Quarkonia production in a Langevin approach

C Greiner, N Krenz and H van Hees

Institut für theoretische Physik, Goethe-Universität Frankfurt am Main, Max-von-Laue-Straße 1, 60438 Frankfurt, Germany

E-mail: krenz@th.physik.uni-frankfurt.de

Abstract. We aim to describe the process of dissociation and recombination of quarkonia in the quark-gluon plasma. Therefore we developed a model which allows to observe the time evolution of a system with various numbers of charm-anticharm-quark pairs at different temperatures. The motion of the heavy quarks is realized within a Langevin approach. We use a simplified version of a formalism developed by Blaizot et al. in which an Abelian plasma is considered where the heavy quarks interact over a Coulomb like potential. We have demonstrated, that the system reaches the expected thermal distribution in the equilibrium limit.

1. Introduction

Heavy quarks are an important tool for the investigation of the quark-gluon plasma (QGP). They are primarily produced in the primordial hard collisions of the heavy ion collision, and their number is conserved until the hadronic freezout. Therefore heavy quarks carry information about the whole evolution of the QGP. Especially the surviving probability of heavy-quark bound states such as J/ψ or Υ can give an insight to the medium properties.

The potential between two heavy quarks is screened by the surrounding medium. As suggested long ago, the suppression of J/ψ could be an evidence for the formation of the deconfined state [1]. Higher temperatures should lead to larger screening effects with a full suppression of J/ψ at very high beam energies. The predicted suppression was found at the SPS at CERN [2] but measurements at RHIC at higher beam energies did not show an increase of the suppression [3]. To explain the results the process of recombination of J/ψ inside the medium has been suggested. At higher beam energies the number of initially produced heavy quarks is larger, leading to a higher possibility that a heavy quark which propagates through the medium meets a partner to form a new quarkonium state. The theoretical investigation of recombination processes is therefore necessary to predict the number of J/ψ-mesons measured in the experiments.

The comparatively large masses of heavy quarks makes their motion accessible by Langevin dynamics [4]. The forces that act on the charm quarks by using the Langevin equation are a drag force and random momentum kicks due to collisions with the medium particles. To enable the formation of bound states we add a potential between the heavy quarks, that leads to a attractive force between charm and anti-charm quarks.

In section 2 we will explain the formalism and the parameters that we have used in our simulation. The current results are presented in section 3 while section 4 gives an outlook on further applications for a description of heavy quarkonia in heavy-ion collisions.

2. Formalism

For the realization of the heavy-quark motion we adopt the formalism by Blaizot et al. [5]. In this description the heavy-quark interaction is reduced to an Abelian model, which means that confinement and color effects are neglected. The Langevin update rules for the coordinate $\boldsymbol{x}$ and the momentum $\boldsymbol{p}$ of a heavy quark with mass M read

$$\frac{\mathrm{d}\boldsymbol{r}}{\mathrm{d}t} = \frac{1}{2M}\boldsymbol{p}, \tag{1}$$

$$\frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = -\gamma\boldsymbol{p} + F\left(\boldsymbol{r} - \bar{\boldsymbol{r}}\right) + \sqrt{2MT\gamma\Delta t}\boldsymbol{\rho}, \tag{2}$$

where γ is the friction coefficient due to the interaction with the medium, $F\left(\boldsymbol{r} - \bar{\boldsymbol{r}}\right)$ is the force resulting from the heavy-quark potential, T is the temperature of the medium, and $\boldsymbol{\rho}$ are Gaussian normal-distributed random numbers. For the pertinent diffusion coefficient the usual Einstein dissipation-fluctuation relation has been employed. The quark-anti-quark potential is given by a screened Coulomb potential with a cut-off for large momenta at small distances. Following [5] the cut-off is taken to be $\Lambda = 4$ GeV. The potential for different temperatures is displayed in Figure 1. The drag force in [5] contains a dependence on the distance between the heavy quarks. For simplicity we neglect this dependence in our simulation and use a constant drag value. In a numerical calculation a cut-off is also necessary for the friction. With the same cut-off as for the potential the drag-coefficient is given by

$$\gamma = \frac{m_\mathrm{D}^2 g^2}{24\pi M}\left[\ln\left(1 + \frac{\Lambda^2}{m_\mathrm{D}^2}\right) - \frac{\frac{\Lambda^2}{m_\mathrm{D}^2}}{\frac{\Lambda^2}{m_\mathrm{D}^2} + 1}\right], \tag{3}$$

where m_D is the Debye screening mass, defined as $m_\mathrm{D}^2 = \frac{4}{3}g^2T^2$, which is the perturbative expression for a two-flavor quark-gluon plasma. The gauge coupling g is given by the relation [6]

$$g^2 = 4\pi\alpha_s = \frac{4\pi\alpha_s(T_C)}{1 + C\ln\left(\frac{T}{T_C}\right)}, \tag{4}$$

with

$$C = 0.76, \quad T_C = 160\,\mathrm{MeV}, \quad \alpha_s(T_C) = 0.5,$$

where T_C is the critical temperature. The value of γ at different temperatures can be seen in Figure 2. The charm-quark mass is set to $M = 1.8\,\mathrm{GeV}$, which within this model results in binding energies of $\bar{\mathrm{c}}c$ pairs leading to bound-state masses close to typical charmonium masses.

3. Results

First the simulation was tested with a single charm-anticharm pair in the medium. The following simulations are done inside a cubic box with side-length 8 fm. At the box boundary the particles are reflected. In case a bound state hits the boundary, we reflect the center-of-mass because we do not want to destroy the binding. To define bound states we use the classical condition that two objects are bound if the energy of the pair is negative. To check this we calculate the relative energy E_rel of the pair, which means subtracting the center-of-mass energy from the total energy. After the system reaches equilibrium the distribution of the relative energy should be given by the classical density of states

$$\frac{\mathrm{d}N}{\mathrm{d}E_\mathrm{rel}} = C\int_{\mathbb{R}^3}\mathrm{d}^3\boldsymbol{r}\int_{\mathbb{R}^3}\mathrm{d}^3\boldsymbol{p}_\mathrm{rel}\,\delta(E_\mathrm{rel} - H_\mathrm{rel})\exp\left(-\frac{H_\mathrm{rel}}{T}\right), \tag{5}$$

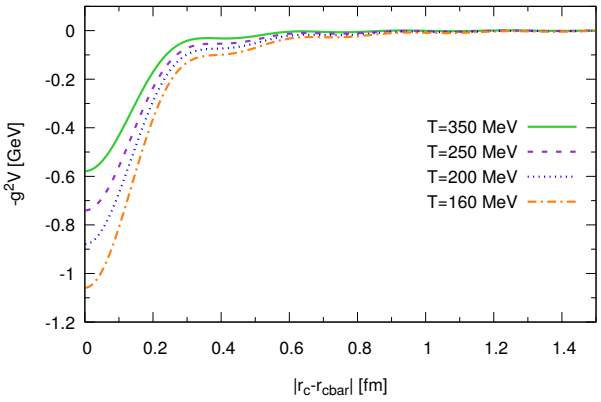

Figure 1. The charm-anti-charm-quark pair potential for different temperatures.

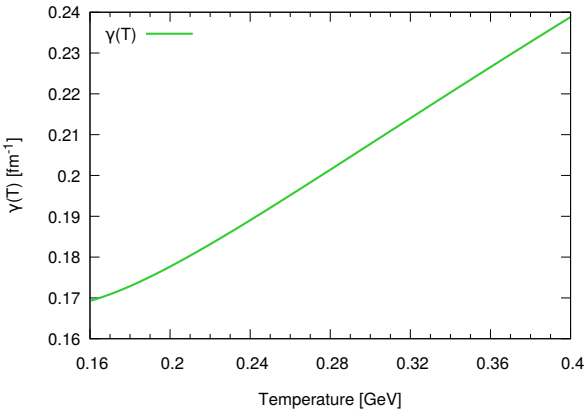

Figure 2. The friction coefficient for different temperatures.

where H_{rel} is the Hamiltonian of the pair and C is a normalization constant. We have solved these integrals numerically for a sphere of radius R with the same volume as our box which leads to

$$\frac{\mathrm{d}N}{\mathrm{d}E_{\mathrm{rel}}} = (4\pi)^2(2\mu)^{3/2}C\int_0^R \mathrm{d}r r^2\sqrt{E_{\mathrm{rel}}-V(r)}\exp\left(-\frac{E_{\mathrm{rel}}}{T}\right), \tag{6}$$

and compare the results with those obtained in the simulation. The results are shown in Figure 3. For this plot both curves are normalized to one. We see that the numerical calculation perfectly fits to the analytic function. We also found this agreement for different temperatures,

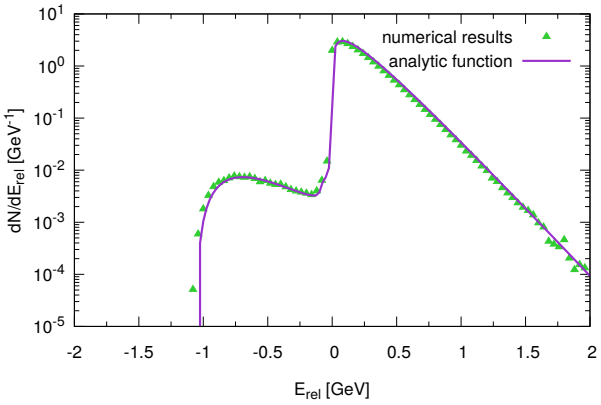

Figure 3. Pair-energy distribution at equilibrium. The simulation is done assuming a cubic box with boxsize 8 fm at a temperature of $T = 160\,\mathrm{MeV}$.

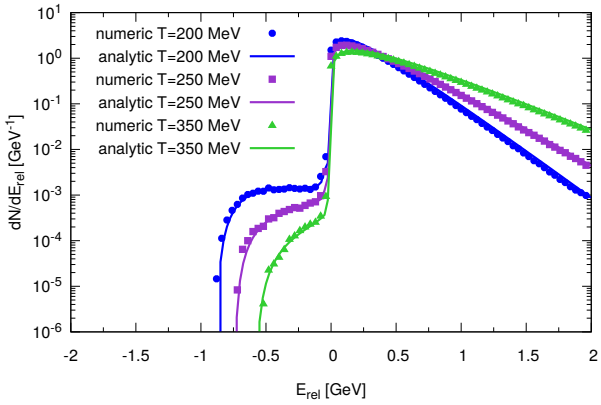

Figure 4. Change of the equilibrium distribution for different temperatures. Due to stronger screening effects higher temperatures lead to a smaller yield of bound states.

as can be seen in Figure 4. For higher temperatures the number of bound states in equilibrium decreases due to stronger screening effects. The different shapes of the box volume, which is a cube in the numerical simulation and a sphere in the analytic calculation seems not to change the result. This indicates that we choose the volume large enough so that the boundary effects can be neglected.

To illustrate the attraction between the charm and the anti-charm quark we plot the distance distribution of a single pair at temperature $T = 160\,\mathrm{MeV}$. The results are shown in Figure

5, where the green (solid) line represents the numerical simulation. The data are taken in the long-time limit when the system is equilibrated. Analytically the distribution can be calculated by evaluating

$$P(r) = \frac{1}{R^6} \int_{R^3} \mathrm{d}^3\boldsymbol{r}_1 \int_{R^3} \mathrm{d}^3\boldsymbol{r}_2 \, \delta\left(r - |\boldsymbol{r}_1 - \boldsymbol{r}_2|\right) \mathrm{e}^{-\frac{V(r)}{T}}, \tag{7}$$

where R^3 is the volume of the box and $V(r)$ is the potential between the heavy quarks. The violet (dashed) line represents a Monte-Carlo evaluation of (7). The numerical and the analytic

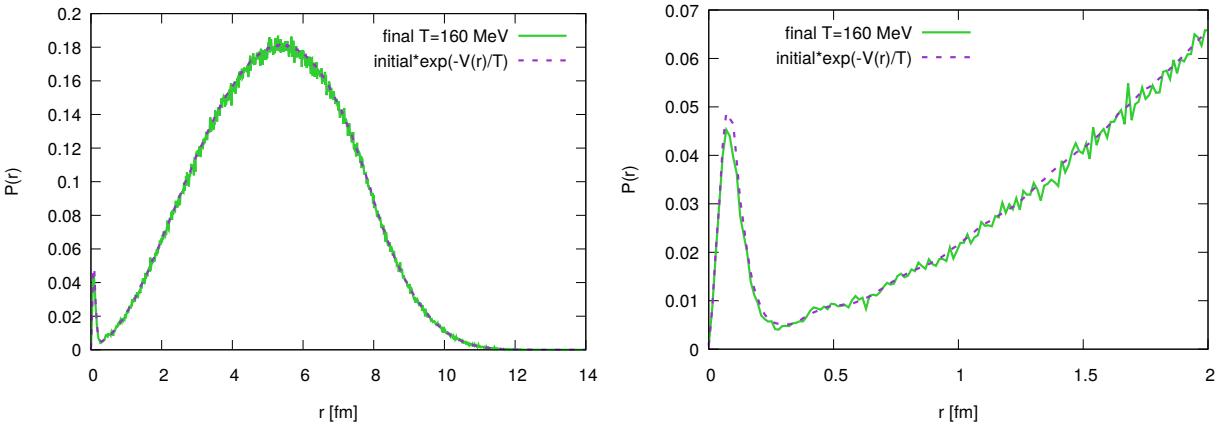

Figure 5. Probability distribution for a single charm anti-charm pair to have the distance r when the system is in equilibrium. The distribution is calculated in the numerical simulation for a single pair in a cubic box with side length 8 fm at temperature $T = 160$ MeV (green solid line) compared to the analytic expectation cf. Eq. (7) (violet dashed line). The right panel zooms into the small-distance region to illustrate the effect of the formation of bound states, leading to a pronounced peak.

function are in good agreement.

We have also investigated the time evolution of the bound states at a temperature of $T = 160\,\mathrm{MeV}$ for two initial conditions.

In the first simulation the charm and anticharm quarks are initially randomly placed inside the box to obtain the time evolution for the formation of bound states. For the second simulation the heavy quarks are initially created as bound states with a pair energy of $-700\,\mathrm{MeV}$, which approximately corresponds to the maximum of the peak on the left panel of Figure 3. The momenta are set back-to-back with a relative velocity that is taken from a Maxwell distribution with its center at the average value of typical relative velocities of charmonium $v_0^2 \sim 0.3$ [7]. As shown in Fig. 6, in both cases the fraction of bound states in the system equilibrates to the same value as expected. At this point the dissociation and recombination rates are equal. The time till the equilibrium value is reached is much larger if the pairs are initially created as bound states. This is due to the strong binding between the quarks. Separating an existing pair requires a large energy transfer from the medium. We notice, that in both cases we found very long timescales for the equilibration.

To see the influence of the medium's temperature on the equilibration time we calculated the time evolution at different temperatures. In this simulation we produced five charm-anticharm pairs, initially created as bound states. The results are displayed in Figure 7.

Larger screening effects at higher temperatures lead to a smaller fraction of bound states at equilibrium. Also the equilibration time decreases. This is expected, because the friction coefficient increases with temperature. The momentum and therefore also the energy transfer

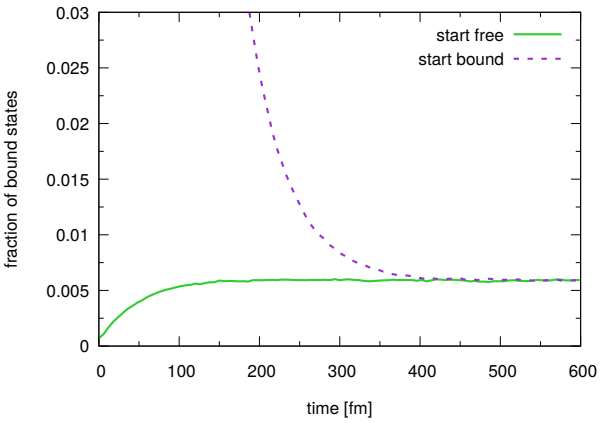

Figure 6. Time evolution of the fraction of bound states with the charm-anticharm pairs produced initially bound (violet dashed line) or placed randomly inside the box (green solid line). Both curves lead to the same equilibrium limit but the equilibration time is longer in case of initial bound states.

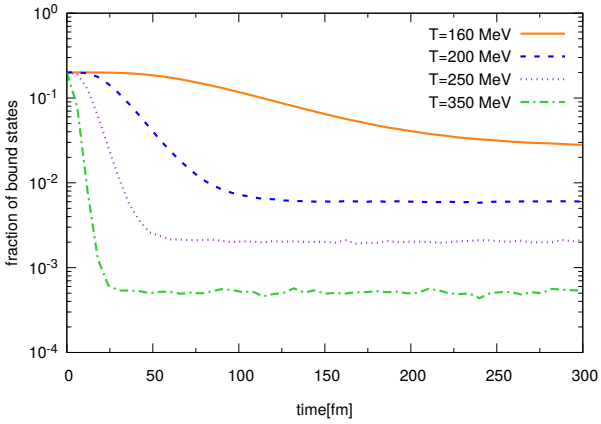

Figure 7. Time evolution of the fraction of bound states for different temperatures. Higher temperatures lead to a less amount of bound states and to a faster equilibration.

in the collisions with the light medium constituents is higher, leading to a faster dissociation of a bound state.

To investigate the relation between the relaxation time and the friction coefficient we have run the simulation for different multiples $k\gamma$. For this simulation we created a single pair as an initial bound state and calculated the time until there is only a fraction of 1/e of the initial pairs left. The results are shown in Figure 8.

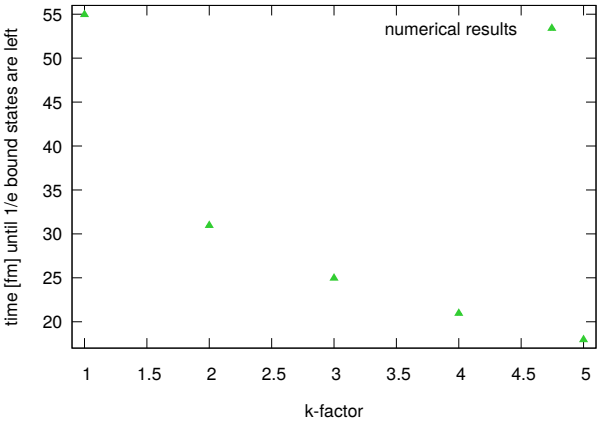

Figure 8. Equilibration time as a function of different multiples k of the friction coefficient γ, defined as the time after which the fraction of bound states has decreased to 1/e of its initial value.

According to the kinetic equilibration time $1/\gamma$ we expect that the decrease of the time also depends on $1/\gamma$. In our simulation the decrease is a bit slower but has the expected shape. To explain the large time scales we need to include chemical effects on the equilibration time in addition to the kinetic scales. Still the behavior of the relaxation time for different friction coefficients is approximately consistent with the underlying theory.

To check if our simulation is also in accordance with an equilibrated thermodynamic system, we calculated the particle number using the grand canonical partition function. In a classical

approximation the particle number is given by

$$N = gV \left(\frac{mT}{2\pi}\right)^{\frac{3}{2}} \mathrm{e}^{-m/T} \mathrm{e}^{\mu/T}, \tag{8}$$

where g is the degeneracy factor of the particles, V is the volume of the system, T is the temperature, m is the mass of the particle, and μ the chemical potential. The factor $\mathrm{e}^{\mu/T}$ is also called the fugacity and will in the following be denoted with λ. Since the dissociation and recombination of a J/ψ-meson is a process of the kind

$$J/\psi \rightleftarrows c + \bar{c},$$

the chemical potentials are connected by $\mu_{J/\psi} = 2\mu_c$ which means $\lambda_{J/\psi} = \lambda_c^2$. The number of J/ψ in the system can then be calculated by

$$\begin{aligned} N_{J/\psi} &= \lambda_c^2 g_{J/\psi} V \left(\frac{m_{J/\psi} T}{2\pi}\right)^{\frac{3}{2}} \mathrm{e}^{-m_{J/\psi}/T} \\ &= N_c^2 \left(\frac{g_{J/\psi}}{g_c^2}\right) \frac{1}{V} \left(\frac{m_{J/\psi}}{m_c^2}\right)^{\frac{3}{2}} \left(\frac{2\pi}{T}\right)^{\frac{3}{2}} \mathrm{e}^{(2m_c - m_{J/\psi})/T} \end{aligned} \tag{9}$$

Divided by the initial number of $c\bar{c}$ pairs equation (9) gives the fraction of bound states in the equilibrated system. We compared the results from our simulation with those obtained by using equation (9) for three different box sizes in Table 1. We find that both values are in the same order of magnitude.

Table 1. Comparison between the fraction of bound states obtained with our simulation and that calculated using equation (9).

Volume	grand canonical fraction of J/ψ	numerical fraction of J/ψ
8^3 fm^3	0.0066	0.0059
10^3 fm^3	0.0035	0.0029
12^3 fm^3	0.002	0.0017

4. Conclusion and Outlook

We have developed a model that allows to investigate the time evolution of a system with multiple pairs of charm and anti-charm quarks using the Langevin equation. We could show that our model passes all equilibrium tests and that the bound-state properties in a box are consistent with the expectations for a grand canonical ensemble.

Various extensions are possible to improve the model. First we want to include a distance dependent friction coefficient as used in [5] to replace our constant γ in equation (3) that we chose for simplicity. To simulate a heavy-ion collision with a hot and dense state at the beginning that expands and cools down until the hadronic freezout we want to describe the medium evolution as an expanding fireball. This enables us to observe the number of J/ψ's at the different stages of a collision. We also want to explore different quarkonia potentials. Especially potentials that include effects from strong interaction such as confinement are important to obtain a more realistic description. The influence of the color charges carried by the charm and anticharm

quarks have to be considered too. Also the classical picture should be replaced by quantum mechanical calculations. A first step could be to decide whether a bound state is created by using the Wigner function as in [8]. In addition we have to replace the continuous energy distribution of the charm-anti-charm pairs with a quantized one that allows only certain energy levels. The energy levels could be comparable with the different excitation states of charmonium to calculate the respective abundance. The long-time goal of this project is to obtain a full in-medium quantum Langevin treatment of quarkonia.

References

[1] Matsui T and Satz H 1986 *Phys. Lett.* B **178** 416.
[2] Baglin C et al. (NA38) 1989 *Phys. Lett.* B **220** 471.
[3] Abelev B I et al. (STAR Collaboration) 2009 *Phys. Rev.* C **80** 041902
[4] Svetitsky B 1988 *Phys. Rev.* D **37** 2484.
[5] Blaizot J P, De Boni D, Faccioli P, and Garberoglio G 2016 *Nucl. Phys.* A **946** 49.
[6] Letessier J and Rafelski J 2004 *Hadrons and Quark-Gluon Plasma* (Cambridge University Press).
[7] Bodwin G T, Kang D, Kim T, Lee J and Yu C 2007 *AIP Conference Proceedings* **892** 315.
[8] Young C and Shuryak 2009 *Phys. Rev.* C **79** 034907.