Gertrude Stein about Oakland, California, ~ 1890:

“There’s no there, there.”
Beam Energy Scan at RHIC:

There is a there, there

But what is it?
The Phases of QCD

Early Universe
Future LHC Experiments

Current RHIC Experiments

Quark-Gluon Plasma

Crossover

~170 MeV

Critical Point

Hadron Gas

1st order phase transition

Future FAIR Experiments

Color Superconductor

Vacuum

Nuclear Matter

Neutron Stars

Baryon Chemical Potential
Beam Energy Scan @ RHIC, down to $\sqrt{s/A} = 7$ GeV

Exp.’y, measure moments of pressure w.r.t. $\mu = \text{quark chemical potential}$:

$$c_n = \frac{\partial^n}{\partial \mu^n} p(T, \mu)$$

Ratio of $4^{th}/2^{nd}$ moments:
~ 1 above 40 GeV, dips below 1, BIG increase from 19 to 7 GeV

The first “there”

N.B.: increase is due to $p_{tr}$ above .8 GeV: weird if critical endpoint
Slice & Dice the moments with convolution correlators

Bill Llope, CPOD ‘17, STAR:

Consider two-particle correlations, along the beam axis (rapidity \( y \)) and w.r.t. angle transverse to the beam (\( \theta \))

\[
R_2 = \frac{\rho_2(y_1, y_2)}{\rho_1(y_1)\rho_2(y_2)} - 1
\]

Integral of \( R_2 \) w.r.t. rapidities \( y_1 \) and \( y_2 \) is related to \( c_2 \) moment

Berger, NPB 85, ‘75; Carruthers & Sarcevic PRL 63, ‘89; M Jacob, Phys Rep 315, ‘99
Bzdak, 1108.0882; Bzdak & Teaney 1210.1965; Jia, Radhakrishnan & Zhou, 1506.03496
Ling & Stephanov, 1512.09125; Bzdak, Koch, & Strodthoff 1607.07375
The there, there

Preliminary

The there, there!
Lattice: *no critical point nearby*

Vovchenko, Steinheimer, Philipsen, Stoecker 1711.0126:
Cluster Expansion Method (CEM) for baryon fluctuations on the lattice: (not Taylor expansion in powers of $\mu$, powers asymptotic behavior in $\mu$.)

*No critical endpoint accessible by experiment: so what is it?*
Matrix models & a (pseudo-) Lifshitz point in QCD

Chiral matrix model: marrying
a linear sigma model, for the chiral transition
plus a “matrix model”, to characterize deconfinement
RDP & VV Skokov, 1604.00002

Quarkyonic chiral spirals and a (pseudo-) Lifshitz point in QCD:
RDP, VV Skokov & A Tsvelik, 1712.x
Fluctuations from a pseudo-Lifshitz point at low energies?

Finite size effects for baryon # cumulants:
G Almasi, VV Skokov, & RDP, 1612.04416

Tetraquarks in QCD: two chiral order parameters, two chiral transitions?
RDP & VV Skokov 1606.04111

Solution for SU(∞): RDP & VV Skokov; 1205.0545
S Lin, RDP & VV Skokov, 1301.7432;
H Nishimura, RDP & VV Skokov, 1712.04465
Matrix model for deconfinement

Polyakov Loop:

\[ \ell = \frac{1}{3} \text{tr} \mathcal{P} \exp \left( i g \int_0^{1/T} A_0 \, d\tau \right) \]

_Simplest_ approximation to give a non-trivial loop: constant, diagonal \( A_0 \):

\[ A_0^{cl} = \frac{2\pi T}{3g} \lambda_3 \, q(T) \quad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \]

Depends upon single function, \( q(T) \), fixed from pressure(T).

Only need two parameters to fit pressure, then compute
Matrix model for pure glue

To one loop order, Stefan-Boltzmann + potential for $q$

$$\mathcal{V}_{\text{pert}}(q) = \frac{2\pi^2}{3} T^4 \left( -\frac{4}{15} + \sum_{a,b} q_{ab}^2 (1 - q_{ab})^2 \right), \quad q_{ab} = |q_a - q_b|_{\text{mod} \, 1}$$

From lattice data for pure glue, assume non-pert. potential $\sim T^2$:

$$\mathcal{V}_{\text{non}}(q) = \frac{2\pi^2}{3} T^2 T_d^2 \sum_{a,b} \left( -c_1 q_{ab} (1 - q_{ab}) - c_2 q_{ab}^2 (1 - q_{ab})^2 + c_3 \right)$$

From lattice for pure glue: $T_d = 270$ MeV.

Constant term $\sim c_3$ most important for $T > 1.2 \, T_d$.

$q$’s only matter for $T < 1.2 \, T_d$: narrow transition region

Dumitru, Guo, Hidaka, Korthals-Altes & RDP, 1011.3820 & 1205.0137 + ....
Chiral symmetry

For 3 flavors of massless quarks,

\[ \mathcal{L}^{q_k} = \bar{q} \not{D} q = \bar{q}_L \not{D} q_L + \bar{q}_L \not{D} q_L, \quad q_{L,R} = \frac{1 \pm \gamma_5}{2} q \]

Classically, global flavor symmetry of SU(3)_L x SU(3)_R x U(1)_A,

\[ q_L \rightarrow e^{-i\alpha/2} U_L q_L, \quad q_R \rightarrow e^{+i\alpha/2} U_R q_R \]

Simplest order parameter for \( \chi \) sym. breaking: \( a,b\ldots = \text{flavor.} \) \( A,B\ldots = \text{color} \)

\[ \Phi^{ab} = \bar{q}_L^{bA} q_R^{aA} \quad \Phi \rightarrow e^{+i\alpha} U_R \Phi U_L^\dagger \]

Quantum mechanically, axial U(1)_A is broken by instantons +\ldots\ to Z(3)_A at T=0
\‘t Hooft instanton vertex is invariant under Z(3)_A:

\[ \det \Phi \rightarrow e^{3i\alpha} \det \Phi \]

As \( T \rightarrow \infty \), U(1)_A approximately restored as \( 1/T^7 \rightarrow 9 \).
Usual linear sigma model

Linear sigma model for $\Phi$:

$$\mathcal{V}_\Phi = m^2 \text{tr} \left( \Phi^\dagger \Phi \right) - c_A \left( \text{det} \ \Phi + \text{c.c.} \right) + \lambda \text{tr} \left( \Phi^\dagger \Phi \right)^2$$

Drop $\text{tr} \left( \Phi^\dagger \Phi \right)^2$: fits show coefficient is really small

Mass, quartic terms $U(1)_A$ invariant; $\text{det} \ \Phi$ only under $Z(3)_A$.

For light but massive quarks, need to add

$$\mathcal{V}_H^0 = - \text{tr} \left( H \left( \Phi^\dagger + \Phi \right) \right)$$

So $m_\pi^2 \sim H$, etc. Standard linear sigma model.
Chiral matrix model

Quarks generate potential in "q", so **must couple Φ to quarks:** \( P_{L,R} = (1 \pm \gamma_5)/2 \)

\[
\mathcal{L}^q_{\Phi} = \overline{q} \left( D + \mu \gamma^0 + y \left( \Phi \mathcal{P}_L + \Phi^\dagger \mathcal{P}_R \right) \right) q
\]

Use matrix model from pure glue, with *same* \( T_d = 270 \text{ MeV} \).

With quarks, \( T_d \) is *just* a parameter in a potential, *not* deconfining \( T_c \).

From quark loop, **need logarithmic term in Φ:**

\[
\mathcal{V}^\text{log}_{\Phi} = \kappa \text{tr} \left( (\Phi^\dagger \Phi)^2 \log \left( \frac{M^2}{\Phi^\dagger \Phi} \right) \right)
\]

To 1 loop order, \( \kappa = 3y^4/(16 \pi^2) \); \( y \) is a free parameter, fit to \( T_\chi \).

Log term complicates things, results similar to that for \( \kappa = 0 \).
New symmetry breaking term

With usual symmetry breaking, at high T,

\[ \mathcal{V}^{eff} \approx -h \phi + \frac{y^2 T^2}{12} \phi^2 + \ldots, \ T \to \infty \]

1\textsuperscript{st} term SB‘g, 2\textsuperscript{nd} quark fluctuations.

But then at high T, no symmetry breaking!

\[ \phi \sim \frac{12h}{y^2 T^2} , \ m_q \sim y \phi \sim \frac{1}{T^2} \]

Solve by adding a new term \textit{by hand}

\[ \mathcal{V}^{eff} \sim h \phi - \frac{y}{6} m_0 T^2 \phi + \frac{y^2 T^2}{12} \phi^2 + \ldots \]

So \( \phi \sim m_0/y \) at high T, \( m_q \sim m_0 \). In QCD,

\[ \mathcal{V}_h^T = -\frac{m_q}{V} \left( \left. \frac{\text{tr}}{D + i \gamma^0 + y \Phi} \right|_{T \neq 0} - \left( T = 0 \right) \right) \]
Solution at $T = 0$

Consider first the SU(3) symmetric case, $h_u = h_d = h_s$.

Spectrum. $0^-$: singlet $\eta'$ & octet $\pi$. $0^+$: singlet $\sigma$ and octet $a_0$.

Satisfy a 't Hooft relation:

$$m_{\eta'}^2 - m_{\pi}^2 = m_{a_0}^2 - m_{\sigma}^2$$

The anomaly moves $\eta'$ up from the $\pi$, but also moves $\sigma$ down from the $a_0$.

QCD: $\langle \Phi \rangle = (\Sigma_u, \Sigma_u, \Sigma_s)$. From:

$$f_\pi = 93, \ m_\pi = 140, \ m_K = 495, \ m_\eta = 540, \ m_{\eta'} = 960$$

Determine:

$$\Sigma_u = 46, \ \Sigma_s = 76, \ h_u = (97)^3, \ h_s = (305)^3, \ c_A = 4560$$

$$m^2 = (538)^2 - 121 y^4, \ \lambda = 18 + 0.04 y^4$$

One free parameter, Yukawa coupling “$y$”, fix from $T_\chi$. 
Varying the Yukawa coupling

\[ T_\chi \uparrow \]

\[ T_\chi \] defined from maximum in light quark susceptibility, \( d\Sigma_u/dT \)

\( \leq \) Grey band: vary \( T_d \) from 260 \( \rightarrow \) 280

\( \leq \) Yellow band = \( y \): 4.5 \( \rightarrow \) 5.5

Grey band: experimental uncertainty in the mass of the \( a_0 \) =>

\[ m_{a0} \uparrow \]
Solution at $T \neq 0$

To eliminate u.v. divergences, lattice uses substracted condensates

$$\Delta_{u,s}^{\text{lattice}}(T) = \frac{\langle \bar{q}q \rangle_{u,T} - (m_u/m_s)\langle \bar{q}q \rangle_{s,T}}{\langle \bar{q}q \rangle_{u,0} - (m_u/m_s)\langle \bar{q}q \rangle_{s,0}}$$

In the chiral-matrix ($\chi$-M) model use this to fix $y = 5$.

$$\Delta_{u,s}^{\chi^{-}M}(T) = \frac{\Sigma_u(T) - (h_u/h_s)\Sigma_s(T)}{\Sigma_u(0) - (h_u/h_s)\Sigma_s(0)}$$

---

Bazavov et al,
1407.6387
1701.03548
Meson masses vs $T$

Usual pattern for $m_u = m_d \neq m_s$. $y = 5$.

U(1)$_A$ breaking persists to high $T$, unphysical.
Pressure and interaction measure, \((e-3p)/T^4\)
\(\chi\)-M model,
Lattice, Bazavov et al, 1407.6387
Hard Thermal Loop (HTL)
(blue region = change ren. scale)
Andersen et al, 1511.04660
Order parameters, chiral and deconfining

Chiral matrix model:

Chiral and deconfining order parameters are strongly correlated

But Polyakov loop from lattice Petreczky & Schadler, 1509.07874 is much smaller than in model.

Persistent discrepancy, also in pure gauge.

*What’s up with lattice loop?*
Susceptibilities, chiral and deconfining

Largest peak for up-up; strange-strange small.
In QCD, notable peaks for loop-up & loop-loop, strongly correlated with up-up

In chiral limit: loop-up suscept. diverges. Sasaki, Friman, Redlich ph/0611147

loop-loop and loop-antiloop finite
6th order baryon susceptibility

In $\chi$-M model, $\chi_6$ shows non-monotonic behavior near $T_\chi$. In HTL, $\chi_6$ is very small (because $m=0$). $\sigma$ model: including change in $\Sigma_u$, but not in loop. Change in $\chi_6$ much smaller.
Baryon susceptibilities: 2nd & 4th

As evaluated at $\mu = 0$, lattice ok.
Baryon $\mu_B = 3 \mu_q$.

$$\chi_n^B(T) = T^{n-4} \left. \frac{\partial^n}{\partial \mu_B^n} p(T, \mu_B) \right|_{\mu_B=0}$$

Lattice: Bazavov et al, 1701.04325
Ratios of moments, vs Columbia lattice

Left: ratio of $\chi_4/\chi_2$ and $\chi_6/\chi_2$ in $\chi$-M model

Bazavov et al, 1701.04325
Lattice moments, Frankfurt

Vovchenko, Steinheimer, Philipsen, Stoecker 1711.0126:

\[ \frac{\chi_4}{\chi_2}, \frac{\chi_6}{\chi_2} \]

![Graph showing LQCD, HotQCD, Wuppertal-Budapest, CEM-LQCD, and CEM-HRG at different temperatures with mu_B = 0.](image)
What’s up with the lattice loop?

Looked at wide variety of variations on $\chi$-M models.

Below: $\chi_2$ from chiral matrix model, lattice, and fitting the loop to the lattice value, then computing $\chi_2$.

If the lattice loop is right, then $\chi_2$ is too small.

Lattice loop: Petreczky & Schadler, 1509.07874
Quarkyonic & 1-D patches

Cold, quark matter as “Quarkyonic” matter: McLerran & RDP 0706.2191
Fermi surface ~ confined, deep in Fermi sea ~ perturbative

Valid at large $N_c$: $N_c = 3$? At $T \neq 0$, $\mu = 0$: $\Lambda_{\text{ren}} \sim 2\pi T$
We suggest: $T = 0$, $\mu \neq 0$: quarkyonic for $\mu_{\text{quark}} < 1 \text{ GeV}$, for any $N_c$, $N_f$
At $\mu \neq 0$, $T \ll \mu$ confining potential $\sim 1/(p^2)^2$ tends to form 1-dim patches
of chiral spirals in effective 1-dim theory
Kojo, Hidaka, McLerran & RDP 0912.3800;
Kojo, RDP & Tsvelik 1007.0248;
Kojo, Hidaka, Fukushima, McLerran, RDP 1107.2124;
RDP, Skokov & Tsvelik 1712.x

Width of patch $\sim \Lambda_{\text{QCD}}$, so for large $\mu$,
Fermi surface is covered with patches
Chiral Spirals in 1+1 dimensions

Chiral Spiral (CS) ~ Migdal’s pion condensate:

\[
(\sigma, \pi^0) = f_\pi(\cos(k_0 z), \sin(k_0 z))
\]

*Ubiquitous* in 1+1 dimensions: Basar, Dunne & Thies, 0903.1868; Dunne & Thies 1309.2443+ ...

*Wealth* of exact solutions, phase diagrams...

\[
\langle \bar{q}q \rangle = 0
\]

\[
\langle \bar{q}q \rangle \neq 0
\]

\[
\downarrow \langle \bar{q}q \rangle_{CS} \neq 0
\]
Chiral Spirals in 3+1 dimensions

In 3+1, common in NJL models: Nickel, 0902.1778 + ... Buballa & Carignano 1406.1367 + ...

In reduction to 1-dim, $\Gamma_{5}^{1-dim} = \gamma_{0} \gamma_{z}$, so chiral spiral between $\bar{q}q$ & $\bar{q}\gamma_{0}\gamma_{z}\gamma_{5}q$

![Graph showing chiral spiral transition](chart.png)
Fluctuations in Chiral Spirals

In Chiral Spiral, $<\varphi> \neq 0$ locally but $<\varphi> = 0$ globally.

Spon. breaking of global symmetry $\Rightarrow$ interactions of Goldstone Bosons $\sim \partial^2$

In CS, spon. bkg’s of global *plus* rotational sym. implies interactions in transverse momenta $\sim \partial^2_\perp$ cancel. Interactions $\sim (\partial^2_\perp)^2 \sim \partial^4_\perp$. $U = GB$:

$$
\mathcal{L}_{CS} = f^2_\pi |(\partial_z - i k_0)U|^2 + \kappa |\partial^2_\perp U|^2 + \ldots
$$

Hidaka, Kamikado, Kanazawa & Noumi 1505.00848; Nitta, Sasaki & Yokokura 1706.02938

Transverse fluctuations disorder: *large fluctuations about* $k_z \sim k_0$:

$$
\int d^2k_\perp dk_z \frac{1}{(k_z - k_0)^2 + (k^2_\perp)^2} \sim \int d^2k_\perp \frac{1}{k^2_\perp} \sim \log \Lambda_{IR}
$$

No true long range order (Landau-Peierls) $\sim$ *smectic liquid crystal*
Varieties of liquid crystals

Nematics: rotational ordering (vector with no direction)

Smectic: rotational ordering and in planes disordered in the planes ("liquid")

Cholesteric: chiral ordering (with twist)

Smectic something like patches in QCD

Smectic – nematic transition has analogy, to follow (1st order from reduction to 1-dim)
Standard phase diagram

Trade $T$ & $\mu$ for $m^2$ & $\lambda$.

Two phases, symmetric & broken

$m^2 = 0$, $\lambda > 0$: usual 2\textsuperscript{nd} order

$m^2 = \lambda = 0$: tricritical point

$m^2 > 0$, $\lambda < 0$: 1\textsuperscript{st} order in mean field theory

$$\mathcal{L} = (\partial_\mu \phi)^2 + m^2 \phi^2 + \lambda \phi^4 + \kappa \phi^6$$

$\langle \phi \rangle = 0$

$\langle \phi \rangle \neq 0$

$\lambda \uparrow$

$\text{2nd}$

$m^2 \rightarrow$

$\text{1st}$

$\uparrow$
Usual critical dimensions

\( \varphi^4: d_{\text{upper}} = 4 : \) expand in \( d = 4 - \varepsilon \) dimensions

\[
\int d^4k \frac{1}{(k^2)^2} \sim \log \Lambda_{\text{UV}}
\]

\( \varphi^4: d_{\text{lower}} = 2 : \) expand in \( 2 + \varepsilon \) dimensions

always disordered when \( d < 2 \)

\[
\int d^2k \frac{1}{k^2} \sim \log \Lambda_{\text{IR}}
\]

\( \varphi^6: d_{\text{critical}} = 3 : \) at tricritical point, log corrections

\[
\int d^3k_1 \int d^3k_2 \frac{1}{(k_1)^2(k_2)^2(k_1 + k_2)^2} \sim \log \Lambda_{\text{UV}}
\]
Lifshitz points

To get a Chiral Spiral (CS):

\[ \mathcal{L}_{CS} = (\partial_0 \phi)^2 + Z (\partial_i \phi)^2 + \frac{1}{M^2} (\partial_i^2 \phi)^2 + m^2 \phi^2 + \lambda \phi^4 \]

Need higher (spatial) derivatives for stability. Then CS occurs when \( Z < 0 \). Cannot have higher derivatives in time or theory is acausal.

In gravity, models with higher derivatives are renormalizable:

\[ \mathcal{L}_{\text{ren. gravity}} = \frac{1}{16\pi G} R + \alpha_1 R^2 + \alpha_2 R^2_{\mu\nu} \]

but acausal. Hořava-Lifshitz gravity: add higher derivatives only in space

Hořava 0901.3775 + ... 

\[ \mathcal{L}_{\text{Hořava–Lifshitz}} = \frac{1}{16\pi G} R + \beta_1 R^2_{ij} + \ldots \]

Only two time derivatives, so causal. Flows into Einstein gravity in the infrared.
Lifshitz phase diagram in mean field theory

Phase diagram in $Z$ & $m^2$: three phases, symmetric, broken, and Chiral Spiral
Hornreich, Luban, Shtrikman, PRL ’75, Hornreich J. Magn. Matter ‘80...Diehl, cond_mat/0205284 + ...

\[ m^2 = 0, \ Z > 0: \text{btwn broken & sym.} \]
usual 2\textsuperscript{nd} order: \[ \longleftrightarrow \]
\[ \langle \phi \rangle \neq 0 \quad \langle \phi \rangle = 0 \]

\[ m^2 < 0, \ Z = 0: \text{broken & CS} \]
1\textsuperscript{st} order in mean field: \[ \downarrow \]
\[ \langle \phi \rangle_{CS} \neq 0 \]

\[ X \text{ at origin, } m^2 = Z = 0: \text{Lifshitz point} \]

\[ m^2 > 0, \ Z < 0: \text{btwn CS & broken} \quad \longleftrightarrow \]
2nd order in mean field, but fluctuations?
Symmetric to CS: 1D (Brazovski) fluctuations

Consider $m^2 > 0, Z < 0$: minimum in propagator at *non*zero momentum

Brazovski ‘75; Hohenberg & Swift ‘95 + ...;
Lee, Nakano, Tsue, Tatsumi & Friman, 1504.03185; Yoshiike, Lee & Tatsumi 1702.01511

\[ \Delta^{-1} = m^2 + Z k^2 + k^4 / M^2 \]
\[ = m^2_{\text{eff}} - 2 Z k_z^2 + (k_{\perp}^2)^2 / M^2 \]

$k = (k_{\perp}, k_z - k_0)$: *no* terms in $k_{\perp}^2$, *only* $(k_{\perp}^2)^2$.

Due to spon. breaking of rotational sym.

\[ \int d^3k \frac{1}{k_z^2 + m^2_{\text{eff}} + \ldots} \sim M^2 \int \frac{dk_z}{k_z^2 + m^2_{\text{eff}}} \sim \frac{M^2}{m_{\text{eff}}} \]

Effective reduction to 1-dim for any spatial dimension $d$, any global symmetry
1\textsuperscript{st} order transition in 1-dim.

*Strong* infrared fluctuations in 1-dim., both in the mass:

\[ \Delta m^2 \sim \lambda \int d^3 k \frac{1}{k_z^2 + m_{\text{eff}}^2 + \ldots} \sim \lambda \frac{M}{m_{\text{eff}}} \]

and for the coupling constant:

\[ \Delta \lambda \sim -\lambda^2 \int \frac{d^3 k}{(k_z^2 + m_{\text{eff}}^2 + \ldots)^2} \sim -\lambda^2 M^3 \int m_{\text{eff}} \frac{dk_z}{k_z^4} \sim -\lambda \frac{M^3}{m_{\text{eff}}^3} \]

*Cannot* tune \( m_{\text{eff}}^2 \) to 0: \( \lambda_{\text{eff}} \) goes negative, 1\textsuperscript{st} order trans. induced by fluctuations

*Not* like other 1st order fluc-ind’d trans’s: just that in 1-d, \( m_{\text{eff}}^2 \neq 0 \) always
Lifshitz phase diagram, with eff. 1-D fluc.’s

$\langle \phi \rangle \neq 0$

$\langle \phi \rangle = 0$

$1^{st} \uparrow$

$2^{nd} \rightarrow$

$Z \uparrow$

$m^2 \rightarrow$

$\langle \phi \rangle_{CS} \neq 0$

What about fluctuations at the Lifshitz point?
Critical dimensions at the Lifshitz point

At the Lifshitz point, $Z=m=0$, massless propagator $\sim 1/k^4$

$$\mathcal{L}_{\text{Lifshitz}} = (\partial^2 \phi)^2 + \lambda \phi^4$$

$d_{\text{upper}} = 8$ : expand in $d = 8 - \varepsilon$ dimensions

$$\int d^8k \frac{1}{(k^4)^2} \sim \log \Lambda_{\text{UV}}$$

$d_{\text{lower}} = 4$ : expand in $d = 4 + \varepsilon$ dimensions

$$\int d^4k \frac{1}{k^4} \sim \log \Lambda_{\text{IR}}$$

$d = 3 < d_{\text{lower}}$ : there is NO (isotropic) Lifshitz point in three dimensions

...+ Bonanno & Zappala, 1412.7046; Zappala, 1703.00791

Infrared fluctuations always generate a mass gap dynamically.
Phase diagram without a Lifshitz point?

Have three phases, three lines of phase transition far from the would be Lifshitz point.  *How can they connect?*
A: looks like Lifshitz point, but isn’t

All three lines connect at a “pseudo”-Lifshitz point.

As terminus of 2nd order line, $m^2 = 0$. So at pseudo-Lifshitz point, $Z \neq 0$

Why do fluctuations drive symmetric-CS transition 1st order if $Z \neq 0$?
B: 1\textsuperscript{st} order line between broken/CS phases ends

Crossover between broken and CS phases? But $\langle \phi \rangle \neq 0$ in the broken phase, and $\langle \phi \rangle = 0$ for a Chiral Spiral. Crossover seems unlikely, unless fluctuations are small (so long range order in CS phase)
Chiral spiral has no long range order, so when fluctuations are large, possible to have just crossover between CS & symmetric phases. Brazikovski 1st order line ends in critical endpoint. Novel tricritical point where 2nd order line joins to 1st order, at small Z.
Lifshitz points in inhomogenous polymers: mean field

Fredrickson & Bates, Jour. Polymer Sci. 35, 2775 (1997);

Polymers A & B, for blend with A, B, & A+B
Have disordered, separated, and “lamellar” phases
Inhomogenous polymers: *no* Lifshitz point

From both experiment & numerical simulations, Lifshitz point wiped out by fluctuations: instead a “bicontinuous microemulsion”, $B\mu E$, appears “structured, fluctuating disordered phase”
Phase diagram for QCD in $T$ & $\mu$: usual picture

Two phases, one Critical End Point (CEP) between crossover and line of 1st order transitions

Ising fixed point, dominated by *massless* fluctuations at CEP
Phase Diagram with Chiral Spirals

Now three phases. If model “C”, two 1\textsuperscript{st} order lines and two CEP’s. “Pseudo” Lifshitz point with large fluctuations.
In CS, large fluctuations at nonzero momenta, \( \sim k_0 \).
Beam Energy Scan and cumulants

To look for Critical End Point, typically compute cumulants

Expectation from theory, to right: corrections to $c_3$ are *positive*

But STAR finds that the corrections to $c_3$, below, are *negative*
Fluctuations at 7 GeV

Beam Energy Scan, down to 7 GeV.
Fluctuations *MUCH* larger when up to 2 GeV than to 0.8 GeV
Trivial multiplicity scaling? ... or first evidence for a Chiral Spiral?!
Suggestion for experiment

For *any* sort of periodic structure (1D, 2D, 3D...),

fluctuations concentrated about some characteristic momentum $k_0$

So “slice and dice”: bin in intervals, 0 to .5 GeV, .5 to 1., etc.

*If* peak in fluctuations in a bin not including zero, *may* be evidence for $k_0 \neq 0$.

*If* periodic structure, fluctuations must go *up* as $\sqrt{s}$ goes *down*, since $\mu$ increases
NJL models and Lifshitz points

Consider Nambu-Jona-Lasino models.
Nickel, 0902.1778 & 0906.5295 + .... + Buballa & Carignano 1406.1367

\[ \mathcal{L}_{\text{NJL}} = \overline{\psi}(\partial + g\sigma)\psi + \sigma^2 \]

Integrating over \( \psi \),

\[
\log(\partial + g\sigma) \sim \ldots + \kappa_1 ((\partial\sigma)^2 + \sigma^4) \\
+ \kappa_2 ((\partial^2\sigma)^2 + \sigma^2(\partial\sigma)^2 + \sigma^6) + \ldots
\]

Consequently, in NJL @ 1-loop, \textit{tricritical} = \textit{Lifshitz point}.

Above due to scaling \( \partial \to \xi \partial \), \( \sigma \to \xi \sigma \).
Special to including only \( \sigma \) at one loop.

Not generic: violated by the inclusion of more fields, to two loop order, etc.