Renormalization of self-consistent Schwinger-Dyson equations^{G}

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The description of particles in a hot and dense medium commonly requires approximations in terms of non-perturbative "dressed" propagators. One of the prominent self-consistent schemes is the so called Φ-derivable approximation [1, 2, 3] or effective action formalism [4]. There the self-energy of the dressed propagator G is generated from a functional Φ by means of a variational principle

$$\Sigma_{12} = 2\mathrm{i}\frac{\delta\Phi[G]}{\delta G_{12}}, \qquad (1)$$

where Φ is given by a truncated set of closed 2-particle irreducible (2PI) diagrams which cannot be disjoined by cutting two lines. Hence Φ is the generating functional for *skeleton diagrams* and thus avoids double counting problems. It was shown by Baym [5] that for so defined approximations the expectation values of Noether charges are exactly conserved, especially those which arise from space-time symmetries (i.e. energy, momentum, angular momentum), while Ward Takahashi identities of external symmetries and even crossing symmetry for the self-energy and higher vertex functions [6] are violated.

For long time the question could not be settled, to which extend self-consistent approximations of this kind can be renormalized by *temperature independent counter terms*. The problem is to isolate the divergent vacuum structure which are inherent and mostly hidden in the self-consistent self-energy due to the implied partial resummation. A further complication is that these hidden sub-divergences can be overlapping. Within the real-time formalism [7, 8], however, we could show that such a separation is possible [9]. In order to isolate the divergent vacuum sub-diagrams from the temperature dependent remainder the use of the real-time formulation and the BPHZ-prescription of renormalization theory is crucial. Besides the renormalization of the *vacuum* self-energy the procedure arrives at a Bethe-Salpeter (BS) type equation for the *vacuum* four-point function which also needs to be renormalized. For this BS-equation, the structure of which is determined by the chosen Φ-functional, overlapping divergences occur. The latter could be resolved and the problem could be cast into a set of renormalized implicit equations for the vacuum four-point function.

Furthermore we could show that also the generating functional can be renormalized by the same counter-terms and finally the renormalized self-energy is defined by Eq. (1) where Φ is to be read as the *renormalized functional*. An analytic continuation from the real time to imaginary time quantities then provides renormalized expressions for thermal equilibrium such as the thermodynamic potentials (like pressure or entropy) which are thermodynamically and dynamically consistent. It is clear that the violation of crossing symmetry also violates the running of the coupling beyond the orders of the expansion parameter used for defining the approximation of the functional. The general problem concerning symmetries in this context is attacked in forthcoming publication [10].

First applications in next to leading order approximation to ϕ^4-theory, including both, the self-consistent tadpole and the sunset-diagram for the self-energy, were presented [11]. We could demonstrate that not only the UV-problem but also the resulting singularities at the on-shell pole of the vacuum propagator could be tackled numerically. The scheme provides fully self-consistent and renormalized self-energies at finite temperature in their full dependence on energy and three-momentum (see the figure). Such self-consistent treatments are important for future applications to QCD or hadronic matter problems at finite temperature and finite baryon densities. There further complications arise due to the gauge structure of the gluon or vector mesons [12]. Further applications concern the derivation of renormalized conserving quantum transport equations [13] which permit to consistently treat broad resonances [14].

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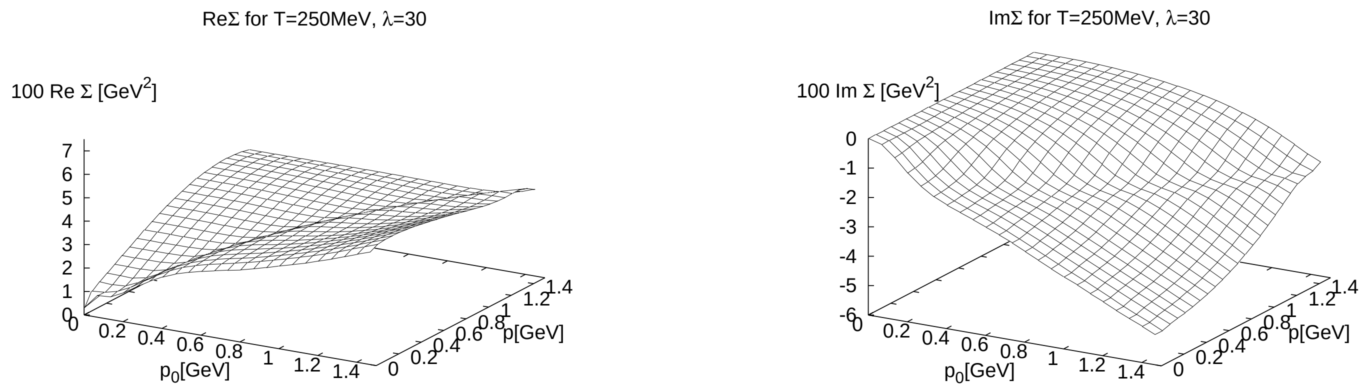

The real and imaginary part of the self-consistent self-energy for $\lambda/24\phi^4$ were calculated. A full calculation without further approximations could be achieved. The main effect of self-consistency is that the higher masses due to the tadpole contribution, which is dominant for the real part of the self-energy lowers the phase-space available for decays into three particles while the growing finite width itself induces a further broadening.