^{\mathrm{p}}\ \ \hat{\mathcal{O}}(\rho^{\mathrm{t}},\ \rho^{\mathrm{p}})\ \ \mathbf{W}^{\mathbf{p}}_{\mathbf{Y}}[\rho^{\mathrm{p}}]$$

Boosting projectile $\quad |\mathbf{P}\rangle_{\mathbf{Y}} \ \rightarrow\ |\mathbf{P}\rangle_{\mathbf{Y}\,+\,\delta\mathbf{Y}} \ = \ \boldsymbol{\Omega}_{\mathbf{Y}}\,|\mathbf{P}\rangle$

evolve with rapidity as $\qquad \mathbf{H}^{\mathbf{RFT}} \ \rightarrow$ **the RFT Hamiltonian**

$$\frac{\mathbf{d}\langle\mathbf{P}|\hat{\mathcal{O}}|\mathbf{P}\rangle}{\mathbf{dY}} \ = \ -\int \mathbf{D}\rho^{\mathrm{p}}\ \ \hat{\mathcal{O}}(\rho^{\mathrm{t}},\ \rho^{\mathrm{p}})\ \ \mathbf{H}^{\mathbf{RFT}}[\rho^{\mathrm{p}},\ \delta/\delta\rho^{\mathrm{p}}]\ \ \mathbf{W}^{\mathbf{p}}_{\mathbf{Y}}[\rho^{\mathrm{p}}]$$

or in other words

$$\frac{\mathbf{dW^{p}}}{\mathbf{dY}} \ = \ -\,\mathbf{H}^{\mathbf{RFT}}\ \ \mathbf{W}^{\mathbf{p}}$$

Spectrum of $\mathbf{H}^{\mathbf{RFT}}$ **defines the energy dependence of the average.**

Dense/Dilute limit

$$\mathbf{H}^{\mathbf{KLWMIJ}} \;=\; \mathbf{H}^{\mathbf{RFT}}(\rho\;\rightarrow\;\mathbf{0})\,; \qquad\qquad \mathbf{H}^{\mathbf{JIMWLK}} \;=\; \mathbf{H}^{\mathbf{RFT}}(\rho\;\rightarrow\;\infty)$$

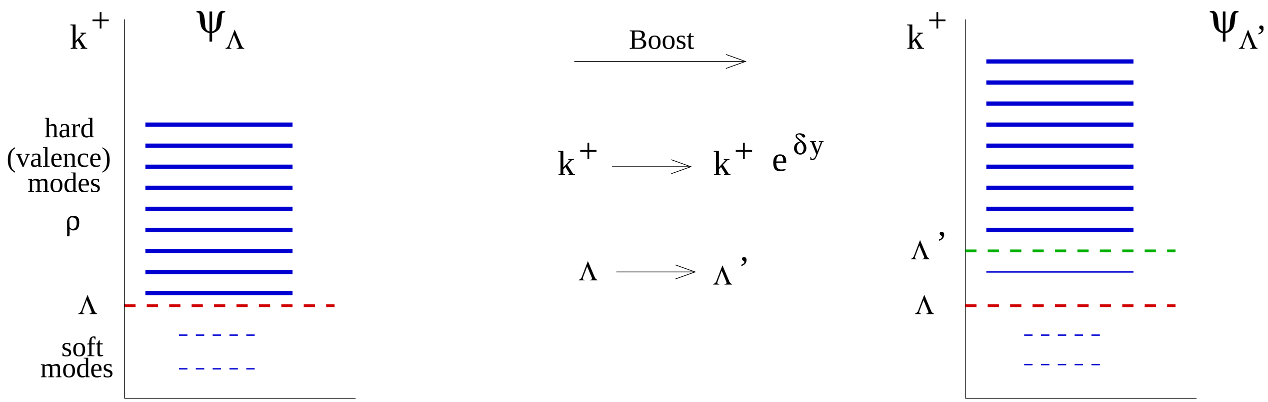
JIMWLK - Jalilian Marian, Iancu, McLerran, Leonidov, Kovner (1997-2002)

KLWMIJ - A. Kovner and M.L., Phys.Rev.D71:085004, 2005

Evolution with Pomeron Loops (model):

$$\mathbf{H}^{\mathbf{RFT}} \;\simeq\; \mathbf{H}^{\mathbf{JIMWLK}}\,(\rho\;\rightarrow\;\infty)\;\;''+''\;\;\mathbf{H}^{\mathbf{KLWMIJ}}\,(\rho\;\rightarrow\;\mathbf{0})$$

High energy evolution of light cone wave function



Hard particles with $k^+ > \Lambda$ scatter off the target. In the eikonal approximation, the scattering amplitude is independent of k^+. Hard (valence) modes are described by the valence density $\rho(x_\perp)$.

Soft modes are not many. They do not contribute much to the scattering amplitude.

The boost opens a window above Λ with the width $\sim \delta y$. The window is populated by soft modes, which became hard after the boost. These newly created hard modes do scatter off the target.

In the dilute limit $\rho \sim 1$; gluon emission $\sim \alpha_s \, \rho$

In the dense limit $\rho \sim 1/\alpha_s$, we have $\alpha_s \, \rho \sim 1$, and the number of gluons in the window can be very large.

Once evolution of the hadronic wave function ($\mathbf{\Omega}$) is computed, we can deduce evolution of an arbitrary observable $\hat{\mathcal{O}}(\rho)$

The evolution of the expectation value

$$\frac{\mathbf{d}\,\langle\mathbf{P}|\hat{\mathcal{O}}|\mathbf{P}\rangle}{\mathbf{d}\,\mathbf{Y}} = \lim_{\mathbf{Y}\to\mathbf{Y_0}} \frac{\langle\mathbf{P}|\mathbf{\Omega}^\dagger_\mathbf{Y}\hat{\mathcal{O}}[\rho+\delta\rho]\,\mathbf{\Omega}_\mathbf{Y}|\mathbf{P}\rangle - \langle\mathbf{P}|\hat{\mathcal{O}}[\rho]|\mathbf{P}\rangle}{\mathbf{Y}\,-\,\mathbf{Y_0}} = -\int \mathbf{D}\rho\,\mathbf{W}[\rho]\,\mathbf{H}^{\mathbf{RFT}}\,\mathcal{O}[\rho]$$

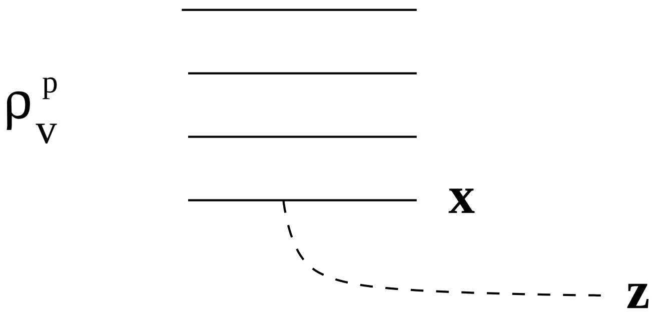
Charge density due to newly produced gluon

$$\delta\rho^{\mathbf{a}}(\mathbf{x}) \;=\; \int_{\mathbf{e}^{\mathbf{Y_0}}\mathbf{\Lambda}}^{\mathbf{e}^{\mathbf{Y}}\mathbf{\Lambda}} \mathbf{dk}^+\,\mathbf{a}_\mathbf{i}^{\dagger\mathbf{b}}(\mathbf{k}^+,\mathbf{x})\,\mathbf{T}^\mathbf{a}_\mathbf{bc}\,\mathbf{a}_\mathbf{i}^\mathbf{c}(\mathbf{k}^+,\mathbf{x})$$

Dual Wilson line (charge density shift operator)

$$\mathbf{R}^{\mathbf{ab}}(\mathbf{x}) \;=\; \left[\mathcal{P}\exp\{\mathbf{T}^\mathbf{c}\,\frac{\delta}{\delta\rho^\mathbf{c}(\mathbf{x})}\}\right]^{\mathbf{ab}}, \qquad\qquad \mathbf{R}\,\hat{\mathcal{O}}[\rho] \;=\; \hat{\mathcal{O}}[\rho\,+\,\mathbf{T}]$$

KLWMIJ Hamiltonian (dilute limit)



Linear evolution means $\delta\rho \, \propto \, \rho_v^p$

Emission amplitude is given by the Weizsaker-Williams field

$$\mathbf{b}_i^a(\mathbf{z}) \;=\; \frac{\mathbf{g}}{\mathbf{2}\,\pi} \int \mathbf{d}^2\mathbf{x} \, \frac{(\mathbf{x} - \mathbf{z})_i}{(\mathbf{x} - \mathbf{z})^2} \, \rho^a(\mathbf{x})$$

Gluon coherent field operator in the dilute limit

$$\boldsymbol{\Omega}_{\mathbf{Y}}(\rho \to \mathbf{0}) \equiv \mathbf{C}_{\mathbf{Y}} = \mathrm{Exp}\left\{ \mathbf{i} \int \mathbf{d}^2\mathbf{z} \, \mathbf{b}_i^a(\mathbf{z}) \int_{e^{Y_0}\Lambda}^{e^{Y}\Lambda} \frac{\mathbf{dk}^+}{\pi^{1/2}|\mathbf{k}^+|^{1/2}} \left[\mathbf{a}_i^a(\mathbf{k}^+, \mathbf{z}) + \mathbf{a}_i^{\dagger a}(\mathbf{k}^+, \mathbf{z}) \right] \right\}$$

The operator $\mathbf{C}$ dresses the valence charges by a cloud of the WW gluons

$$\mathbf{H}^{\mathrm{RFT}}(\rho \to \mathbf{0}) \;=\; \mathbf{H}^{\mathrm{KLWMIJ}}[\rho, \delta/\delta\rho] \;=\; \int_{\mathbf{z}} \mathbf{b}_{\mathbf{z}}^a[\rho] \, [\mathbf{1} - \mathbf{R}_{\mathbf{z}}]^{\mathbf{ab}} \, \mathbf{b}_{\mathbf{z}}^b[\rho]$$

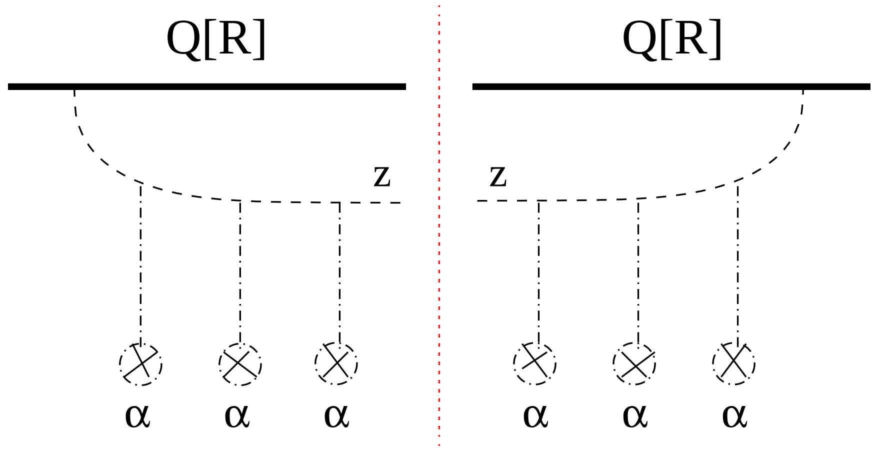
$$\mathbf{H}^{\mathbf{KLWMIJ}} = \frac{\alpha_s}{2\,\pi^2} \int \mathbf{d^2z}\;\; \mathbf{Q^a_i(z)\, Q^a_i(z)} \;\; \geq \;\; \mathbf{0}$$

The "amplitudes" $\mathbf{Q^a_i(z)}$ are defined as

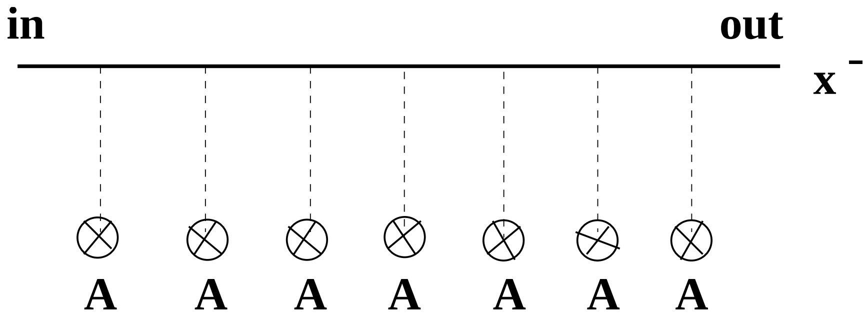
$$\mathbf{Q^a_i(z)} \;=\; \int \mathbf{d^2x}\, \frac{\mathbf{(x-z)_i}}{\mathbf{(x-z)^2}} \;\left[\mathbf{R^{ab}(z)\, J^b_R(x)} \;-\; \mathbf{J^b_L(x)}\right]$$

The generators of the right/left color rotations

$$\mathbf{J^a_R(x)} \;=\; -\,\mathrm{tr}\left\{\mathbf{R(x)\, T^a}\, \frac{\delta}{\delta \mathbf{R^\dagger(x)}}\right\}, \qquad \mathbf{J^a_L(x)} \;=\; -\,\mathrm{tr}\left\{\mathbf{T^a\, R(x)}\, \frac{\delta}{\delta \mathbf{R^\dagger(x)}}\right\}$$



Eikonal scattering approximation



Eikonal scattering is a color rotation

Eikonal factor does not depend on rapidity

In the light cone gauge ($\mathbf{A}^+ = 0$) the large target field component is $\mathbf{A}^- = \alpha^t$.

$$\mathbf{S}(\mathbf{x}) = \mathcal{P} \exp\left\{ \mathbf{i} \int \mathbf{dx}^+ \, \mathbf{T}^a \, \alpha_t^a(\mathbf{x}, \mathbf{x}^+) \right\} . \qquad \text{"}\boldsymbol{\Delta}\text{"} \alpha^t = \rho^t \quad (\mathbf{YM})$$

$$|\mathbf{in}\rangle = |\mathbf{z}, \mathbf{b}\rangle ; \qquad |\mathbf{out}\rangle = |\mathbf{z}, \mathbf{a}\rangle ; \qquad |\mathbf{out}\rangle = \mathbf{S}\,|\mathbf{in}\rangle$$

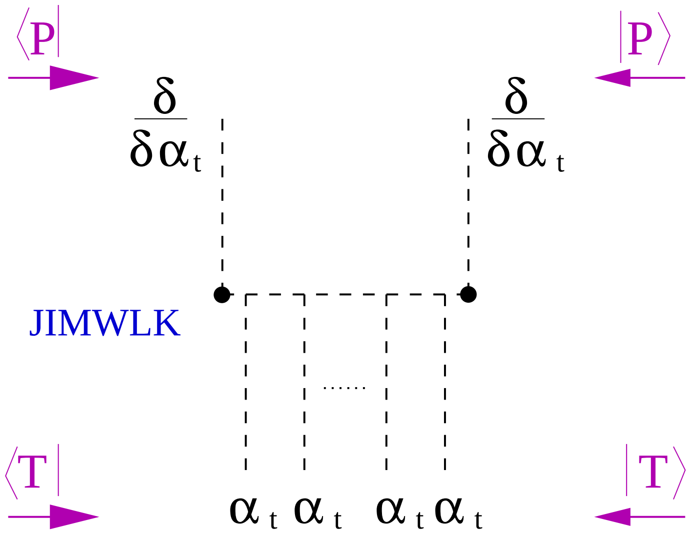
JIMWLK Hamiltonian (dense limit)

In the dense regime: $\mathbf{\Omega}(\rho \sim \mathbf{1}/\alpha_s) \; = \; \mathbf{C}\,\mathbf{B}$ $\qquad$ $\mathbf{B}$ **is a Bogolyubov operator**

$$\mathbf{B} \;=\; \exp[\,\mathbf{\Lambda}(\rho)\,(\mathbf{a}^{\mathbf{2}} + \mathbf{a}^{\dagger\,\mathbf{2}}) + \cdots]$$

$\mathbf{B}$ **defines quasiparticles above the WW background**

$$H^{JIMWLK}\left[\alpha^t, \frac{\delta}{\delta\alpha^t}\right] \;=\; \int_z b_i\left[\frac{\delta}{\delta\alpha^t}\right]\;[1 \;-\; S(z)]\; b_i\left[\frac{\delta}{\delta\alpha^t}\right]$$



Eikonal scattering matrix for projectile's gluon

$$S^{ab}(z) \;=\; P\,exp\left\{i\,\alpha^t(z)\right\}^{ab}$$

DDD - Dense Dilute Duality

$$H^{KLWMIJ}\,(\rho \to 0) \;=\; \alpha_s \int_{x,y,z} \frac{(z-x)_i(z-y)_i}{(z-x)^2(z-y)^2}\;\; \rho^a(x)\,[1\;-\;R(z)]^{ab}\,\rho^b(y)$$

linear emission + multiple rescatterings

$$H^{JIMWLK}\,(\rho \to \infty) \;=\; \alpha_s \int_{x,y,z} \frac{(z-x)_i(z-y)_i}{(z-x)^2(z-y)^2}\;\; \frac{\delta}{\delta\alpha^a(x)}\,[1\;-\;S(z)]^{ab}\,\frac{\delta}{\delta\alpha^b(y)}$$

non-linear emission + double gluon exchange

DDD transformation:

$$i\,\alpha \;\to\; \frac{\delta}{\delta\,\rho}\,; \qquad\qquad \frac{\delta}{\delta\alpha} \;\to\; i\,\rho \qquad\qquad S \;\to\; R \qquad\qquad \boxed{H^{JIMWLK} \;\leftrightarrow\; H^{KLWMIJ}}$$

Self-Duality of High Energy Evolution

- Lorentz Invariance (LI)
- Eikonal Approximation (EA)
- Projectile - Target Democracy (PTD)

$$\boxed{\mathbf{H}^{\mathbf{RFT}}(\mathbf{i}\,\alpha\ \ ,\delta/\delta\,\alpha)\ \ =\ \ \mathbf{H}^{\mathbf{RFT}}(\delta/\delta\,\rho,\ \ \mathbf{i}\,\rho)}$$

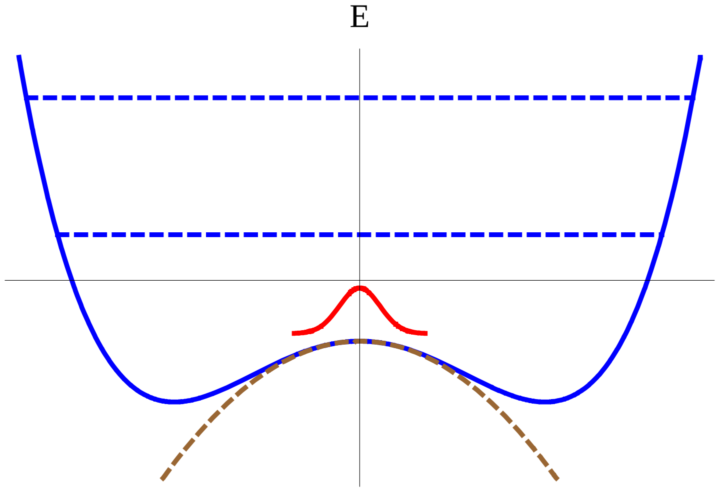
Self-Duality $\quad = \quad$ t-channel unitarity?

KLWMIJ (JIMWLK) vs BFKL

JIMWLK/KLWMIJ has non-negative spectrum (Unitarity!)

BFKL has a negative eigenvalue (Unitarity is violated!)

BFKL is a limit of JIMWLK/KLWMIJ?!



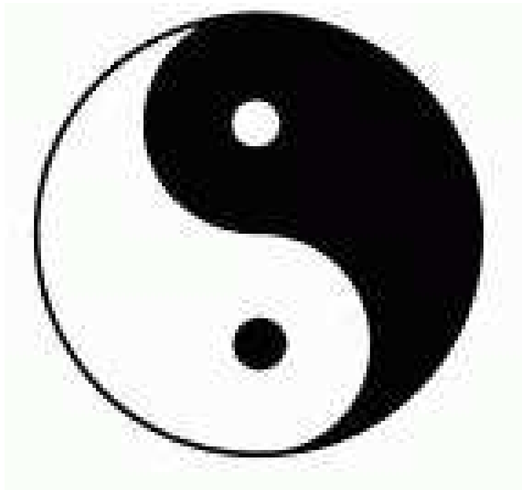
Initial wave packet localized at the origin (dilute regime $\rho \sim 0$) can be expanded in both KLWMIJ and BFKL eigenfunctions.

BFKL eigenfunctions are non-normalizable

At small times (rapidities) the evolutions are similar but at late times BFKL drives the system towards unitarity violation.

Reggeon Field Theory in QCD – Summary

- Hamiltonian (2+1) dimensional interacting non-local field theory.
- The basic "quantum Reggeon field" is the unitary matrix R (S).
- Symmetry: DDD, Self-duality
- Two zero energy degenerate vacua ("Yang" and "Yin"), DDD is spontaneously broken.
- Spectrum of excitations is twice degenerate (gluons and "holes")



- More symmetries: $\mathrm{SU_V(N)}$, $\mathrm{Z_2}$; 2-d Conformal invariance?
- BFKL Pomeron is a tachyon

Phenomenology (LHC, HERA, RHIC, TeVatron, EIC, LHeC, ILC)

There are Postdoc Fellowships at the BGU

Projecting KLWMIJ onto singlets

$\mathbf{H_{KLWMIJ}}$ defines a 2+1 dimensional non-local QFT of unitary matrix R, but not a QFT of Reggeons. Reggeons are physical scattering amplitudes - color singlets.

Is it possible to project $\mathbf{H_{KLWMIJ}}$ onto color singlets and derive the RFT ?

First step is to choose effective degrees of freedom and make sure to preserve symmetries

$\mathbf{SU_L(N)} \times \mathbf{SU_R(N)}$ – effective degrees of freedom must be scalars.

Charge conjugation Z_2: $\mathbf{R(x)} \rightarrow \mathbf{R^*(x)}$

Time reversal (Signature) Z_2: $\mathbf{R(x)} \rightarrow \mathbf{R^\dagger(x)}$

Natural condition: in the linearized regime $(\mathbf{R = 1 - T\frac{\delta}{\delta\rho}...})$ we shell reduce to BKP.

In a sense, we study the low energy limit of high energy QCD.

There is infinite number of independent color singlets, but there is a natural hierarchy

Dipole: $\mathbf{d(x,y) \;=\; \frac{1}{N_c}\mathrm{Tr}[R(x)R^\dagger(y)]}$

Qudrupole: $\mathbf{Q(x,y,u,v) \;=\; \frac{1}{N_c}\mathrm{Tr}[R(x)\,R^\dagger(y)\,R(u)\,R^\dagger(v)]}$

Naturally decomposes into

Pomeron: - C, T even $\mathbf{P(1,2) = \frac{1}{2}[2 - d(1,2) - d(2,1)]}$

Odderon: - C, T odd $\mathbf{O(1,2) = \frac{1}{2}[d(1,2) - d(2,1)]}$

B-Reggeon: C,T even, perturbatively orthogonal to $\mathbf{P}$

$$\mathbf{B_{1,2,3,4} = \frac{1}{4}\left[4 - Q_{1,2,3,4} - Q_{4,1,2,3} - Q_{3,2,1,4} - Q_{2,1,4,3}\right] - \left[P_{12} + P_{14} + P_{23} + P_{34} - P_{13} - P_{24}\right]}$$

Other 'ONs

C-Reggeon odd, T even: $\mathbf{C_{1,2,3,4} = \frac{1}{4}\left[Q_{1,2,3,4} + Q_{4,1,2,3} - Q_{3,2,1,4} - Q_{2,1,4,3}\right]}$

T odds: $\mathbf{D^{\pm}_{1,2,3,4} = \frac{1}{4}\left[Q_{1,2,3,4} - Q_{4,1,2,3}\right] \pm \frac{1}{4}\left[Q_{3,2,1,4} - Q_{2,1,4,3}\right]}$

And higher multipoles

$$H_{KLWMIJ} = H_P + H_O + H_B + H_C + H_D + \cdots$$

$$H_P = -\frac{\bar{\alpha}_s}{2\,\pi} \int_{x,y,z} \frac{(x\, -\, y)^2}{(x\, -\, z)^2\,(y\, -\, z)^2} \Big\{ [P_{x,z} + P_{z,y}\, -\, P_{x,y} - P_{x,z}P_{z,y} + O_{x,z}O_{z,y}]\; P^\dagger_{x,y} \Big\}$$

$$H_O = -\frac{\bar{\alpha}_s}{2\,\pi} \int_{x,y,z} \frac{(x\, -\, y)^2}{(x\, -\, z)^2\,(y\, -\, z)^2} \Big\{ [O_{x,z} + O_{z,y}\, -\, O_{x,y} - O_{x,z}P_{z,y} - P_{x,z}O_{z,y}]\; O^\dagger_{x,y} \Big\}$$

$$H_B = -\frac{\bar{\alpha}_s}{2\,\pi} \int_{xyuvz} \Bigg\{ \Big[-\; [M_{x,y;z}\; +\; M_{u,v;z}\; -\; L_{x,u,v,y;z}]\;\; B_{xyuv} + 4L_{x,v,u,v;z}B_{xyuz} \Big] B^\dagger_{xyuv}$$

$$-2L_{x,y,u,v;z}\Big[P_{xv}P_{uy} + O_{xv}O_{uy}\Big]B^\dagger_{xyuv} - 2P_{xz}P_{yz}\Big[2L_{x,y,u,v;z}B^\dagger_{xyuv} - \Big(L_{x,u,y,v;z} + L_{x,v,y,u;z}\Big)B^\dagger_{xuyv}\Big]$$

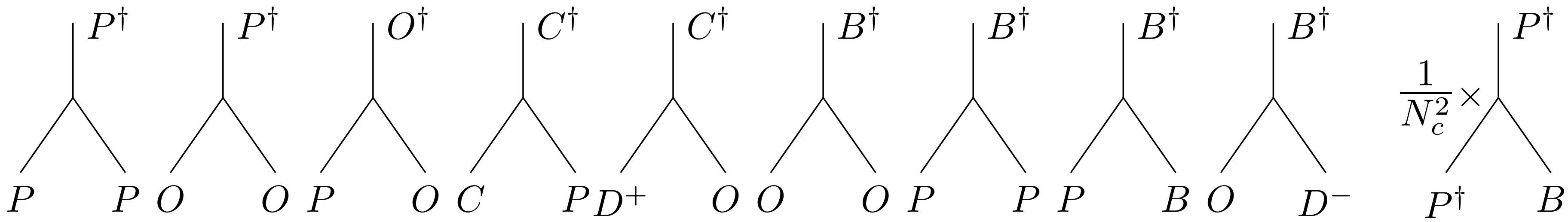
$$-4P_{xz}P_{yu}\Big[2L_{x,y,x,v;z}B^\dagger_{xyuv} - L_{x,y,x,u;z}B^\dagger_{xyvu}\Big] - 4B_{xyuz}P_{zv}L_{x,v,u,v;z}B^\dagger_{xyuv}$$

$$-4D^+_{xyuz}O_{zv}L_{x,v,u,v;z}B^\dagger_{xyuv} \Bigg\}$$

All vertices allowed by the symmetries

At leading N_c all of them have the nature of splitting: one Reggeon going into two

$$PPP^\dagger, \quad OOP^\dagger; \quad POO^\dagger; \quad PPB^\dagger; \quad BPB^\dagger; \quad CPC^\dagger \quad \ldots$$



At subleading N_c one gets also merging vertices

QCD

QCD Lagrangian

$$\mathcal{L} \;=\frac{1}{4}\, G_{\mu\nu}\, G^{\mu\nu} \;+\; \overline{\psi}\,(i\;\partial\!\!\!/ \;-\; g\; A\!\!\!/ - m)\;\psi$$

The field strength

$$G_a^{\mu\nu} \;=\; \partial^\mu\, A_a^\nu \;-\; \partial^\nu\, A_a^\mu \;-\; g\, f^{abc}\, A_b^\mu\, A_c^\nu$$

Equations of motion:

Maxwell equation:

$$\partial_\mu\, G^{\mu\nu} \;=\; g\; J^\nu\,; \qquad\qquad J_a^\nu \;=\; \bar{\psi}\,\gamma^\nu\,\tau^a\,\psi \;-\; f^{abc}\, G_b^{\nu\mu}\, A_c^\mu$$

Dirac equation

$$(i\,\gamma^\mu\, D_\mu \;-\; m)\,\psi \;=\; 0$$

Light Cone

LC time $x^{+} = (t + z)/\sqrt{2}$ $\quad$ $x^{-} = (t - z)/\sqrt{2}$

LC gauge

$$A^{+} = \frac{1}{\sqrt{2}}(A^{0} + A^{3}) = 0$$

The Gauss law constraint

$$\partial_{\mu} G^{\mu +} = g J^{+}$$

is solved for the A^{-} field

$$-(\partial^{+})^{2} A_{a}^{-} + \partial^{+}\partial_{i} A_{a}^{i} = g J_{a}^{+}$$

$$A_{a}^{-} = -\frac{\partial^{i}}{\partial^{+}} A_{a}^{i} + \frac{g}{(\partial^{+})^{2}} J_{a}^{+}$$

Same story with quarks

Light Cone Hamiltonian

Canonical variables: $A^i, \quad \Pi^i = \frac{\delta L}{\delta(\partial^- A^i)} = G^{+i} = \partial^+ A^i$

Light Cone Hamiltonian:

$$H^{LC} = \int dx^- d^2 x_\perp \left[\Pi^i \, \partial^- A^i - L \right] = H_E + H_M$$

The electric and magnetic parts

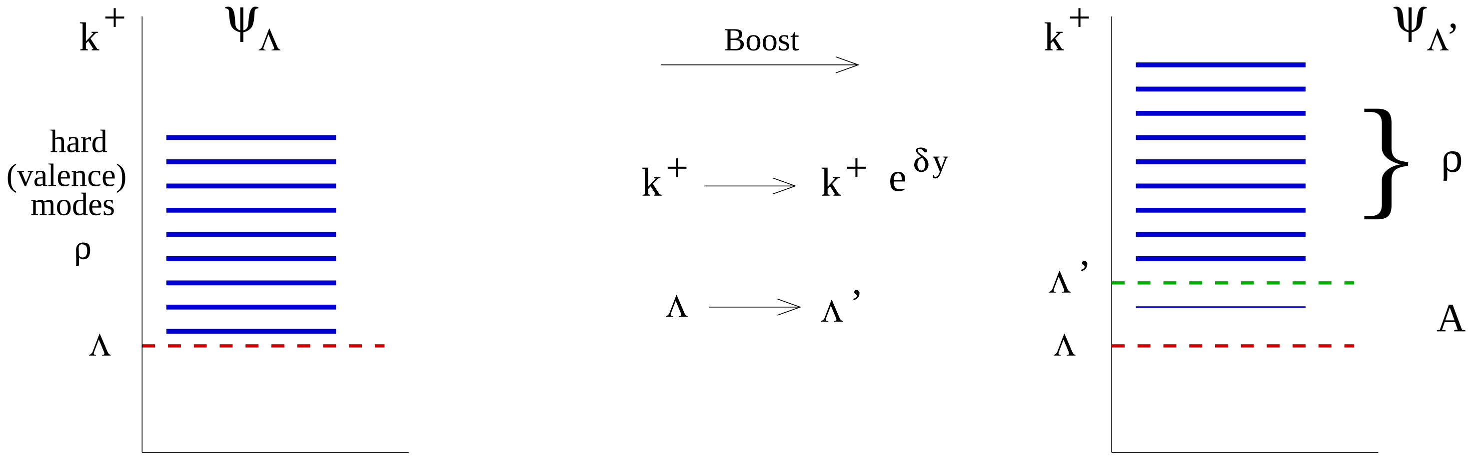
$$H_E = \frac{1}{2} \int \frac{dk^+}{(2\,\pi)} \, d^2 x \;\; \Pi_a^-(k^+, x) \, \Pi_a^-(-k^+, x)$$

$$H_M = \frac{1}{4} \int \frac{dk^+}{(2\,\pi)} \, d^2 x \;\; G_{ij}^a(k^+, x) \, G_{ij}^a(-k^+, x)$$

The chromoelectric field

$$\Pi_a^-(k^+, x) = \partial^+ A^- = -\,\partial^i A_i^a + \frac{g}{\partial^+} J_a^+$$

We split the modes into hard and soft: The hard modes act as an external current $j_a^+ = \delta(x^-)\,\rho^a$ **for the soft modes.** $J^+ = j^a + g\,A\,A + (quark\ current)$



$$H^{LC} = H_A^{LC} + H_\rho^{LC}\,; \qquad H_\rho^{LC}\,|\Psi_\Lambda\rangle = E\,|\Psi_\Lambda\rangle\,; \qquad H^{LC}\,|\Psi_{\Lambda'}\rangle = E'\,|\Psi_{\Lambda'}\rangle$$

$|\Psi_\Lambda\rangle$ **is a vacuum of the soft modes** A**.**

$$H_A^{LC} = H_0 + \delta H^\rho + g\,AAA + \ldots\,; \qquad \delta H^\rho \sim g\,\rho\,A$$

Quantization

$$A_i^a(x^-, \mathbf{x}_\perp) = \int_0^\infty \frac{dk^+}{2\pi} \frac{1}{\sqrt{2k^+}} \left\{ a_i^a(k^+, \mathbf{x})\, e^{-ik^+x^-} + a_i^{a\,\dagger}(k^+, \mathbf{x})\, e^{ik^+x^-} \right\}$$

$$\left[a_i^a(k^+, k), a_j^{b\,\dagger}(p^+, p) \right] = (2\pi)^3\, \delta^{ab}\, \delta_{ij}\, \delta^3(k - p)$$

The free part of the LCH

$$H_0 \;=\; \int_{k^+>0} \frac{dk^+}{2\pi} \frac{d^2k_\perp}{(2\pi)^2} \frac{k_\perp^2}{2k^+}\; a_i^{\dagger a}(k^+, k_\perp)\; a_i^a(k^+, k_\perp)$$

The vacuum of the LCH is simply the Fock space vacuum of the operators a

$$a_q|0\rangle = 0\,; \qquad\qquad E_0 \;=\; 0$$

The one particle state

$$|k, a, i\rangle \;=\; \frac{1}{(2\pi)^{3/2}}\, a_i^{a\,\dagger}(k^+, k)\, |0\rangle \qquad\qquad E_g \;=\; k^- \;=\; \frac{k_\perp^2}{2k^+}$$

Perturbation Theory

$$\delta H^{\rho} \;=\; -\int \frac{dk^+}{2\pi}\frac{d^2k_\perp}{(2\pi)^2}\frac{g\,k_i}{\sqrt{2}\,|k^+|^{3/2}}\;\left[a_i^{\dagger a}(k^+,\,k_\perp)\;\hat\rho^{a}(-k_\perp)\;+\;a_i^{a}(k^+,\,-k_\perp)\;\hat\rho^{a}(k_\perp)\right]$$

The first order perturbation theory

$$|\theta\rangle \;=\; \beta\,|0\rangle \;-\; \sum_i\,|i\rangle\,\frac{\langle i|\,\delta H^{\rho}\,|0\rangle}{E_i} \qquad\qquad \langle\theta|\theta\rangle \;=\; 1\;\rightarrow\;\beta$$

This Hamiltonian creates only one particle state from the vacuum

$$\langle 1\,gluon|\,\delta H^{\rho}\,|0\rangle \;=\; \langle k_\perp, k^+, a, i|\,\delta H^{\rho}\,|0\rangle \;=\; \frac{g\,k_i}{4\,\pi^{3/2}\,|k^+|^{3/2}}\,\rho^{a}(-k_\perp)$$

We can write the soft gluon vacuum state to the first order in the coupling as

$$|\theta\rangle \;=\; C_{\delta Y}\,|0\rangle\,; \qquad\qquad |\Psi_{\Lambda'}\rangle \;=\; C_{\delta Y}\,|\Psi_{\Lambda}\rangle$$

The Coherent operator having the form

$$C_{\delta Y} \; = \; \mathrm{Exp}\left\{ i \int d^2x \, b_i^a(x) \, \int_{\Lambda}^{e^{\delta Y}\,\Lambda} \frac{dk^+}{\sqrt{2}\,\pi |k^+|^{1/2}} \left[a_i^a(k^+, x) \; + \; a_i^{\dagger a}(k^+, x) \right] \right\}$$

The "classical" field b_i is the Weizsaker-Williams field of the color charge density ρ^a

$$b_i^a(k) \; = \; g \, \frac{-i\, k_i}{k_\perp^2} \, \rho^a(-k) \, ; \qquad\qquad b_i^a(x) \; = \; \frac{g}{2\pi} \int d^2y \frac{(x-y)_i}{(x-y)^2} \, \rho^a(y) \, .$$

NLO

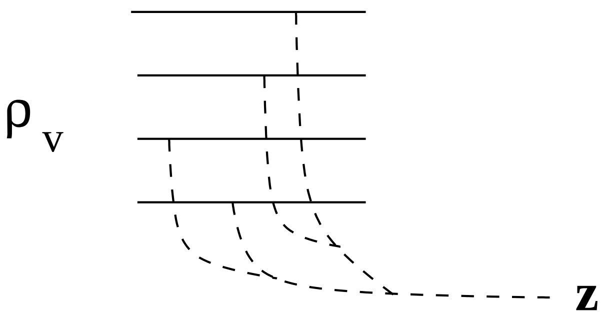
- **NLO: g^3 + normalization up g^4**

$$|\theta\rangle \;=\; \beta\,|0\rangle \;+\; \sum_i \,|i\rangle \left[-\frac{\langle i|\,\delta H\,|0\rangle}{E_i} \;+\; \frac{\langle i|\,\delta H\,|j\rangle\,\langle j|\,\delta H\,|0\rangle}{E_i\,E_j} + \right.$$

$$\left. +\;\frac{\langle i|\,\delta H\,|0\rangle\,\langle j|\,\delta H\,|0\rangle^2\,(2\,E_j\,-\,E_i)}{2\,E_i^2\,E_j^2} \;-\; \frac{\langle i|\,\delta H\,|j\rangle\,\langle j|\,\delta H\,|k\rangle\,\langle k|\,\delta H\,|0\rangle}{E_i\,E_j\,E_k} \right]$$

Beyond JIMWLK: $JIMWLK+$

A. Kovner and M.L., JHEP 0503:001,2005

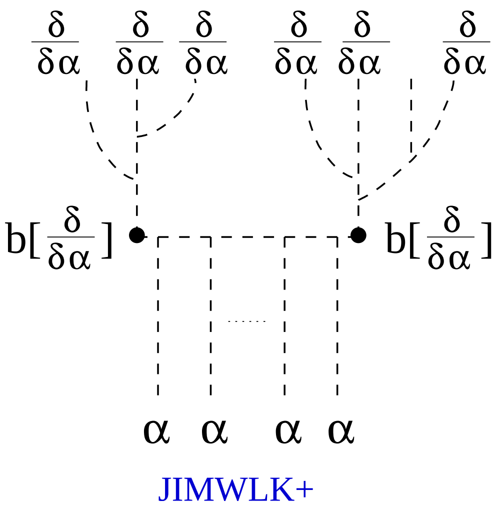


Coherent emission of a single gluon

$$D_i[b]\, b_i^a \;=\; \rho_p^a$$

b **is non-linear in** $\rho^p \;=\; \rho_v \;\geq\; 1$

$$H^{JIMWLK+} \;=\; \int_z b_i \;\left[1 \;-\; S(z)\right]\; b_i$$



Semi-inclusive reactions

The wave function coming into the collision region at time $t = 0$

$$|\mathbf{\Psi_{in}}\rangle \;=\; \mathbf{\Omega_Y}\,|\rho, \mathbf{0_a}\rangle\;.$$

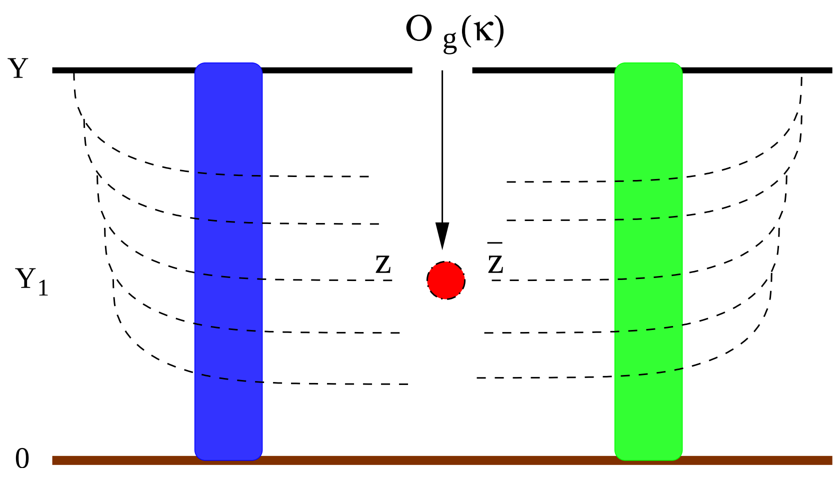
The system emerges from the collision region with the wave function

$$|\mathbf{\Psi_{out}}\rangle \;=\; \hat{\mathbf{S}}\,\mathbf{\Omega_Y}\;|\rho, \mathbf{0_a}\rangle\;.$$

The system keeps evolving after the collision to the asymptotic time $t \rightarrow +\infty$, at which point the measurement of an observable $\hat{\mathcal{O}}$ is made

$$\langle\hat{\mathcal{O}}\rangle_{\mathbf{P,T}} \;=\; \langle\,\mathbf{\Omega_Y^\dagger}\;(\mathbf{1}\;-\;\hat{\mathbf{S}}^\dagger)\,\mathbf{\Omega_Y}\;\;\hat{\mathcal{O}}\;\;\mathbf{\Omega_Y^\dagger}\;\;(\mathbf{1}\;-\;\hat{\mathbf{S}})\,\mathbf{\Omega_Y}\;\rangle_{\mathbf{P,T}}$$

Single inclusive gluon production



The observable

$$\hat{\mathcal{O}}_g \;\sim\; \mathbf{a}_i^{\dagger\, a}(\mathbf{k})\, \mathbf{a}_i^{a}(\mathbf{k})$$

$$\frac{dN}{d^2k\,dy} \;=\; \langle\, \sigma(\mathbf{k})\, \rangle_{P,T}$$

$$\sigma(\mathbf{k}) = \int_{z,\bar{z},x_1,\bar{x}_1} e^{ik(z-\bar{z})}\vec{f}(\bar{z}-\bar{x}_1)\cdot\vec{f}(x_1-z)\;\left\{\rho(x_1)[S^{\dagger}(x_1)-S^{\dagger}(z)][S(\bar{x}_1)-S(z)]\rho(\bar{x}_1)\right\}$$

Here

$$f_i(x-y) = \frac{(x-y)_i}{(x-y)^2}$$

Yin and Yang

Yang (white) vacuum: $H^{KLWMIJ}\,|\,Yang\,\rangle\;=\;0$ $\qquad\omega^{Yang}\;=\;0$

$$|\,Yang\,\rangle\;=\;\delta(\rho)\,;\qquad S^{ab}\,|\,Yang\,\rangle\;=\;\delta^{ab}\,|\,Yang\,\rangle\,;\qquad \frac{\delta}{\delta R}\,|\,Yang\,\rangle\;=\;0$$

Yin (black) vacuum: $H^{JIMWLK}\,|\,Yin\,\rangle\;=\;0$ $\qquad\omega^{Yin}\;=\;0$

$$|\,Yin\,\rangle\;=\;1\,;\qquad \langle\,Yin\,|\,S^{ab}(x)|\,Yin\,\rangle\;=\;0\,;\qquad \frac{\delta}{\delta S}\,|\,Yin\,\rangle\;=\;0$$

DDD transforms $|\,Yin\,\rangle$ **into** $|\,Yang\,\rangle$

$$H^{RFT}\,|\,Yin\,\rangle\;=\;H^{RFT}\,|\,Yang\,\rangle\;=\;0$$

Vacuum is doubly degenerate: DDD is spontaneously broken

There are two degenerate towers of excited states:

"GLUONS" - live above Yang

$$g_n = R(x_1) \cdots R(x_n) \,|\, Yang \,\rangle$$

$S = 1$ **at all points in the transverse plane except** $x_1, \cdots x_n$

"HOLES" - live above Yin

$$h_n = S(x_1) \cdots S(x_n) \,|\, Yin \,\rangle$$

$S = 0$ **at all points in the transverse plane except** $x_1, \cdots x_n$

Projectile averaged S-matrix:

$$\mathbf{\Sigma^p_{Y-Y_0}}(\rho^t) \;=\; \langle\, \mathbf{P}|\; \hat{\mathbf{S}}(\rho^t,\; \rho^p)\; |\mathbf{P}\rangle \;=\; \int \mathbf{D}\rho^p\;\; \hat{\mathbf{S}}(\rho^t,\; \rho^p)\;\; \mathbf{W^p_{Y-Y_0}}[\rho^p]$$

$$\mathbf{S(Y)} \;=\; \int \mathbf{D}\rho^t\;\; \mathbf{\Sigma^p_{Y-Y_0}}[\rho^t]\;\; \mathbf{W^t_{Y_0}}[\rho^t]$$

$$\frac{\mathbf{dS}}{\mathbf{dY}} \;=\; -\int \mathbf{D}\rho^t\;\; \mathbf{\Sigma^P_{Y-Y_0}}[\rho^t]\;\; \mathbf{H^{RFT}}[\rho^t,\; \delta/\delta\rho^t]\;\; \mathbf{W^T_{Y_0}}[\rho^t]$$

$$\mathbf{H^{RFT}}\;|\,\mathbf{\Psi_i}\,\rangle \;=\; \omega_{\mathbf{i}}\;|\,\mathbf{\Psi_i}\,\rangle \qquad\qquad |\,\mathbf{\Psi_i}\,\rangle_{\mathbf{Y}} \;=\; \mathbf{e}^{-\,\omega_{\mathbf{i}}\;\mathbf{Y}}\;|\,\mathbf{\Psi_i}\,\rangle$$

$$\mathbf{\Sigma^P_0} \;=\; \sum_{\mathbf{i}} \gamma^{\mathbf{P}}_{\mathbf{i}}\;|\,\mathbf{\Psi_i}\,\rangle \qquad\qquad \mathbf{W^T_0} \;=\; \sum_{\mathbf{i}} \gamma^{\mathbf{T}\,*}_{\mathbf{i}}\;\langle\;\mathbf{\Psi_i}\,|$$

$$\mathbf{S(Y)} \;=\; \sum_{\mathbf{i}} \gamma^{\mathbf{P}}_{\mathbf{i}}\, \gamma^{\mathbf{T}\,*}_{\mathbf{i}}\; \mathbf{e}^{\,-\,\omega_{\mathbf{i}}\,\mathbf{Y}}$$

Unitarity $\Longleftrightarrow \quad \omega_{\mathbf{i}} \;\geq\; \mathbf{0}$ **;** $\qquad \omega(BFKL) \;<\; 0$

RFT beyond JIMWLK/KLWMIJ

$$\mathbf{H^{RFT}} = \frac{\mathbf{1}}{\mathbf{2}\pi}\,[\mathbf{b} - \bar{\mathbf{b}}]\,\mathbf{R}^\dagger\,(\mathbf{1} - \mathbf{l} - \mathbf{L})\,(\mathbf{1} - \mathbf{2l})\,\mathbf{R}\,(\mathbf{1} - \mathbf{2l})\,(\mathbf{1} - \mathbf{l} - \mathbf{L})\,[\mathbf{b} - \bar{\mathbf{b}}]$$

$\mathbf{b} \equiv \mathbf{b}[\rho]$ is the WW field of the incoming state

$\bar{\mathbf{b}} \equiv \mathbf{R}^\dagger\,\mathbf{b}[\mathbf{R}\,\rho]$ is the WW field of the outgoing state

Projectors:

$$\mathbf{l} \equiv \frac{\partial_i\,\partial_j}{\partial^2} \qquad\qquad \mathbf{L} \equiv \frac{\mathbf{D}[\mathbf{b}]_i\,\mathbf{D}[\mathbf{b}]_j}{\mathbf{D}[\mathbf{b}]^2}$$

Expanding in either small ρ or small $\delta/\delta\rho$ we reproduce KLWMIJ and JIMWLK.

Denote soft glue creation and annihilation operators as $\mathbf{a}$ and $\mathbf{a}^\dagger$.

$$\mathbf{H_{QCD} = H(\rho,\, a,\, a^\dagger)}$$

Hadron wave function in the soft gluon Fock space

$$|\mathbf{\Psi}\rangle_{\mathbf{Y_0}} = |v\rangle \;=\; |\rho\rangle_{\mathbf{valence}} \otimes |\mathbf{0_a}\rangle_{\mathbf{soft}}$$

The evolved wave function

$$|\mathbf{\Psi}\rangle_{\mathbf{Y}} \;=\; \mathbf{\Omega_Y}(\rho,\, \mathbf{a})\, |\mathbf{\Psi}\rangle_{\mathbf{Y_0}}\,;$$

or equivalently

$$\mathbf{\Omega^\dagger\; H\; \Omega \;=\; H_{diagonal}}$$

The major challenge is to find $\mathbf{\Omega}$ that does the job