Unterstützt von / Supported by
Alexander von Humboldt
Stiftung / Foundation

Electrical Conductivity of QGP in Strong Magnetic Field

Daisuke Satow

Collaborator: Koichi Hattori (Fudan Uni, Shanghai 🇨🇳)
Shiyong Li, Ho-Ung Yee (Uni. Illinois at Chicago 🇺🇸)

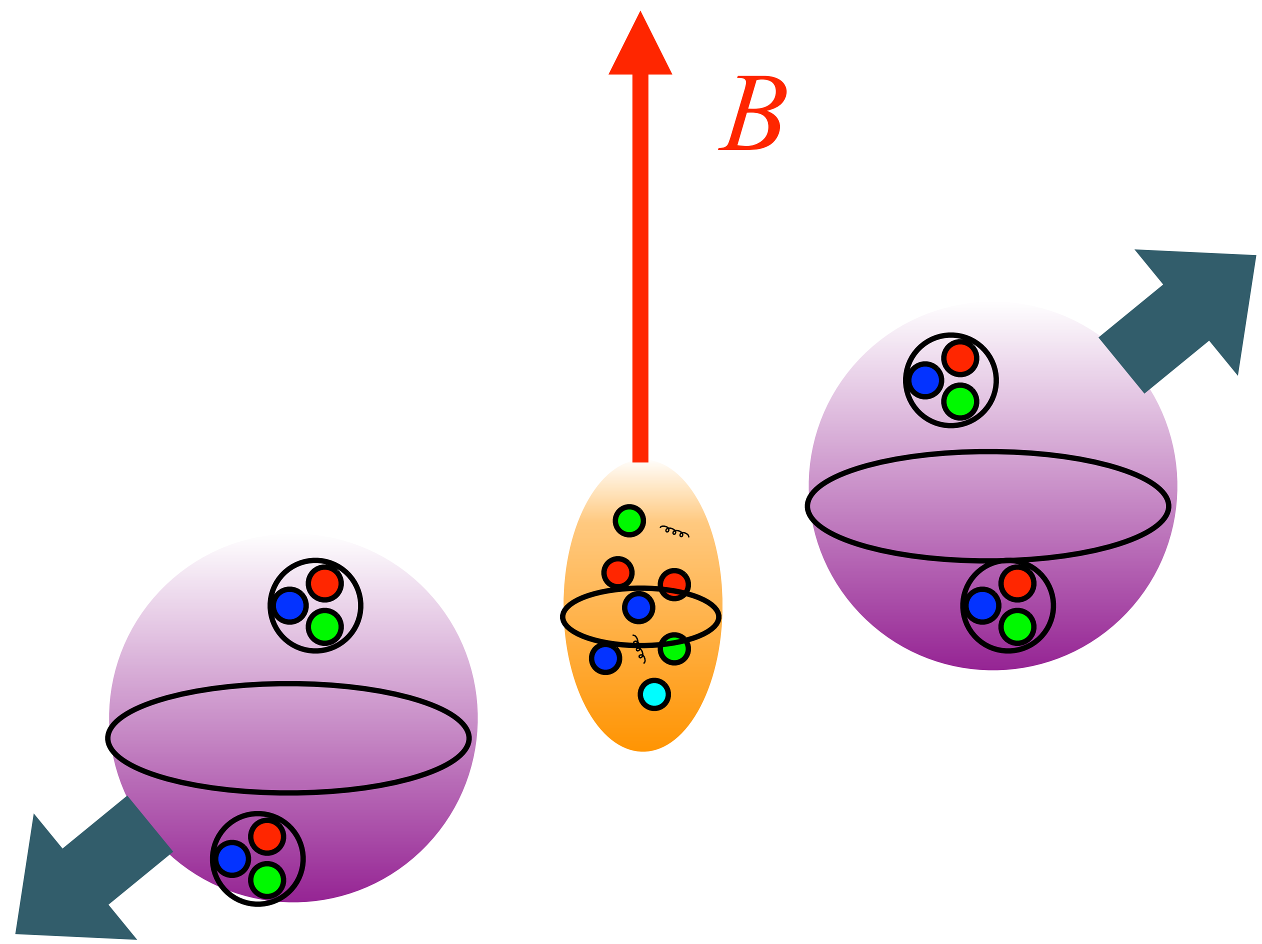
K. Hattori and **D. S.**, arXiv: 1610.06818 [hep-ph].
K. Hattori, S. Li, **D. S.**, H. U. Yee, arXiv: 1610.06839 [hep-ph].

Outline

- (Long) Introduction
—quarks and gluons in strong B
- Calculation of Conductivity and Results
- Possible Phenomenological Implications (Very Brief, more like future works)
- Summary and future perspective

Introduction

Strong magnetic field (B) may be generated in heavy ion collision due to Ampere's law.

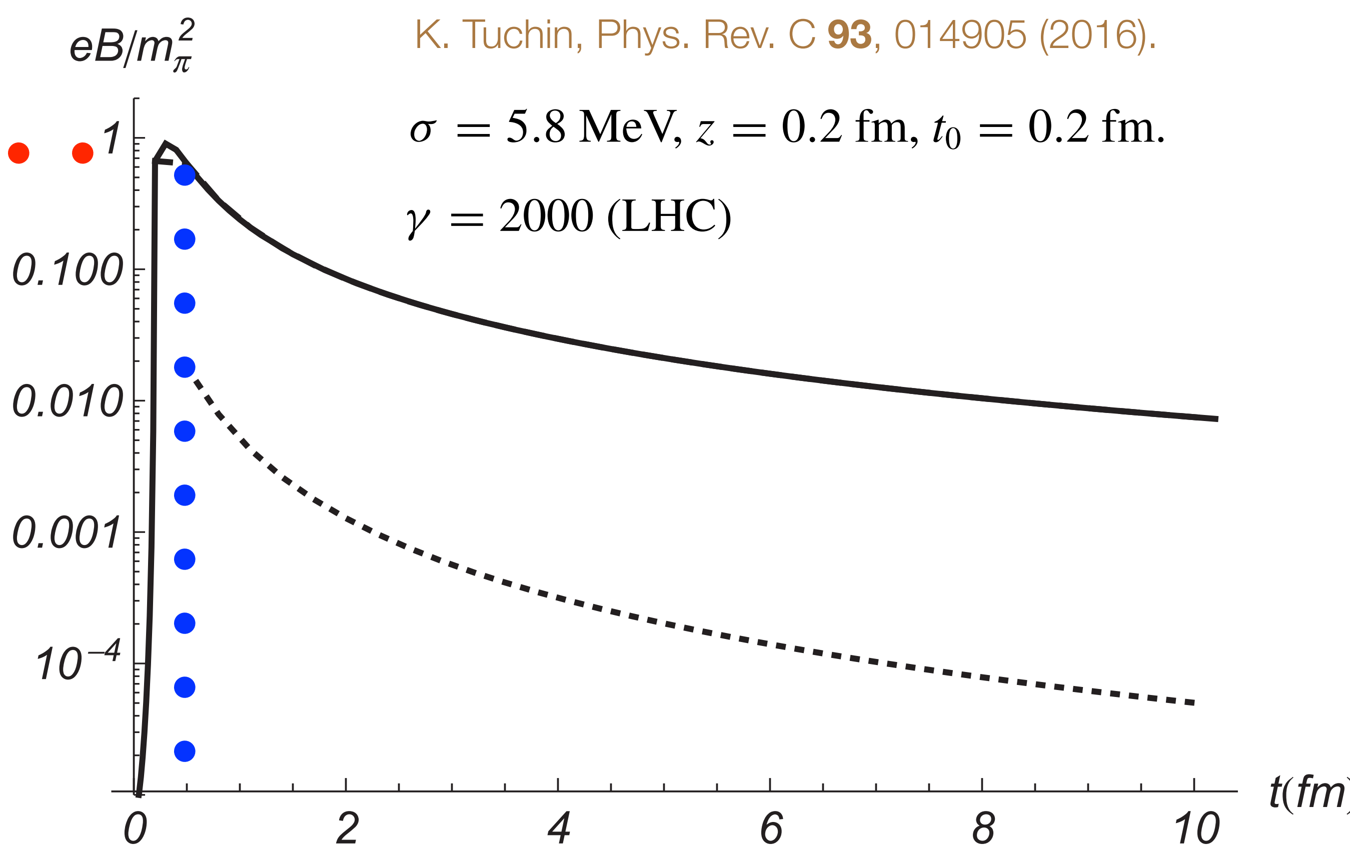


Introduction

strength:

$$\sqrt{eB} \sim 100\ [\mathrm{MeV}]$$

(Comparable with T)



lifetime: $\sim 0.3\,\mathrm{fm}$

At thermalization time (~0.5 fm), there still may be strong B.

Heavy ion collision may give a chance to investigate QCD matter at **finite temperature** in **strong magnetic field**.

Introduction

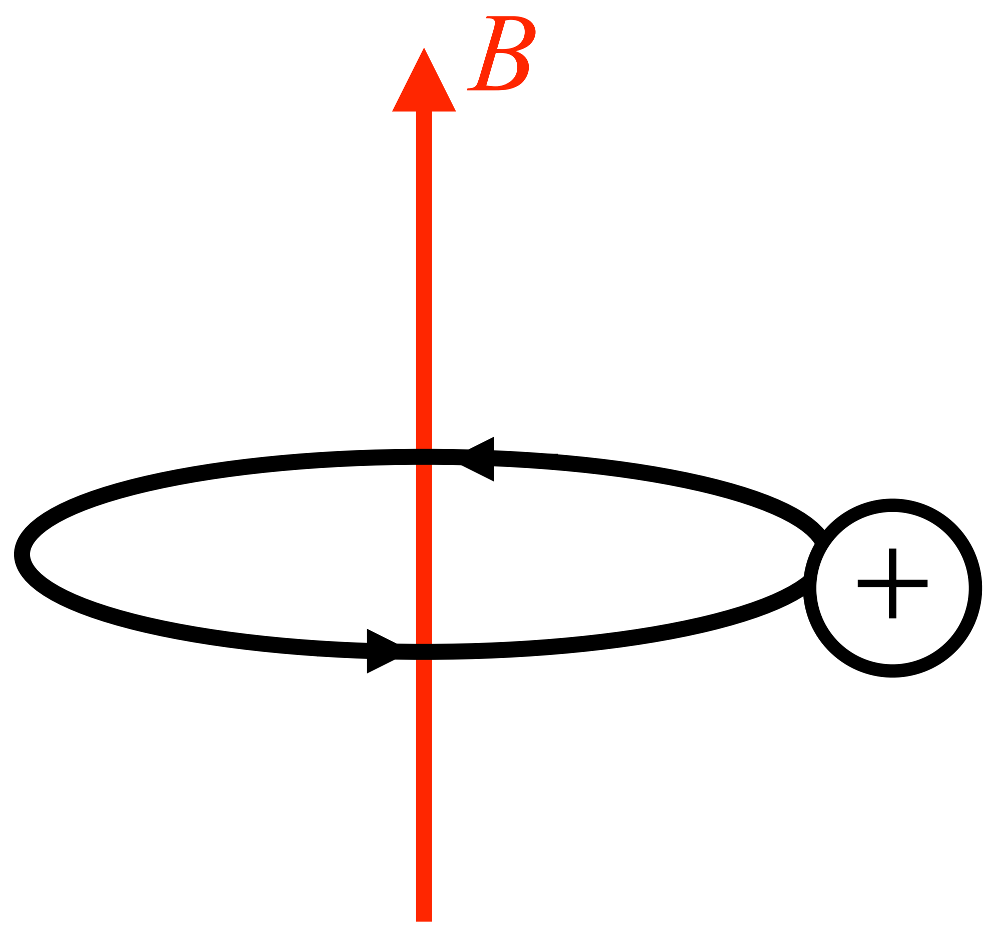
How the system behaves when the energy scale of B is much larger than the typical energy scale of the system?

$(\sqrt{eB} >> T, m, \Lambda_{\mathrm{QCD}} \ldots)$

Quark in Strong B

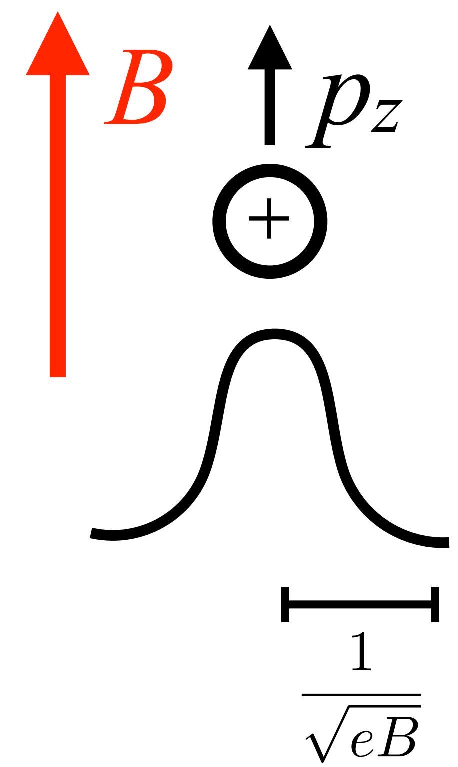
One-particle state of quark in magnetic field

Classical: Cyclotron motion due to Lorentz force



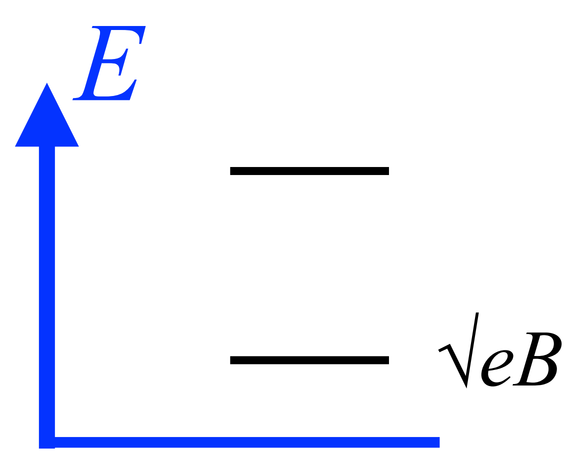
Quark in Strong B

Quantum: Landau Quantization



Longitudinal: Plane wave
Transverse: Gaussian

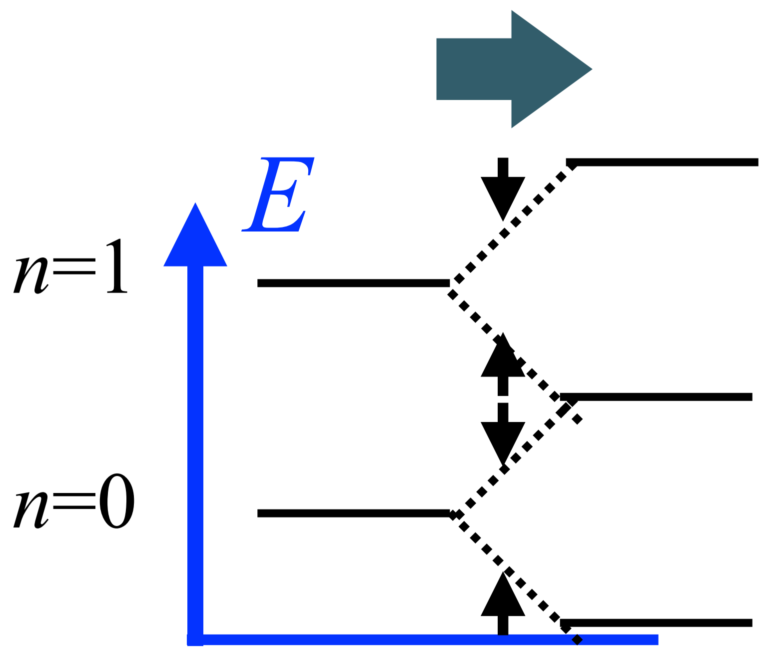
$$E_n = \sqrt{(p_z)^2 + m^2 + 2eB\left(n + \frac{1}{2}\right)}$$

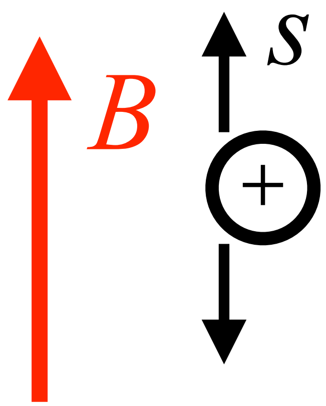


The gap (~$\sqrt{eB}$) is generated by zero-point oscillation.

Quark in Strong B

For spin-1/2 particle, we have Zeeman effect:





$$E_n = \sqrt{(p_z)^2 + m^2 + 2eB\left(n + \frac{1}{2} \mp \frac{1}{2}\right)}$$

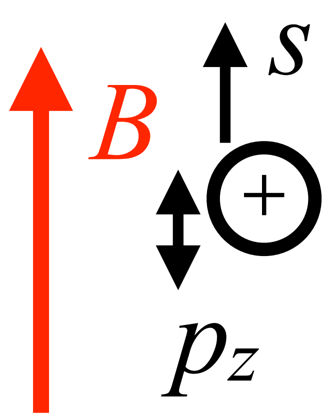
When n=0 (LLL), gap is small (m~1MeV).
When n>0, gap is large (~$\sqrt{eB}$~100MeV)

Lowest Landau Level (LLL) Approximation

When the typical energy of particle (T) is much smaller than gap ($\sqrt{eB}$), the higher LL does not contribute ($\sim\exp(-\sqrt{eB}/T)$), so **we can focus on the LLL**.

Confined in one direction fermion, no spin degrees of freedom.

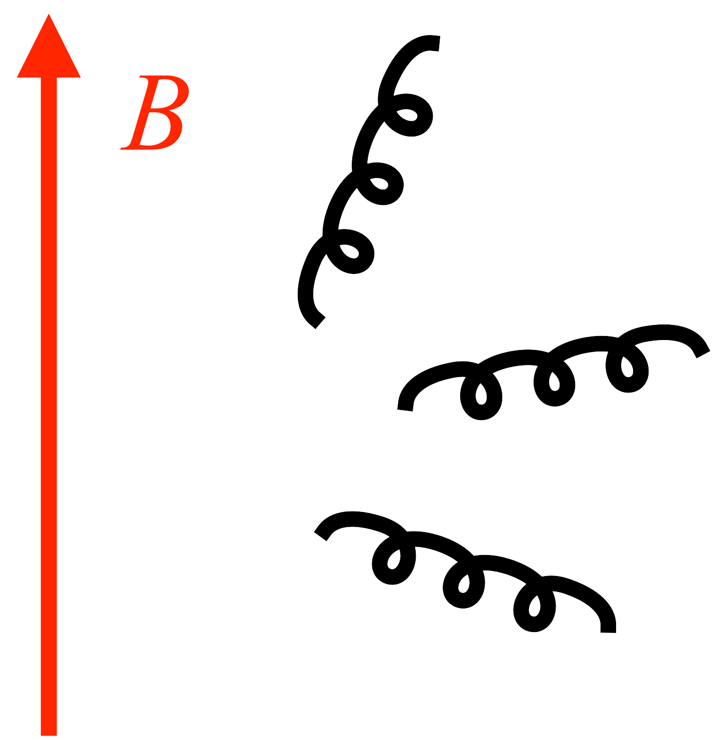
$$E_n = \sqrt{(p_z)^2 + m^2}$$



In heavy-ion collision, this condition may be marginally realized ($T\sim\sqrt{eB}\sim$100MeV).
But in Weyl semi-metal, it is already realized ($T\sim$1meV, $\sqrt{eB}\sim$10eV).

Gluon in Strong B

Gluon does not have charge, so it does not feel B in the zeroth approximation.

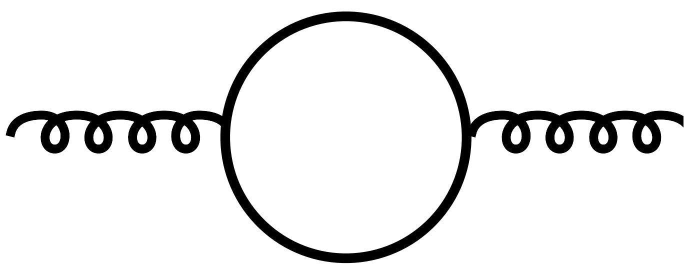


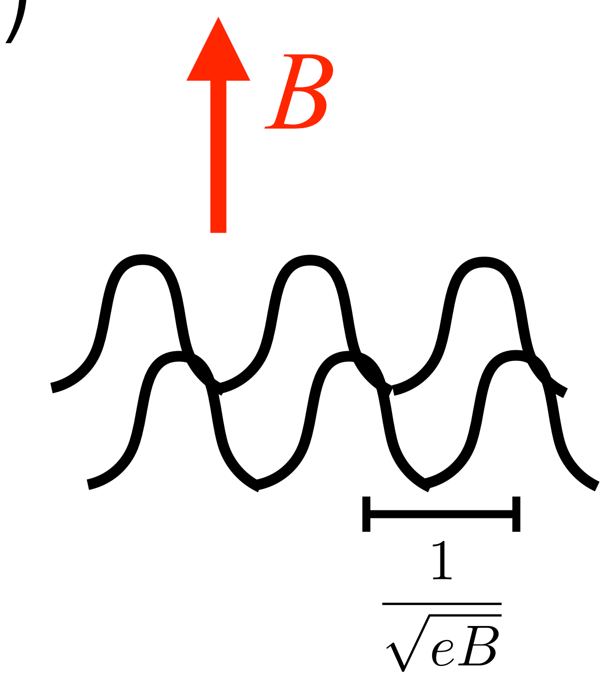
Massless boson in (3+1)D

Gluon in Strong B

K. Fukushima, Phys. Rev. D **83**, 111501 (2011).

Coupling with (1+1)D quarks generates gluon mass.
(Schwinger mass generation)





(surface density)
~(average distance)$^{-2}$~eB

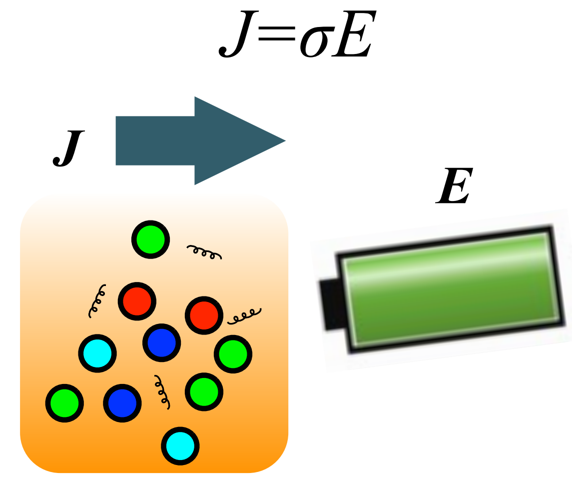
Color factor

$$M^2 \equiv \frac{1}{2} \times \frac{g^2}{\pi} \sum_f \frac{|eB|}{2\pi} \sim g^2 eB$$

Schwinger mass **Landau degeneracy**

Motivation to Discuss Electrical Conductivity

Electrical conductivity is phenomenologically important because



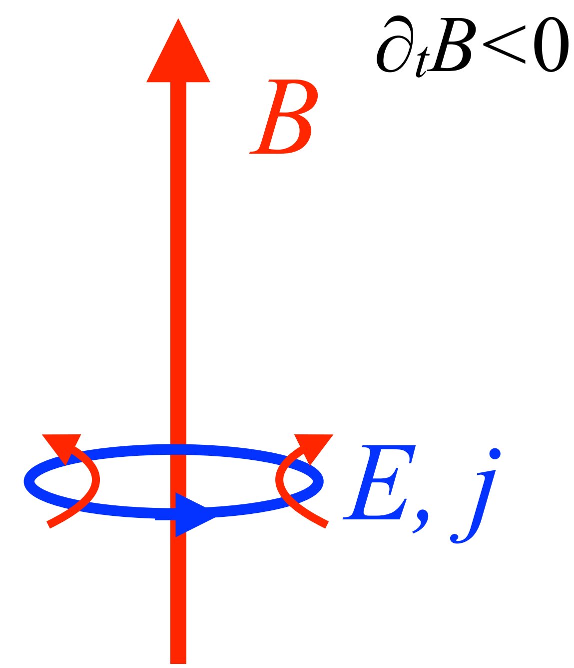
- Input parameter of magnetohydrodynamics (transport coefficient)

- May increase lifetime of $\boldsymbol{B}$ (Lenz's law)

$$\nabla \times \mathbf{E} = -\partial_t \mathbf{B}$$

$$\partial_t \mathbf{E} = \nabla \times \mathbf{B} - \mathbf{j}$$

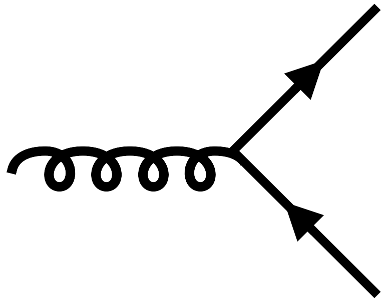
When σ is large



Possible Scattering Process for Conductivity

***B*=0**

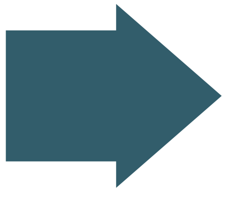
1 to 2 scattering is kinematically forbidden;
one massless particle can not decay to
two massless particles



strong *B*

Gluon is effectively massive in (1+1)D

$$E = \sqrt{p_z^2 + p_\perp^2 + M^2}$$



Decay of a gluon into two quarks becomes kinematically possible.

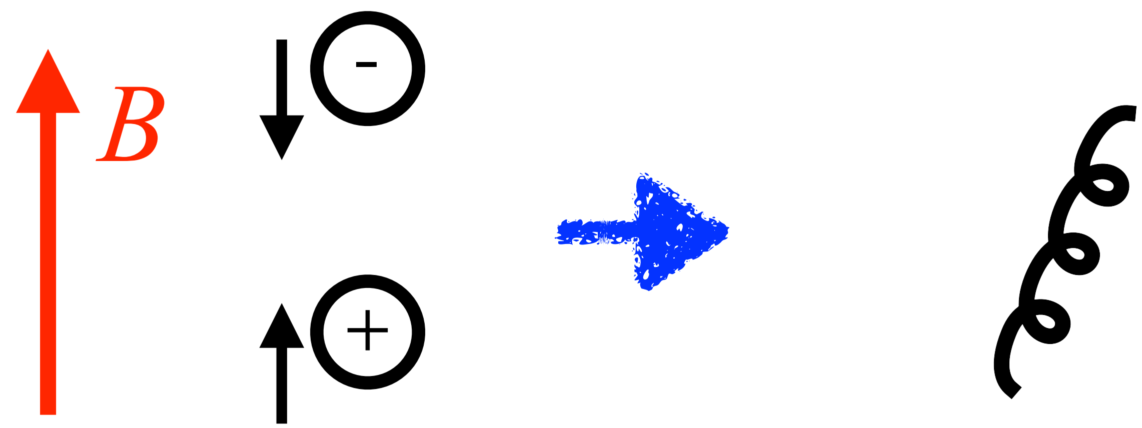
Chirality in (1+1)D

Spin is always up.

When m=0, the direction of p_z determines chirality.

Chirality in (1+1)D

Chirality is conserved at m=0:



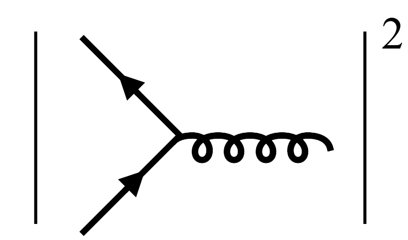
χ=+1+1=2 χ=0

1 to 2 scattering is forbidden at m=0.

Motivation to Discuss Electrical Conductivity

Electrical conductivity is also theoretically interesting: the scattering process is very different from that in B=0.

- Because the kinematics is non-standard (1+1 D for quark, 3+1D for gluon), the 1 to 2 scattering is the leading process, instead of 2 to 2.



- At m=0, the 1 to 2 process is forbidden due to chirality conservation. Thus, we need to include finite m effect to have non-divergent conductivity.

Outline

- (Long) Introduction
—quarks and gluons in strong B
- **Calculation of Conductivity and Results**
- Possible Phenomenological Implications (Very Brief, more like future works)
- Summary and future perspective

Hierarchy of Energy Scale at LLL

E

$\sqrt{eB}$ **Energy gap of higher LL**

$\gg$

T **Typical energy of quark and gluon**

$\gg$

m **Chirality flipping amplitude**

Hierarchy of Energy Scale at LLL

For ordering of m and M, we consider the both cases.

($m \ll M$ and $m \gg M$) ($M \sim g\sqrt{eB}$)

E: $\sqrt{eB} \gg T \gg M \gg m$

E: $\sqrt{eB} \gg T \gg m \gg M$

Electrical Conductivity

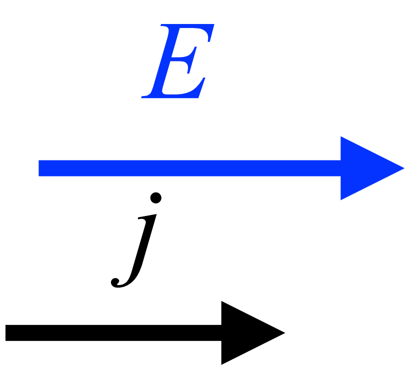
D. S., Phys. Rev. D, **90**, 034018 (2014).

Conductivity at weak B ($\sqrt{eB} << T$)

B=0

$$j^i = \sigma^{ij} E^j$$

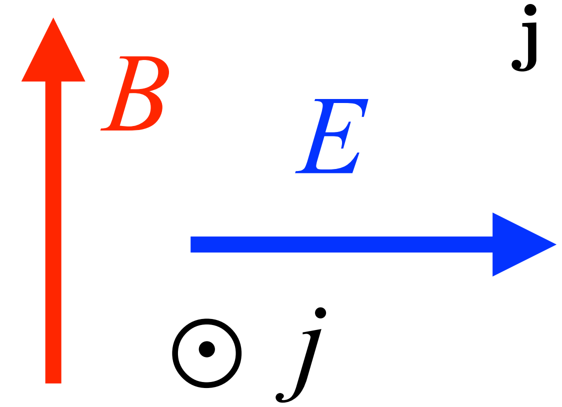
$$\mathbf{j} = \sigma_0 \mathbf{E}$$



$$\sigma^{ij} = \begin{bmatrix} \sigma_0 & 0 & 0 \\ 0 & \sigma_0 & 0 \\ 0 & 0 & \sigma_0 \end{bmatrix}$$

linear in B

$$\mathbf{j} = \sigma_1 \mathbf{E} \times \mathbf{B}$$



$$\sigma^{ij} = \begin{bmatrix} \sigma_0 & \sigma_1 & 0 \\ -\sigma_1 & \sigma_0 & 0 \\ 0 & 0 & \sigma_0 \end{bmatrix}$$

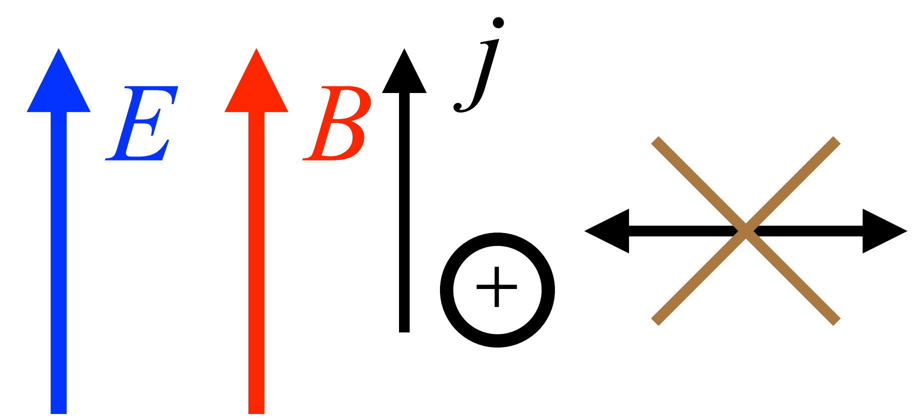
σ_0 is independent from B. ($\sigma_0 \sim e^2 T/g^4$)

σ_1 is linear in B. ($\sigma_1 \sim e^3 B\mu/g^8 T^2$)

Electrical Conductivity

Strong B (LLL)

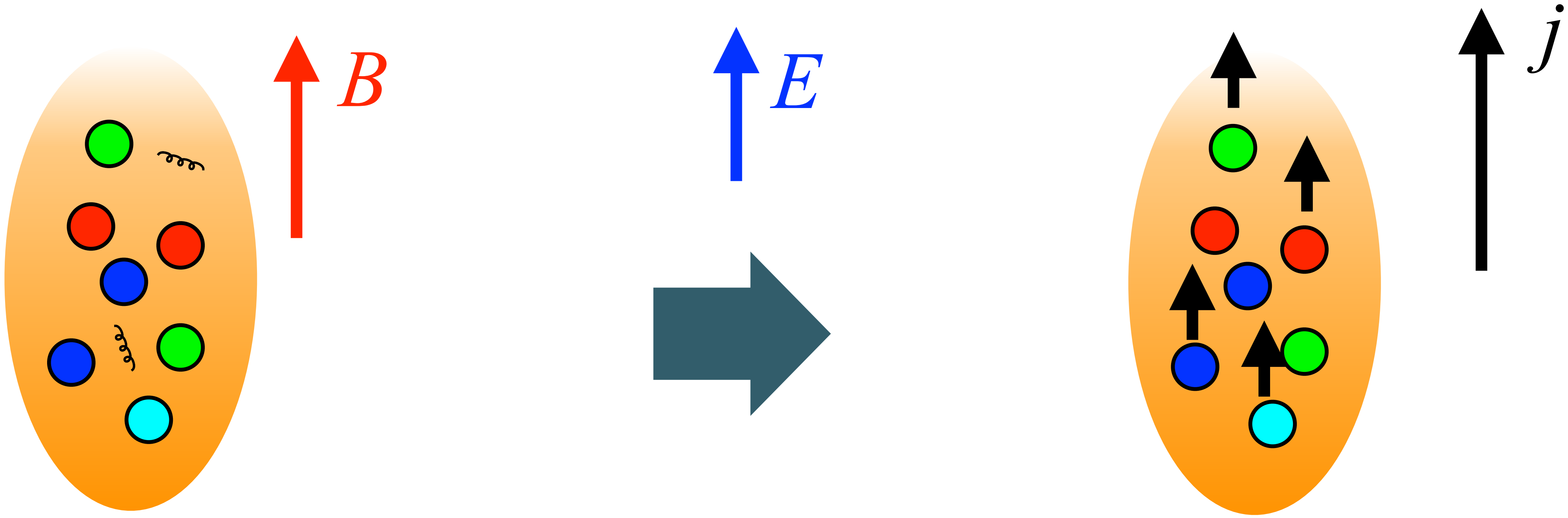
Quarks are confined in the direction of B,
so there is no current in other directions.



σ^{33} is finite, other components are zero.
(Very different from weak B case)

$$j^i = \sigma^{ij} E^j \qquad \sigma^{ij} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_{33} \end{bmatrix}$$

Calculation of conductivity



Thermal equilibrium in strong B

Linear response against E

Slightly non-equilibrium, finite j

$$n^f(k^3, T, Z) = n_F(\epsilon_k^L)$$

$$\epsilon_k^L \equiv \sqrt{(k^3)^2 + m^2}$$

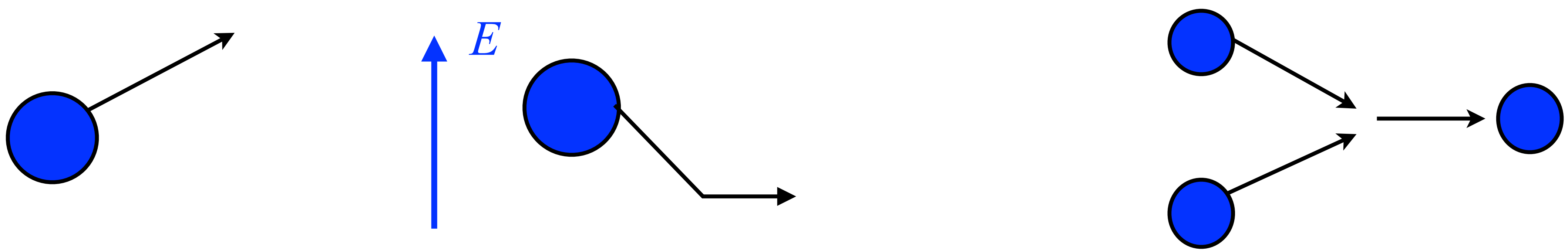
$$n^f(k^3, T, Z) = n_F(\epsilon_k^L) + \delta n^f(k^3, T, Z)$$

$$j^3(T, Z) = 2e \sum_f q_f N_c \frac{|eq_f B|}{2\pi} \int \frac{dk^3}{2\pi} v^3 \delta n^f(k^3, T, Z) = \sigma^{33} E^3$$

Evaluation of δn_F is necessary.

Calculation of conductivity

Evaluate n_F with (1+1)D Boltzmann equation



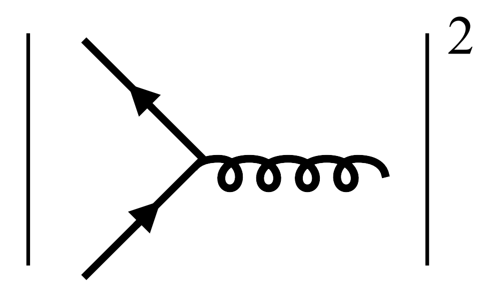
$$[\partial_T + v^3\partial_Z + eq_f E^3(T,Z)\partial_{k^3}]n^f(k^3,T,Z) = C[n]$$

$$v^3 \equiv \partial\epsilon_k^L/(\partial k^3) = k^3/\epsilon_k^L$$

1 to 2 collision:

$$C[n] = \frac{1}{2\epsilon_k^L}\int |M|^2[n_B^{k+l}(1-n_F^k)(1-n_F^l) - (1+n_B^{k+l})n_F^k n_F^l]$$

$$|M|^2 = 4g^2 C_f m^2$$



Vanishes at m=0!

(chirality conservation)

Calculation of conductivity

$$[\partial_T + v^3\partial_Z + eq_f E^3(T,Z)\partial_{k^3}]n^f(k^3,T,Z) = C[n]$$

linearize $n^f(k^3,T,Z) = n_F(\epsilon_k^L) + \delta n^f(k^3,T,Z)$

Constant E: ∂_T, ∂_Z=0

$$eq_f E^3\beta v^3 n_F(\epsilon_k^L)[1-n_F(\epsilon_k^L)] = C[\delta n^f(k^3,T,Z)]$$

$$C[n] = \frac{1}{2\epsilon_k^L}\int |M|^2[n_B^{k+l}(1-n_F^k)(1-n_F^l) - (1+n_B^{k+l})n_F^k n_F^l]$$

linearize

$$C[\delta n] = -\frac{1}{2\epsilon_k^L}\int_l |M|^2\left[\delta n_F^k\left(n_B^{k+l}+n_F^l\right) - \delta n_F^l\left(n_B^{k+l}+n_F^k\right)\right]$$

damping rate of quark ($=-2\xi_k\delta n^k{}_F$)

Calculation of conductivity

Solution for δn^F with damping rate ξ_k

$$\delta n_F^k = -\frac{1}{2\xi_k} e q_f E^3 \beta v^3 n_F(\epsilon_k^L)[1 - n_F(\epsilon_k^L)]$$

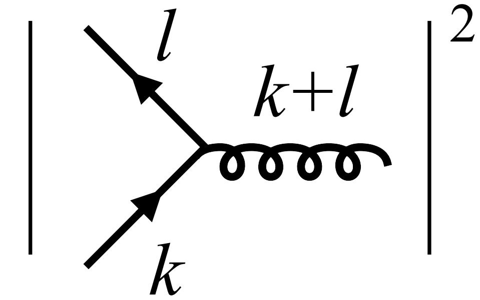
$$j^3(T,Z) = 2e \sum_f q_f N_c \frac{|B_f|}{2\pi} \int \frac{dk^3}{2\pi} v^3 \delta n^f(k^3, T, Z)$$

$$\Rightarrow \quad j^3 = e^2 \sum_f q_f^2 N_c \frac{|eq_f B|}{2\pi} 4\beta \int \frac{dk^3}{2\pi} (v^3)^2 \frac{1}{2\xi_k} n_F(\epsilon_k^L)[1 - n_F(\epsilon_k^L)] E^3$$

$$\sigma^{33}$$

Quark Damping Rate

$$\epsilon_k^L \xi_k = \frac{g^2 C_F m^2}{4\pi} \int_m^\infty dl^0 \frac{n_F(l^0) + n_B(l^0 + \epsilon_k^L)}{\sqrt{(l^0)^2 - m^2}}$$



leading-log approximation ($\ln[T/m] >> 1$)

$l^0 << T$ dominates

$$\epsilon_k^L \xi_k \simeq \frac{g^2 C_F m^2}{4\pi} \left[\frac{1}{2} + n_B(\epsilon_k^L)\right] \int_m^\infty dl^0 \frac{1}{\sqrt{(l^0)^2 - m^2}}$$

$$\simeq \frac{g^2 C_F m^2}{4\pi} \left[\frac{1}{2} + n_B(\epsilon_k^L)\right] \ln\left(\frac{T}{m}\right)$$

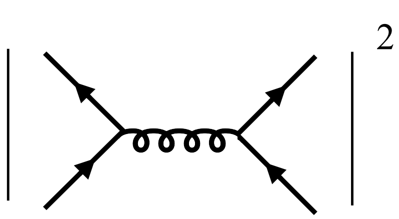
matrix element

log divergence in phase space integral

UV cutoff: T

IR cutoff: m

cf: 2 to 2

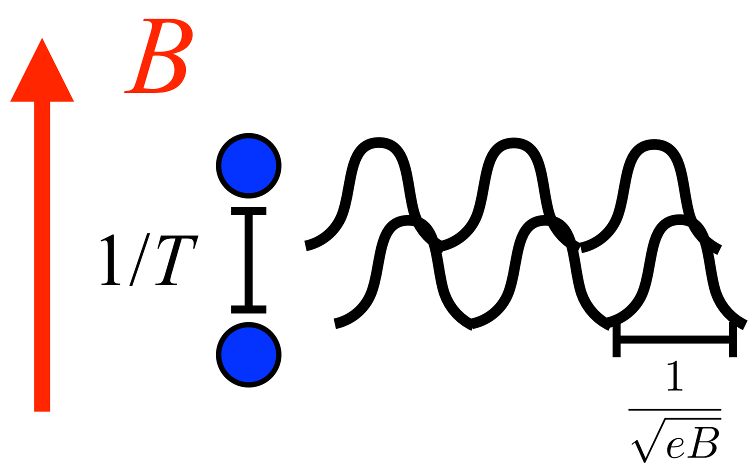


$\sim g^4$

soft fermion and hard boson

$n_F(1+n_B)+(1-n_F)n_B=n_F+n_B$

Results



(average distance among quarks)~$1/T$
→(quark density in 1D)~T

Quark density in 1D

$$\sigma^{33} = e^2 \sum_f q_f^2 N_c \frac{|eq_f B|}{2\pi} \frac{4T}{g^2 C_F m^2 \ln(T/m)}$$

Landau degeneracy

Quark damping rate

Due to chirality conservation, collision is forbidden when m=0. Thus, σ~$1/m^2$. Despite T>>m, it is very sensitive to m!!

When M>>m, $\ln(T/m) \rightarrow \ln(T/M)$.

Other Term Does Not Contribute

$$C[\delta n] = -\frac{2g^2 C_F m^2}{\epsilon_k^L} \int_l [\delta n_F^k (n_B^{k+l} + n_F^l) - \delta n_F^l (n_B^{k+l} + n_F^k)]$$

Other Term

$$\delta n_F^l = -\frac{eq_f}{2\xi_l} E^3 \partial_{l^3} n_F(\epsilon_l^L)$$:odd in l^3

function of $(\varepsilon^L{}_k + \varepsilon^L{}_l)$

(Other term)~ $\int_l (n_B^{k+l} + n_F^k) \delta n_F^l$ → 0

even in l^3

Same for $m << M$ case.

Our result (only retaining quark damping rate) **is correct.**

Equivalent Diagrams

Our calculation is based on (unestablished) (1+1)D kinetic theory, but actually **we can reproduce the same result by field theory calculation**.

J. -S. Gagnon and S. Jeon, Phys. Rev. D **75**, 025014 (2007); **76**, 105019 (2007).

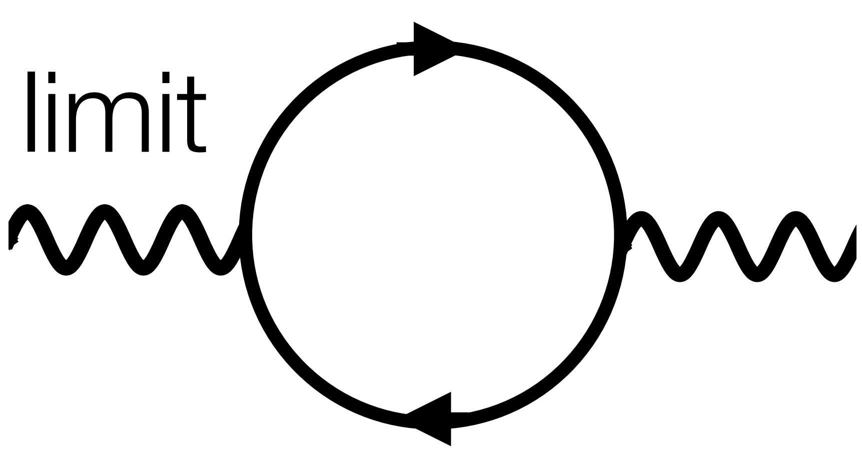
Kubo formula: $\sigma^{ij} \equiv \lim_{\omega\to 0} \frac{\Pi^{Rij}(\omega)}{i\omega}$

$j^\mu \equiv e \sum_f q_f \overline{\psi}_f \gamma^\mu \psi_f$

f: flavor index, q_f: electric charge

$\Pi^{R\mu\nu}(x) \equiv i\theta(x^0)\langle[j^\mu(x), j^\nu(0)]\rangle$

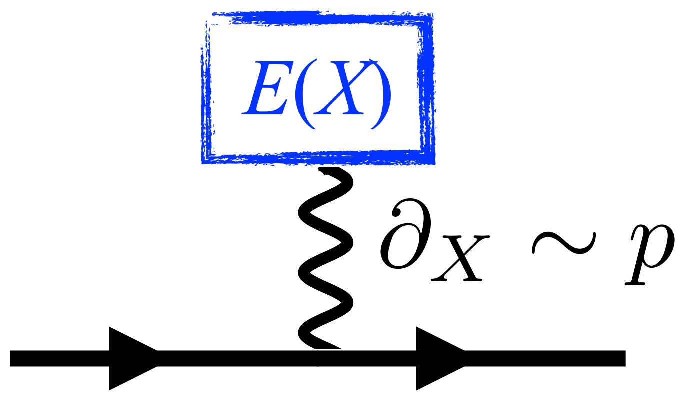
$p\to 0$ limit



Equivalent Diagrams

J. -P. Blaizot and E. Iancu, Nucl. Phys. B **557**, 183 (1999).

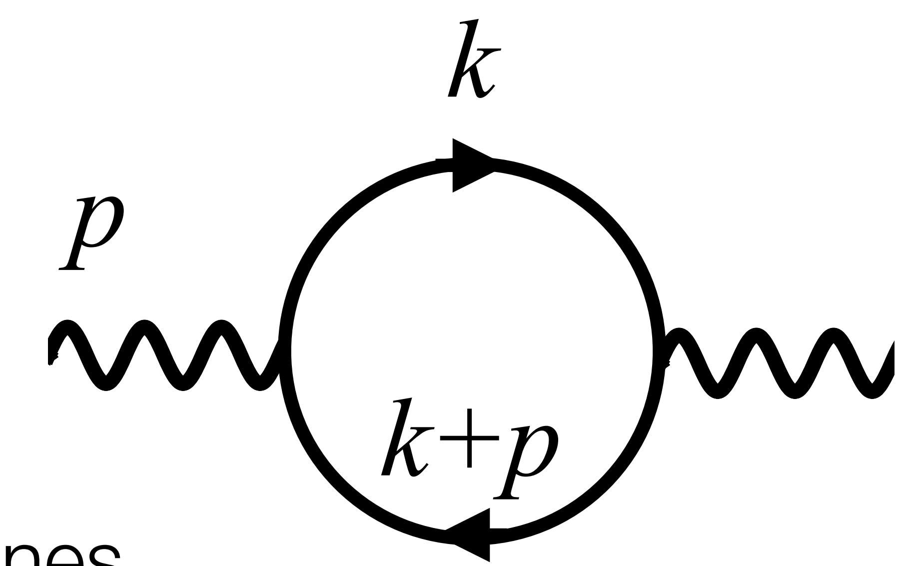
Kinetic eq.



$k=(\varepsilon^L{}_k,\vec{k})=\varepsilon^L{}_k(1,\vec{v})$

Connect the ends of lines

Field Theory



$$\frac{1}{(k+p)^2 - m^2 + 2i\xi_{k+p}(k^0+p^0)}$$

on-shell condition: $k^2\text{-}m^2 = 2i\xi_k k^0$,
$p \rightarrow 0$ limit

$$\frac{1}{2\left(k\cdot p + 2i\xi_k k^0\right)}$$

$$(v\cdot\partial_X + 2\xi_k)\delta n^f = -eq_f E^3 \partial_k n^f(k)$$

Outline

- (Long) Introduction
—quarks and gluons in strong B
- Calculation of Conductivity and Results
- **Possible Phenomenological Implications (Very Brief, more like future works)**
- Summary and future perspective

Possible Phenomenological Implications

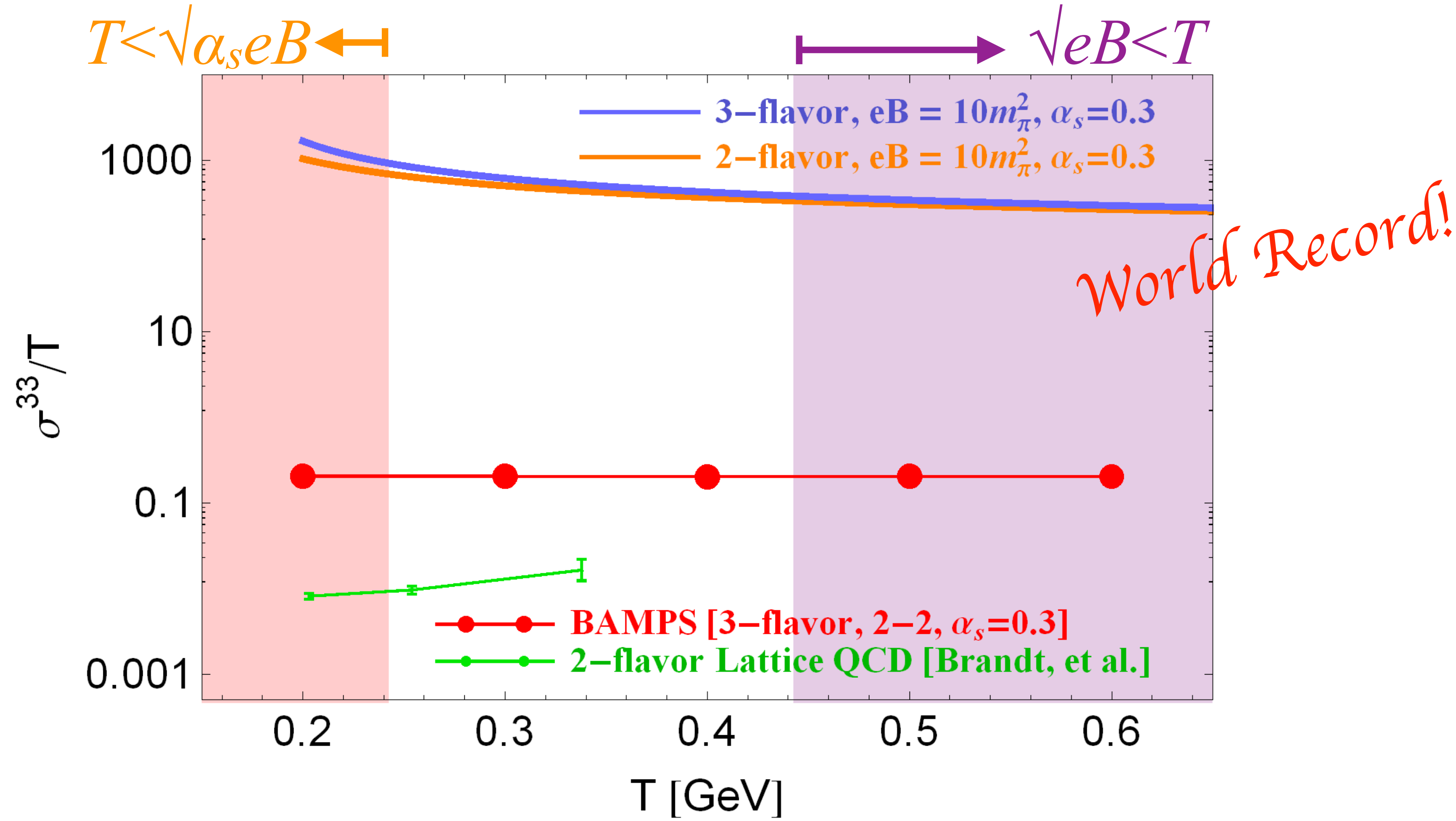
1. Order Estimate

$$\sigma^{33} = e^2 \sum_f q_f^2 N_c \frac{|eq_f B|}{2\pi} \frac{4T}{g^2 C_F m^2 \ln(T/M)}$$

Because of m^{-2} dependence, s contribution is very small.

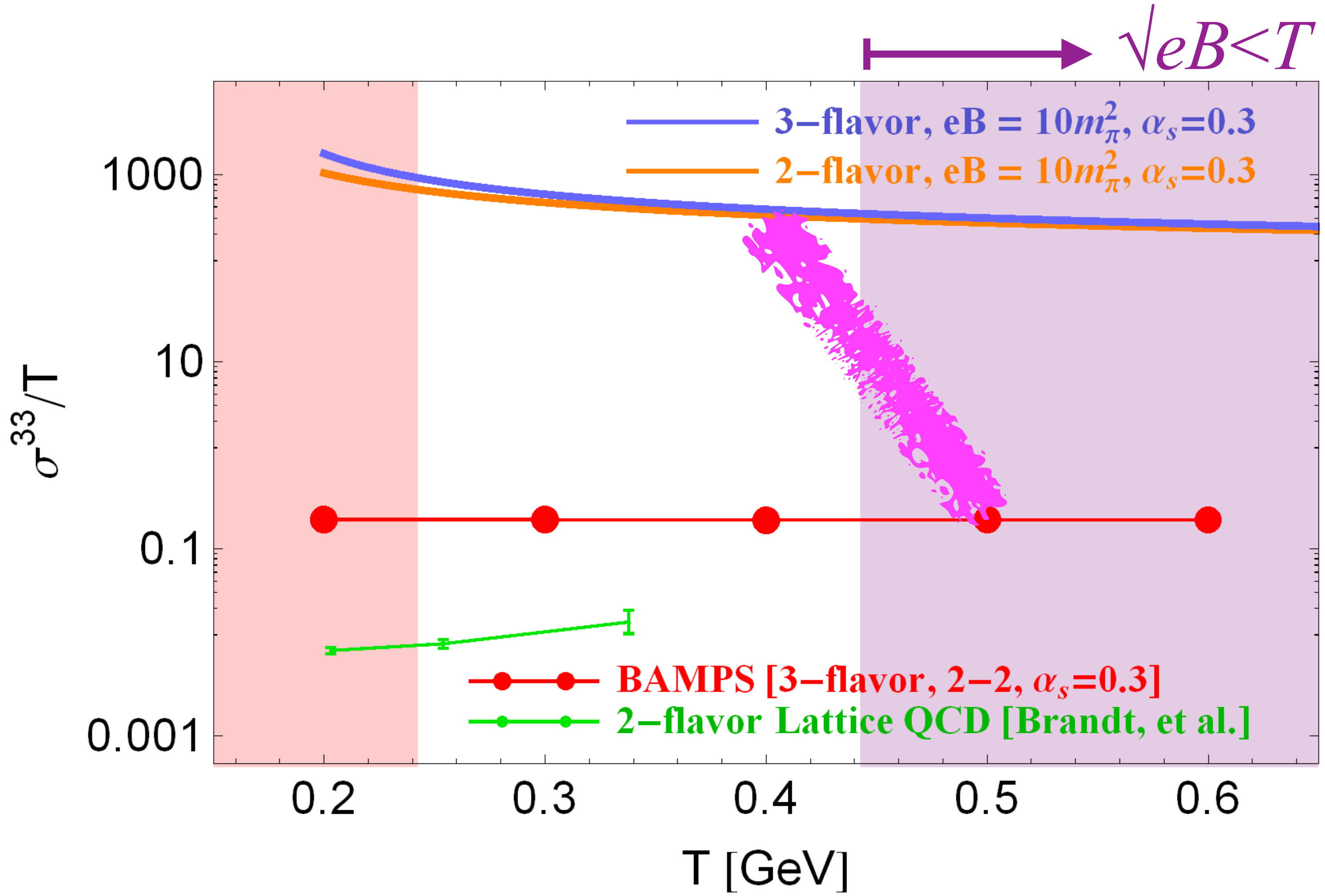
$$\alpha_s = \frac{g^2}{4\pi} = 0.3,$$
$$m = 3\text{MeV}(u,d), 100\text{MeV}(s),$$
$$eB = 10m_\pi^2 = (440\text{MeV})^2.$$

M=160MeV>>m

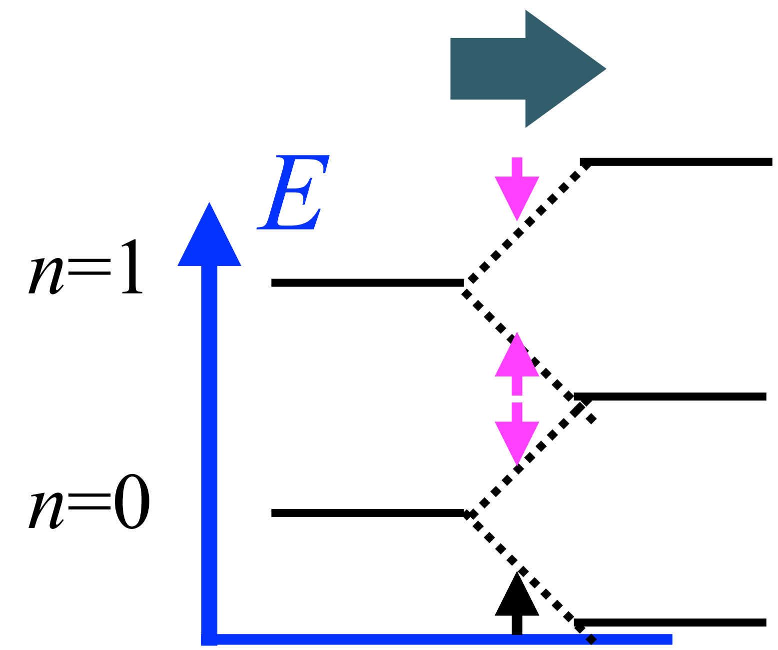


BAMPS: M. Greif, I. Bouras, C. Greiner and Z. Xu, Phys. Rev. D **90**, 094014 (2014).
Lattice: B. B. Brandt, A. Francis, B. Jaeger and H. B. Meyer, Phys. Rev. D **93**, 054510 (2016).

Possible Phenomenological Implications



$$\sigma^{33} = e^2 \sum_f q_f^2 N_c \frac{|eq_f B|}{2\pi} \frac{4T}{g^2 C_F m^2 \ln(T/M)}$$

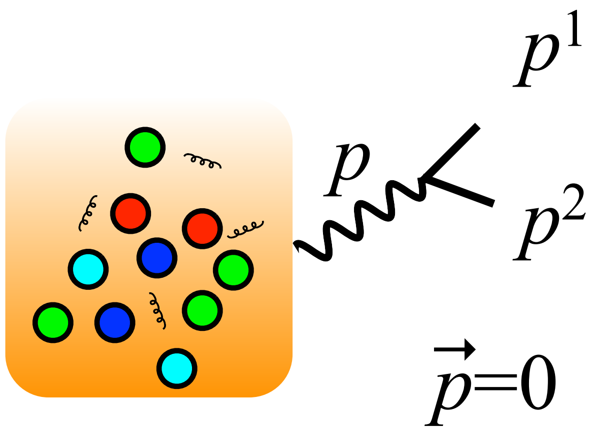


Beyond LLL approximation, there are also spin down particles, so the scattering is not suppressed by m^2.

σ^{33} is expected to be smaller at large T, so that it smoothly connects with B=0 result.

Possible Phenomenological Implications

2. Soft Dilepton Production



G. D. Moore and J. -M. Robert, hep-ph/0607172.

$$\frac{d\Gamma}{d^4p} = \frac{\alpha}{12\pi^4\omega^2} T\sigma^{33}$$

∵(virtual photon emission rate)~$n_B(\omega)\mathrm{Im}\Pi^{\mu}{}_{\mu} \sim T\sigma^{33}$

(photon interaction energy w leptons) (quark mean free path)$^{-1}$

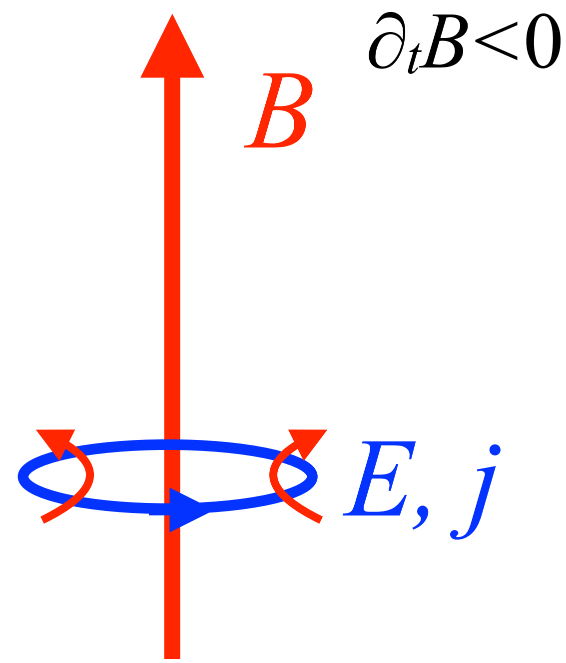
$$e\sqrt{eB} \ll \omega \ll \frac{g^2m^2}{T}\ln\left(\frac{T}{M}\right)$$

σ^{33} is large

Soft dilepton production is enhanced by B?

Possible Phenomenological Implications

3. Back Reaction to EM Fields



<u>Bad news</u>:

In LLL approximation, we have **no current in transverse plane**, so **Lenz's law does NOT work!**

The lifetime of B does not increase…

Summary

- We **calculated electrical conductivity in strong B using the LLL approximation,** for $m<<M$ and $m>>M$ cases.

Quark density in 1D

$$\sigma^{33} = e^2 \sum_f q_f^2 N_c \frac{|eq_f B|}{2\pi} \frac{4T}{g^2 C_F m^2 \ln(T/m)}$$

Landau degeneracy

Quark damping rate

When $M>>m$, $\ln(T/m) \to \ln(T/M)$.

- We **found that the conductivity is enhanced by large B, and small m**. The sensitivity to m was explained **in terms of chirality conservation.**

Future Perspective

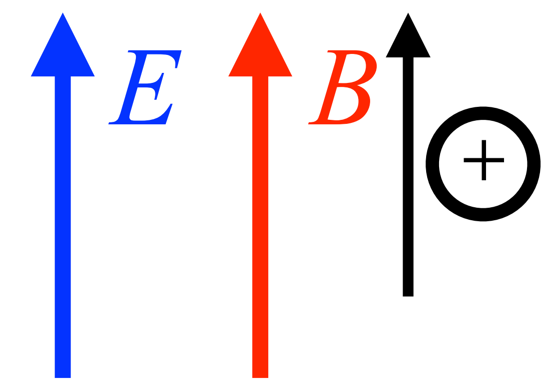
- Go beyond LLL approximation (more realistic B)

- Calculate other transport coefficients (viscosity, heat conductivity…)

Back Up

Electrical Conductivity

Axial anomaly: $\partial_t n_A = \frac{e^2 N_c N_F}{2\pi^2}\boldsymbol{E}\cdot\boldsymbol{B} - \frac{1}{\tau_R} n_A$



Stationary solution: $n_A = \frac{e^2 N_c N_F}{2\pi^2}\boldsymbol{E}\cdot\boldsymbol{B}\tau_R$

Chiral magnetic effect: $\boldsymbol{J} = \frac{e^2 N_c N_F}{2\pi^2}\mu_A \boldsymbol{B} = \frac{e^2 N_c N_F}{2\pi^2 \chi} n_A \boldsymbol{B}$

$$\sigma_{zz} = \frac{e^4 (N_c N_F)^2 B^2}{4\pi^4 \chi}\tau_R \qquad \chi = N_c \frac{1}{2\pi}\left(\frac{eB}{2\pi}\right)$$

1. Sphaleron $\frac{1}{\tau_{R,s}} = \frac{(2N_F)^2 \Gamma_s}{2\chi T}$ $\qquad \Gamma_s \sim \alpha_s^5 \log(1/\alpha_s) T^4$

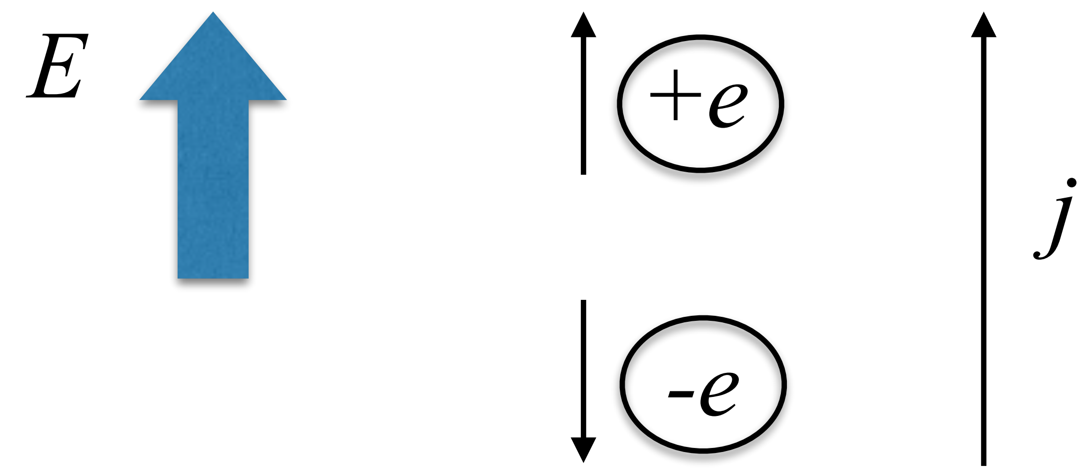
$\wedge\wedge$ $\quad \alpha_s m_q^2 \gg \alpha_s^5 T^2$

2. Current quark mass $\frac{1}{\tau_{R,m}} \sim \alpha_s m_q^2 / T$

$$\sigma_{zz} \sim e^2 N_c (eB) T \frac{1}{\alpha_s m_q^2}$$

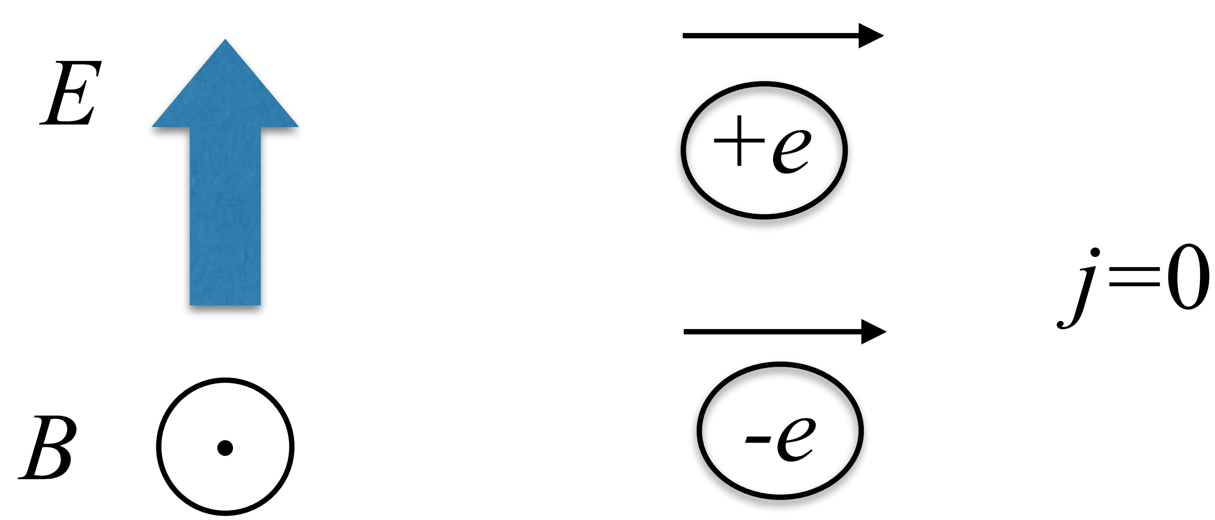
Collision-dominant case —Boltzmann equation

<u>Ohmic current</u>



exists even when $\mu=0$.

<u>Hall current</u>



cancels when $\mu=0$.

Motivation

➡ **Compute electrical conductivity in strong B limit!**

Hierarchy of Energy Scale at LLL

E

$\sqrt{eB}$ **Energy gap of higher LL**

$\gg$

T **Typical energy of quark and gluon**

$\gg$

m **Chirality flipping amplitude**

We also assume that **the gluon screening mass (*M*~*g*√*eB*) is much smaller than *T***, so that the gluon is thermally excited.