

# Kinetics of the chiral phase transition\*

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We study the dynamics of the chiral phase transition in a linear quark-meson  $\sigma$  model using a novel approach based on semiclassical wave-particle duality. The quarks are treated as test particles in a Monte-Carlo simulation of elastic collisions and the coupling to the  $\sigma$  meson, which is treated as a classical field. The exchange of energy and momentum between particles and fields is described in terms of appropriate Gaussian wave packets. It has been checked that energy-momentum conservation and the principle of detailed balance are fulfilled, and that the dynamics leads to the correct equilibrium limit. First schematic studies of the dynamics of matter produced in heavy-ion collisions are presented.

## 1. Introduction

One of the prime goals of contemporary heavy-ion experiments, as the ongoing beam-energy scan at the Relativistic Heavy Ion Collider (RHIC) at the Brookhaven National Laboratory (BNL) and in the future the Facility for Anti-Proton and Ion Research (FAIR) in Darmstadt and the Nuclotron-based Ion Collider Facility (NICA) in Dubna, is the exploration of the phase diagram of strongly interacting matter [1]. In the low-energy regime QCD is governed by the approximate chiral symmetry of the light-quark sector. Due to the formation of a quark condensate  $\langle\bar{\psi}\psi\rangle \neq 0$  the symmetry is spontaneously broken at low temperatures and densities and is expected to be restored at high temperatures and densities. From effective chiral models one expects that the cross-over nature of the phase transition at  $\mu_B = 0$  (as known from lattice-QCD calculations) becomes of first order at  $\mu_B \neq 0$ , with the first-order transition line ending in a critical point (second-order phase transition). One possible observable indicating a critical point in the phase diagram are critical (grand-canonical) fluctuations of conserved

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\* Presented at CPOD 2016, Wroclaw, Poland, May 30 – June 4, 2016

charges (like baryon number or electric charge) and the divergence of the corresponding susceptibilities [2–8].

The main challenge is to find experimental signatures for this structure of the phase diagram, particularly the existence of a critical point, from observations of the quickly expanding and cooling “fireballs” of strong-interaction matter created in heavy-ion collisions. Here we use a novel approach to numerically simulate this dynamics in terms of kinetic theory, based on a linear quark-meson  $\sigma$  model [9–11]. By adjusting the coupling constants and evaluating the model at finite temperature and baryo-chemical potential the different kinds of phase transition (cross-over, 1<sup>st</sup> and 2<sup>nd</sup> order) can be realized. In our kinetic simulation the  $\sigma$  meson is described as a classical mean field and the quarks and antiquarks in terms of Monte-Carlo test particles. Besides the usual binary elastic collision term  $qq \rightarrow qq$  also decay and recombination processes,  $\sigma \leftrightarrow q\bar{q}$  have to be described. To this end we employ a kind of “wave-particle duality” approach, which we describe in the next Sec.

## 2. Wave-particle dynamics

The linear quark-meson  $\sigma$  model with the chiral  $SU(2)_L \times SU(2)_R$  symmetry is defined by the Lagrangian

$$\mathcal{L} = \bar{\psi} [i\cancel{D} - g(\sigma + i\gamma_5 \mathbf{p} \cdot \boldsymbol{\tau})] \psi + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi}) - U(\sigma, \vec{\pi}), \quad (2.1)$$

where  $\psi$  denotes the two-flavor quark field  $(u, d)$ , and  $(\sigma, \vec{\pi})$  the scalar and pseudo-scalar meson fields, transforming according to the  $SO(4)$  representation of the chiral group. The meson-field potential is given by

$$U(\sigma, \vec{\pi}) = \frac{\lambda^2}{4} (\sigma^2 + \vec{\pi}^2 - \nu^2)^2 - f_\pi m_\pi^2 \sigma - U_0. \quad (2.2)$$

With  $\nu = f_\pi^2 - m_\pi^2/\lambda$  the minimum of the potential is given by  $\sigma_0 = f_\pi$ , and the approximate chiral symmetry is spontaneously broken to the  $SU(2)_V$  isospin symmetry. The mass of the  $\sigma$  meson becomes  $m_\sigma^{(0)} = 2\lambda^2 f_\pi^2 + m_\pi^2$ . For  $\lambda \simeq 20$  one obtains  $m_\sigma^{(0)} \simeq 600$  MeV. Through the Yukawa couplings of the quark fields to the mesons in (2.1) the quarks acquire a constituent-quark mass  $m_q = g^2 \sigma_0$ . Varying  $g \in (3.3, 5.5)$  leads to a cross-over ( $g = 3.3$ ), a 2<sup>nd</sup>-order ( $g = 3.63$ ), or a 1<sup>st</sup> order phase transition for  $\mu_B = 0$ .

Schematically the kinetic theory is defined by a coupled system of stochastic mean-field and Boltzmann-Vlasov transport equations,

$$\square \sigma + \lambda(\sigma^2 - \nu^2)\sigma - f_\pi m_\pi^2 + g \langle \bar{\psi} \psi \rangle = I(\sigma \leftrightarrow \bar{q}q), \quad (2.3)$$

$$\left[ \partial_t + \mathbf{p}/E_q \cdot \vec{\nabla}_{\vec{x}} - \vec{\nabla}_{\vec{x}} E_\psi \cdot \vec{\nabla}_{\vec{p}} \right] f_q(t, \vec{x}, \vec{p}) = C(\psi \psi \rightarrow \psi \psi, \sigma \leftrightarrow \bar{q}q), \quad (2.4)$$

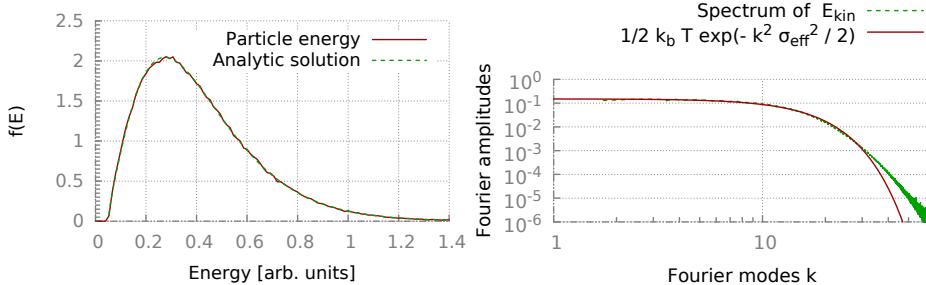


Fig. 1. (Color online) **Left:** The energy distribution of the quarks and anti-quarks follow the expected relativistic Maxwell-Boltzmann distribution  $f_q(E) \propto \exp(-E/T)$ , demonstrating the numerical stability of the kinetic equations as well as the validity of the principle of detailed balance. **Right:** The spectrum of the kinetic energy density of the mean field,  $\dot{\phi}^2/2$ . For low  $k$  (long wavelengths) the distribution follows the expected equipartition theorem for a classical field. At higher energies the finite interaction volume encoded in the finite width of the Gaussian wave packets leads to an effective cut-off.

where  $I$  and  $C$  denote collision terms contributing in a stochastic way to the time evolution of the mean field  $\sigma$  and the quark phase-space distribution function  $f$ , respectively.

The decay and quark-annihilation processes  $\sigma \leftrightarrow q\bar{q}$  is described by the quantum-field theoretical  $S$ -matrix element, via the Breit-Wigner cross section with the corresponding on-shell decay width. The annihilation process is described via Monte-Carlo simulation. To determine the local distribution of  $\sigma$  particles to describe the decay process, the local energy-momentum densities are calculated from the  $\sigma$  field and mapped to a corresponding local thermal-equilibrium relativistic Boltzmann-Jüttner distribution with the  $\sigma$ -meson mass determined self-consistently with the temperature according to the corresponding equilibrium value. In the decay and annihilation processes the corresponding energy and momentum are transferred to the mean field by adding an appropriate relativistic Gaussian wave packet, guaranteeing energy-momentum conservation in each local reaction process. We have demonstrated that such a description also obeys the principle of detailed balance and thus leads to the correct equilibrium limit for the quark distribution function as well as the  $\sigma$  field (cf. Fig. 1).

### 3. Simulation of an expanding fireball

To study a situation more close to the rapidly expanding and cooling medium produced in heavy-ion collisions, we initialize the system as a spherically symmetric droplet with a Woods-Saxon like spherically sym-

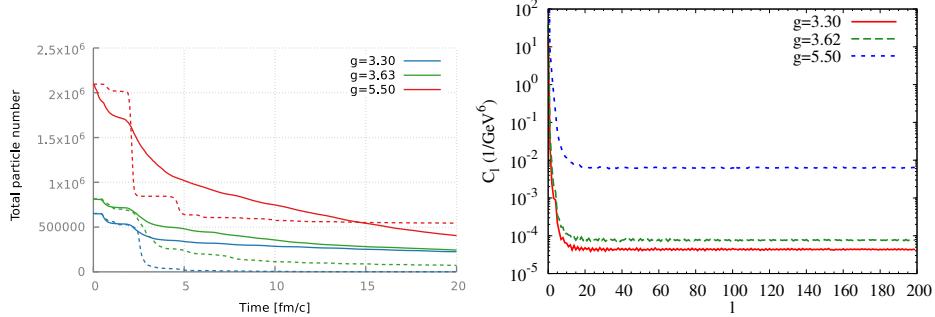


Fig. 2. (Color online) **Left:** Total quark number in the matter-droplet scenario. Solid lines show the simulations with chemical processes, the dashed lines the simulations without the  $\sigma \leftrightarrow \bar{q}q$  processes. In the scenario without chemical processes the droplet radiates most of the quarks in shell-like structures, as reflected in the quark-number plateaus which drop suddenly. In the calculations with chemical processes the systems lose quarks in a steady and continuous way; the formation of quark-number plateaus is washed out. **Right:** Angular power spectrum of quark-density fluctuations. The qualitative features of the spectrum are independent of the nature of the phase transition but their absolute size depends on the coupling strength  $g$ .

metric temperature distribution,  $T(\mathbf{x}) = T_0 \{1 + \exp[(|\mathbf{x}| - R_0)/\alpha]\}^{-1}$  with  $R_0 = 0.45$  fm and  $\alpha = 0.1$  fm. The total system size has been chosen to 5 fm, and the boundary conditions have been adapted such as to allow for an expanding-fireball scenario. For the quarks we have introduced a “distance cutoff”  $r_c = 2.75$  fm from the center, where the particles are considered to leave the system.

We have simulated this expanding-fireball scenario with and without implementing the chemical processes,  $\sigma \leftrightarrow q\bar{q}$  (cf. Fig. 2). Neglecting these processes leads to a rapidly oscillating  $\sigma$ -field in the beginning of the expansion, losing energy by radiating long-wavelength oscillations propagating out of the system, and despite small spatial fluctuations the system stays isotropic. For larger couplings, particularly at  $g = 5.5$ , according to a 1<sup>st</sup>-order phase transition in equilibrium, the system can stay in a metastable state where cold quarks are trapped in a potential well of the chiral field for some time. Taking the chemical processes into account, the strong fluctuations of the  $\sigma$  field are damped by the decay process, while the annihilation process creates strong local fluctuations on the field, increasing with stronger coupling constants  $g$ , which also leads to spatial anisotropies of the fireball. In accordance with the findings using a Langevin approach to describe the fluctuations on a hydrodynamical background [12–14], one finds the strongest fluctuations in quark density for the largest coupling corre-

sponding to a 1<sup>st</sup>-order phase-transition rather than the expected maximum at the 2<sup>nd</sup>-order critical point. However, here the exspected equilibrium phenomena cannot be observed since the system size is in the order of the scattering length and thus also of the correlation length of the fluctuations. Also the total lifetime of the system is of the order of the equilibration time.

In an attempt to characterize the different phase-transition scenarios in the expanding-fireball systems we evaluated the angular power spectrum of the quarks leaving the fireball. As can be seen in the right panel of Fig. 2, the power spectra do not show qualitative differences for the different phase-transition scenarios but just a strong dependence of their absolute magnitude with the Yukawa coupling  $g$ , leading to the largest correlations for the 1<sup>st</sup>-order scenario at  $g = 5.5$ .

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