Heavy Quarks in the Quark-Gluon Plasma

Hendrik van Hees

Justus-Liebig Universität Gießen

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with

M. Mannarelli, V. Greco, L. Ravagli and R. Rapp
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Heavy Quarks in Heavy-Ion collisions

- Hard production of HQs described by PDF’s + pQCD (PYTHIA)
- \( c, b \) quark
- HQ rescattering in QGP: Langevin simulation
drag and diffusion coefficients from microscopic model for HQ interactions in the sQGP

- Hadronization to \( D, B \) mesons via quark coalescence + fragmentation
  V. Greco, C. M. Ko, R. Rapp, PLB 595, 202 (2004)

- Semileptonic decay \( \Rightarrow \)
  “non-photonic” electron observables
Heavy-Quark diffusion

- Fokker-Planck Equation

\[ \frac{\partial f(t, \vec{p})}{\partial t} = \frac{\partial}{\partial p_i} \left[ p_i A(t, p) + \frac{\partial}{\partial p_j} B_{ij}(t, \vec{p}) \right] f(t, \vec{p}) \]

- drag (friction) and diffusion coefficients

\[ p_i A(t, \vec{p}) = \langle p_i - p_i' \rangle \]

\[ B_{ij}(t, \vec{p}) = \frac{1}{2} \langle (p_i - p_i')(p_j - p_j') \rangle \]

\[ = B_0(t, p) \left( \delta_{ij} - \frac{p_ip_j}{p^2} \right) + B_1(t, p) \frac{p_ip_j}{p^2} \]

- transport coefficients defined via \( \mathcal{M} \)

\[ \langle X(\vec{p}') \rangle = \frac{1}{\gamma_c} \frac{1}{2E_p} \int \frac{d^3 \vec{q}}{(2\pi)^3 2E_q} \int \frac{d^3 \vec{q}'}{(2\pi)^3 2E_q'} \int \frac{d^3 \vec{p}'}{(2\pi)^3 2E_{p'}} \sum |\mathcal{M}|^2 (2\pi)^4 \delta^{(4)}(p + q - p' - q') \hat{f}(\vec{q}) X(\vec{p}') \]

- correct equil. lim. \( \Rightarrow \) Einstein relation: \( B_1(t, p) = T(t) E_p A(t, p) \)
Meaning of Fokker-Planck coefficients

- non-relativistic equation with constant $A = \gamma$ and $B_0 = D_1 = D$

$$\frac{\partial f}{\partial t} = \gamma \frac{\partial}{\partial \vec{p}} (\vec{p} f) + D \frac{\partial^2 f}{\partial \vec{p}^2}$$

- Green’s function:

$$G(t, \vec{p}; \vec{p}_0) = \left\{ \frac{\gamma}{2\pi D [1 - \exp(-2\gamma t)]} \right\}^{3/2} \times \exp\left\{ -\frac{\gamma}{2D} \frac{[\vec{p} - \vec{p}_0 \exp(-\gamma t)]^2}{1 - \exp(-2\gamma t)} \right\}$$

- Gaussian with

$$\langle \vec{p}(t) \rangle = \vec{p}_0 \exp(-\gamma t),$$

$$\langle \vec{p}^2(t) \rangle - \langle \vec{p}(t) \rangle^2 = \frac{3D}{\gamma} [1 - \exp(-2\gamma t)] \approx 6Dt$$

- $\gamma$: friction (drag) coefficient; $D$: diffusion coefficient
- equilibrium limit for $t \to \infty$: $D = mT\gamma$

(Einstein’s dissipation-fluctuation relation)
Relativistic Langevin process

- Fokker-Planck equation equivalent to \textit{stochastic differential equation}
- \textit{Langevin process}: friction force + Gaussian random force
- In the (local) rest frame of the heat bath

\[
\begin{align*}
\frac{d\vec{x}}{dt} &= \frac{\vec{p}}{E_p} dt, \\
\frac{d\vec{p}}{dt} &= -A \vec{p} dt + \sqrt{2} dt \left[ \sqrt{B_0} P_{\perp} + \sqrt{B_1} P_{\parallel} \right] \vec{w}
\end{align*}
\]

- \(\vec{w}\): normal-distributed random variable
- Dependent on realization of stochastic process
- To guarantee correct equilibrium limit: Use Hänggi-Klimontovich calculus, i.e., use \(B_{0/1}(t, \vec{p} + d\vec{p})\)
- For constant coefficients: Einstein dissipation-fluctuation relation \(B_0 = B_1 = E_p T A\).
- To implement flow of the medium
  - Use Lorentz boost to change into local “heat-bath frame”
  - Use update rule in heat-bath frame
  - Boost back into “lab frame”
Elastic pQCD processes

- Lowest-order matrix elements [Combridge 79]

- Debye-screening mass for $t$-channel gluon exch. $\mu_g = gT$, $\alpha_s = 0.4$
- not sufficient to understand RHIC data on “non-photonic” electrons
Non-perturbative interactions: effective resonance model

- General idea: Survival of $D$- and $B$-meson like resonances above $T_c$
- Chiral symmetry $SU_V(2) \otimes SU_A(2)$ in light-quark sector of QCD

\[
\mathcal{L}_D^{(0)} = \sum_{i=1}^{2} [(\partial_\mu \Phi_i^\dagger)(\partial^\mu \Phi_i) - m_D^2 \Phi_i^\dagger \Phi_i] + \text{massive (pseudo-)vectors } D^*
\]

- $\Phi_i$: two doublets: pseudo-scalar $\sim (\overline{D^0}_D)$ and scalar
- $\Phi^*_i$: two doublets: vector $\sim (\overline{D^0}_{D^*})$ and pseudo-vector

\[
\mathcal{L}_{qc}^{(0)} = \overline{q} i \slashed{\partial} q + \overline{c}(i \slashed{\partial} - m_c)c
\]

- $q$: light-quark doublet $\sim (u_d)$
- $c$: singlet
Interactions determined by chiral symmetry

For transversality of vector mesons:

heavy-quark effective theory vertices

\[ \mathcal{L}_{\text{int}} = -G_S \left( \frac{q}{2} \left( 1 + \gamma^\mu \right) \Phi_1^\mu c_v + \frac{q}{2} i \gamma^5 \Phi_2 c_v + \text{h.c.} \right) \]

\[ -G_V \left( \frac{q}{2} \left( 1 + \gamma^\mu \right) \Phi_1^\mu c_v + \frac{q}{2} i \gamma^\mu \gamma^5 \Phi_2^\mu c_v + \text{h.c.} \right) \]

\( v \): four velocity of heavy quark

in HQET: spin symmetry \( \Rightarrow G_S = G_V \)
Resonance Scattering

- elastic heavy-light-(anti-)quark scattering

\[ \bar{q} c \rightarrow D, D', D_s \]

- \( D \)- and \( B \)-meson like resonances in sQGP

\[ \bar{q} c \rightarrow D, D', D_s \]

- parameters
  - \( m_D = 2 \text{ GeV}, \Gamma_D = 0.4 \ldots 0.75 \text{ GeV} \)
  - \( m_B = 5 \text{ GeV}, \Gamma_B = 0.4 \ldots 0.75 \text{ GeV} \)
Cross sections

- total pQCD and resonance cross sections: comparable in size
- BUT pQCD forward peaked $\leftrightarrow$ resonance isotropic
- resonance scattering more effective for friction and diffusion
Transport coefficients: pQCD vs. resonance scattering

- three-momentum dependence

\[ \gamma \,[1/\text{fm}] \]

\[ \begin{align*}
T=200 \text{ MeV} & \quad \text{resonances } \Gamma=0.3 \text{ GeV} \\
& \quad \text{resonances } \Gamma=0.4 \text{ GeV} \\
& \quad \text{resonances } \Gamma=0.5 \text{ GeV} \\
pQCD: \alpha_s=0.3 & \\
pQCD: \alpha_s=0.4 & \\
pQCD: \alpha_s=0.5 & \\
\end{align*} \]

- resonance contributions factor \( \sim 2 \ldots 3 \) higher than pQCD!
Transport coefficients: pQCD vs. resonance scattering

- Temperature dependence

![Graphs showing temperature dependence of transport coefficients](image)

- Resonances: $\Gamma = 0.4 \text{ GeV}$
- pQCD: $\alpha_s = 0.4$
- Total

Hendrik van Hees (JLU Gießen)
Heavy Quarks in the QGP
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Time evolution of the fire ball

- **Elliptic fire-ball parameterization**
  fitted to hydrodynamical flow pattern [Kolb ’00]
  \[ V(t) = \pi(z_0 + \dot{v}_z t)a(t)b(t), \quad a, b: \text{half-axes of ellipse}, \]
  \[ v_{a,b} = v_\infty[1 - \exp(-\alpha t)] \mp \Delta v[1 - \exp(-\beta t)] \]

- Isentropic expansion: \( S = \text{const (fixed from } N_{ch}) \)

- QGP Equation of state:
  \[ s = \frac{S}{V(t)} = \frac{4\pi^2}{90} T^3(16 + 10.5n_f^*), \quad n_f^* = 2.5 \]

- obtain \( T(t) \Rightarrow A(t,p), B_0(t,p) \) and \( B_1 = TEA \)

- for semicentral collisions \( (b = 7 \, \text{fm}) \): \( T_0 = 340 \, \text{MeV}, \)
  QGP lifetime \( \simeq 5 \, \text{fm}/c. \)

- simulate FP equation as relativistic Langevin process
need initial $p_T$-spectra of charm and bottom quarks

(modified) PYTHIA to describe exp. D meson spectra, assuming δ-function fragmentation

exp. non-photonic single-$e^{\pm}$ spectra: Fix bottom/charm ratio

\[
\frac{1}{2\pi p_T} \frac{dN}{dp_T} dy \ [\text{a.u.}]
\]

\[
\sigma_{bb}/\sigma_{cc} = 4.9 \times 10^{-3}
\]
Spectra and elliptic flow for heavy quarks

- $\mu_D = gT$, $\alpha_s = g^2/(4\pi) = 0.4$
- resonances $\Rightarrow$ $c$-quark thermalization without upscaling of cross sections
- Fireball parametrization consistent with hydro

$\mu_D = 1.5T$ fixed

spatial diff. coefficient:

$D = D_s = \frac{T}{m_A}$

$2\pi T D \approx \frac{3}{2\alpha_s^2}$
Spectra and elliptic flow for heavy quarks

Au-Au $\sqrt{s}=200$ GeV (b=7 fm)

$c$, reso ($\Gamma=0.4-0.75$ GeV)
$c$, pQCD, $\alpha_s=0.4$
$b$, reso ($\Gamma=0.4-0.75$ GeV)

LO QCD
[Moore, Teaney '04]
Observables: $p_T$-spectra ($R_{AA}$), $v_2$

- **Hadronization**: Coalescence with light quarks + fragmentation
  \[ \leftrightarrow c\bar{c}, b\bar{b} \text{ conserved} \]
- Single electrons from decay of $D$- and $B$-mesons

- Without further adjustments: data quite well described
  
  [HvH, V. Greco, R. Rapp, Phys. Rev. C 73, 034913 (2006)]
Observables: $p_T$-spectra ($R_{AA}$), $v_2$

- Hadronization: Fragmentation only
- single electrons from decay of $D$- and $B$-mesons
Observables: $p_T$-spectra ($R_{AA}$), $v_2$

- Central Collisions
- Single electrons from decay of $D$- and $B$-mesons

Coalescence+Fragmentation

Fragmentation only
Comparison to newer data

(a) 0−10% central
- Armesto et al. (I)
- van Hees et al. (II)
- \( \frac{3}{2T} \pi \)
- \( \frac{12}{2T} \pi \)
- Moore & Teaney (III)

(b) minimum bias
- \( \pi^0 R_{AA}, p_T > 4 \text{ GeV/c} \)
- \( \pi^0 v_2, p_T > 2 \text{ GeV/c} \)
- \( e^\pm R_{AA}, e^\pm v_2^{HF} \)

PHENIX Collaboration
PRL 98 172301 (2007)
Microscopic model: Static potentials from lattice QCD

- color-singlet free energy from lattice
- use internal energy

\[ U_1(r, T) = F_1(r, T) - T \frac{\partial F_1(r, T)}{\partial T}, \]
\[ V_1(r, T) = U_1(r, T) - U_1(r \to \infty, T) \]

- Casimir scaling for other color channels \[ [\text{Nakamura et al 05; Döring et al 07}] \]

\[ V_3 = \frac{1}{2} V_1, \quad V_6 = -\frac{1}{4} V_1, \quad V_8 = -\frac{1}{8} V_1 \]
T-matrix

- Brueckner many-body approach for elastic $Qq$, $Q\bar{q}$ scattering

\[ T = V T_c + V \]

- reduction scheme: 4D Bethe-Salpeter $\rightarrow$ 3D Lipmann-Schwinger
- $S$- and $P$ waves
- same scheme for light quarks (self consistent!)
- Relation to invariant matrix elements

\[ \sum_q |\mathcal{M}(s)|^2 \propto \sum_q d_a \left( |T_{a,l=0}(s)|^2 + 3 |T_{a,l=1}(s)|^2 \cos \theta_{cm} \right) \]
\textbf{T-matrix}

- **resonance formation** at lower temperatures \( T \simeq T_c \)
- **melting** of resonances at higher \( T \)! \( \Rightarrow \) sQGP
- \( P \) wave smaller
- resonances near \( T_c \): natural connection to quark coalescence

[Ravagli, Rapp 07]

- model-independent assessment of elastic \( Qq, Q\bar{q} \) scattering
- problems: uncertainties in extracting potential from IQCD

in-medium potential \( V \) vs. \( F \)?
from non-pert. interactions reach $A_{\text{non-pert}} \simeq 1/(7 \text{ fm}/c) \simeq 4A_{\text{pQCD}}$

A decreases with higher temperature

higher density (over)compensated by melting of resonances!

spatial diffusion coefficient

$$D_s = \frac{T}{mA}$$

increases with temperature
Non-photonic electrons at RHIC

- same model for bottom
- quark coalescence + fragmentation → $D/B \rightarrow e + X$

- T-matrix
  - pQCD, $\alpha_s=0.4$

- $R_{AA}$ for Au-Au, $\sqrt{s}=200$ GeV (central)

- $v_2$ for Au-Au, $\sqrt{s}=200$ AGeV

- $v_2$ for Au+Au, $\sqrt{s}=200$ AGeV

- coalescence crucial for explanation of data
- increases both, $R_{AA}$ and $v_2$ ⇔ “momentum kick” from light quarks!
- “resonance formation” towards $T_c$ ⇒ coalescence natural [Ravagli, Rapp 07]
Properties of the sQGP

- Measure for coupling strength in plasma: $\eta/s$
- Relation to spatial diffusion coefficient

$$\frac{\eta}{s} \simeq \frac{1}{2} T D_s \quad \text{(AdS/CFT)}, \quad \frac{\eta}{s} \simeq \frac{1}{5} T D_s \quad \text{(wQGP)}$$
successes of quark-coalescence models in HI phenomenology

- high baryon/meson ration in heavy-ion compared to $pp$ collisions compared
- Constituent-quark number scaling of $v_2$

$$v_{2,\text{had}}(p_T) = n_q v_{2,q}(p_T/n_q)$$

- experiment: CQNS better for KE$_T$ than $p_t$

problems with “naive” coalescence models

- violates conservation laws (energy, momentum!)
- violates 2nd theorem of thermodynamics (entropy)

Resonance structures close to $T_c$

- transport process with $q\bar{q}(qq) \leftrightarrow R$
Resonance-Recombination Model

\[ \frac{\partial}{\partial t} f_M(t, p) = -\frac{\Gamma}{\gamma_p} f_M(t, p) + g(p) \Rightarrow f_M^{(eq)}(p) = \frac{\gamma_p g(p)}{\Gamma} \]

\[ g(p) = \int \frac{d^3p_1 d^3p_2}{(2\pi)^6} \int d^3x f_q(x, p_1) f_{\bar{q}}(x, p_2) \sigma(s) v_{rel} \delta^3(p - p_1 - p_2) \]

\[ \sigma(s) = g\sigma \frac{4\pi}{k_{cm}^2} \frac{(\Gamma m)^2}{(s - m^2)^2 + (\Gamma m)^2} \]

\[ \frac{E dN}{d^3p} (\text{GeV}^{-2}) \]

T=180 MeV, \( \beta_0=0.55 \)
Meson spectra

- $q\bar{q}$ input: Langevin simulation
- Meson output: resonance-recombination model

![Graph of phi meson](image1)

![Graph of J/psi meson](image2)

- PHENIX
- STAR
- STAR (new)
Constituent-quark number scaling

- usual coalescence models: factorization ansatz

$$f_q(p, x, \varphi) = f_q(p, x)[1 + 2v^q_2(p_T) \cos(2\varphi)]$$

- CQNS usually not robust with more realistic parametrizations of $v_2$
- here: $q$ input from Langevin simulation
Summary

- Heavy quarks in the sQGP
- non-perturbative interactions
  - mechanism for strong coupling: resonance formation at $T \gtrsim T_c$
  - IQCD potentials parameter free
  - res. melt at higher temperatures $\Leftrightarrow$ consistency betw. $R_{AA}$ and $v_2$!
- also provides “natural” mechanism for quark coalescence
- resonance-recombination model
- problems
  - extraction of $V$ from lattice data
  - potential approach at finite $T$: $F$, $V$ or combination?

Outlook

- include inelastic heavy-quark processes (gluon-radiation processes)
- other heavy-quark observables like charmonium suppression/regeneration