Contents

A quantum field theoretical renormalizable model for the $\pi\rho$-system

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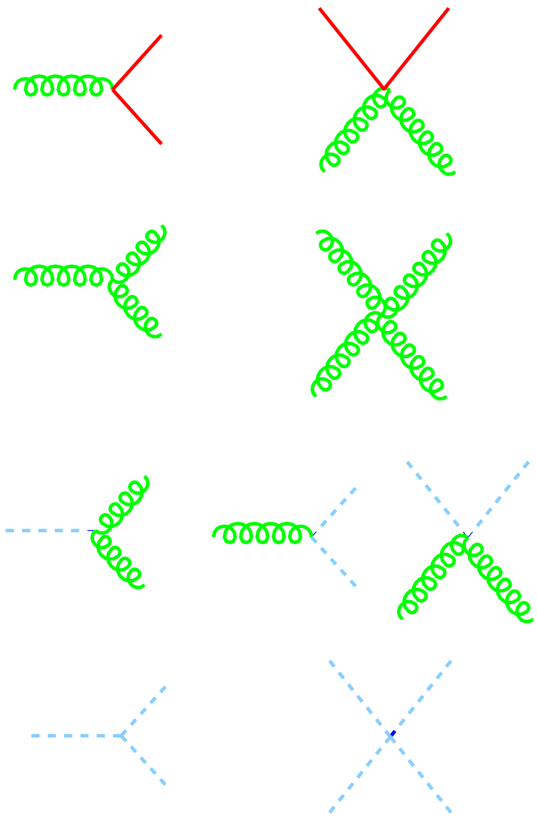
The model

- $U(1) \times SU(2)$ gauge model
- spontaneously broken with Higgs mechanism to $U(1) \Rightarrow 3$ ρ-mesons and 1 photon
- Price for renormalizability: 1 Higgs boson
- Couple the pions as a $SO(3)$-Triplett minimally to the gauge fields $\Rightarrow$ vector meson dominance

Unitary Gauge - Physical Vertices



$\pi\rho$-interactions

$\rho\rho$-interactions

ρ-Higgs-interactions

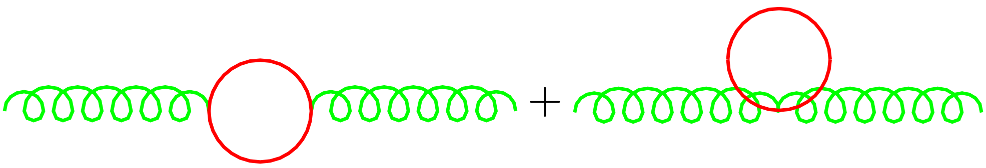
Higgs-Higgs-interactions

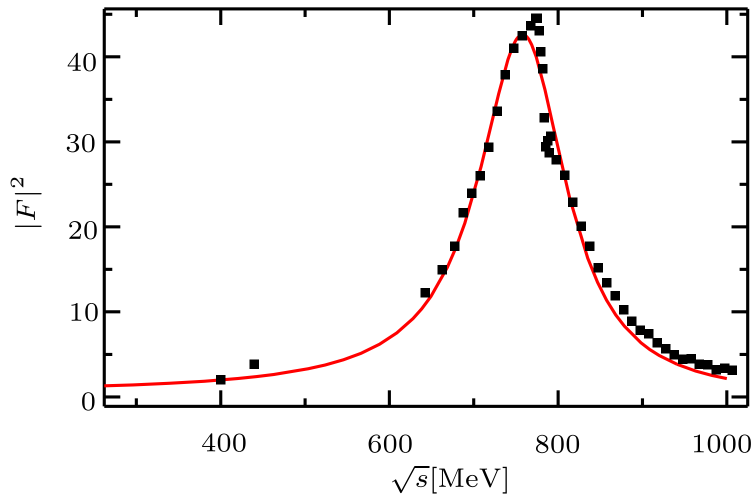
- No problem to add π-Higgs-interactions $\Rightarrow$ Theory can be seen as a gauged linear σ-model

Fit of the parameters

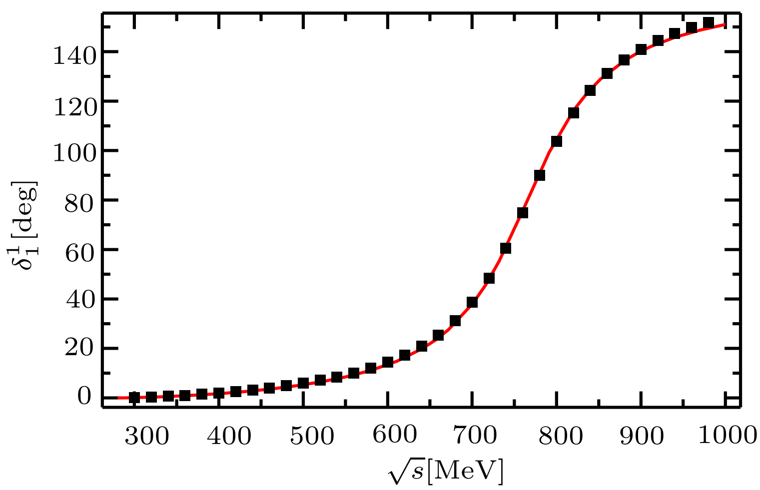
Form factor and Phase Shift

- Using dimensional regularization and renormalization of the one-loop-self-energy diagrams

$-i\Sigma =$ 



Data: Amendolia et al. Phys. Lett. **138B** (1984) 454
Barkov et al. Nucl. Phys. **B256** (1985) 365

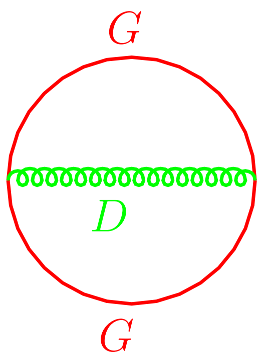


Data: Frogatt, Petersen, Nucl. Phys. **B129** (1977) 89

Selfconsistent approximations

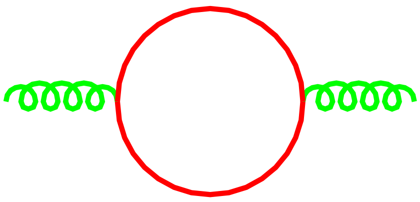
Generating functional

- $\Phi[G, D]$: sum over all 2PI closed diagrams with at least two loops

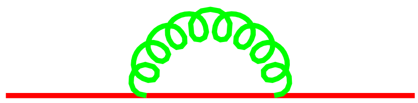
$$i\Phi[G, D] =$$ 

$$+ \cdots$$

- Variation with respect to Green's functions $\Rightarrow$ self energies fulfilling Dyson's equations

$$\frac{\delta i\Phi}{\delta D} = -i\Pi_\rho =$$ 

$$\Pi_\rho = D_0^{-1} - D^{-1}$$

$$\frac{\delta i\Phi}{\delta G} = -i\Sigma_\pi =$$ 

$$\Sigma_\pi = G_0^{-1} - G^{-1}$$

- Sum up to a certain loop order $\Rightarrow$ Selfconsistent effective approximation
- Respects any conservation law basing on global symmetries
- In thermal field theory: Thermodynamically consistent approximation

Renormalization

Renormalizing the selfconsistent approximation

- Can be seen as resummation of all self energy insertions ⇒ Infinities to all orders
- Renormalizable theory ⇒ finite by renormalizing parameters already present in Lagrangian
- Physical renormalization conditions

$$\Sigma_\pi(m_\pi^2) = \partial_s \Sigma_\pi(m_\pi^2) = 0, \; \Pi_\rho(0) = \partial_s \Pi_\rho(0) = 0$$

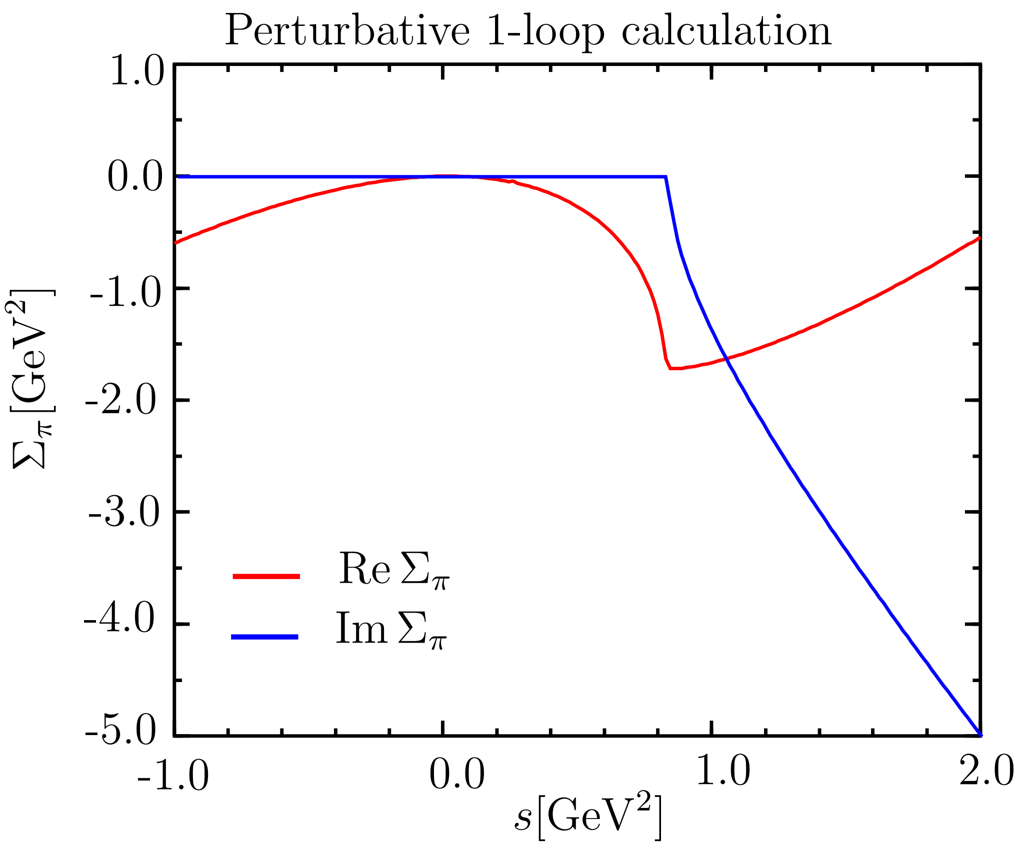
- Analytical properties of Green's functions

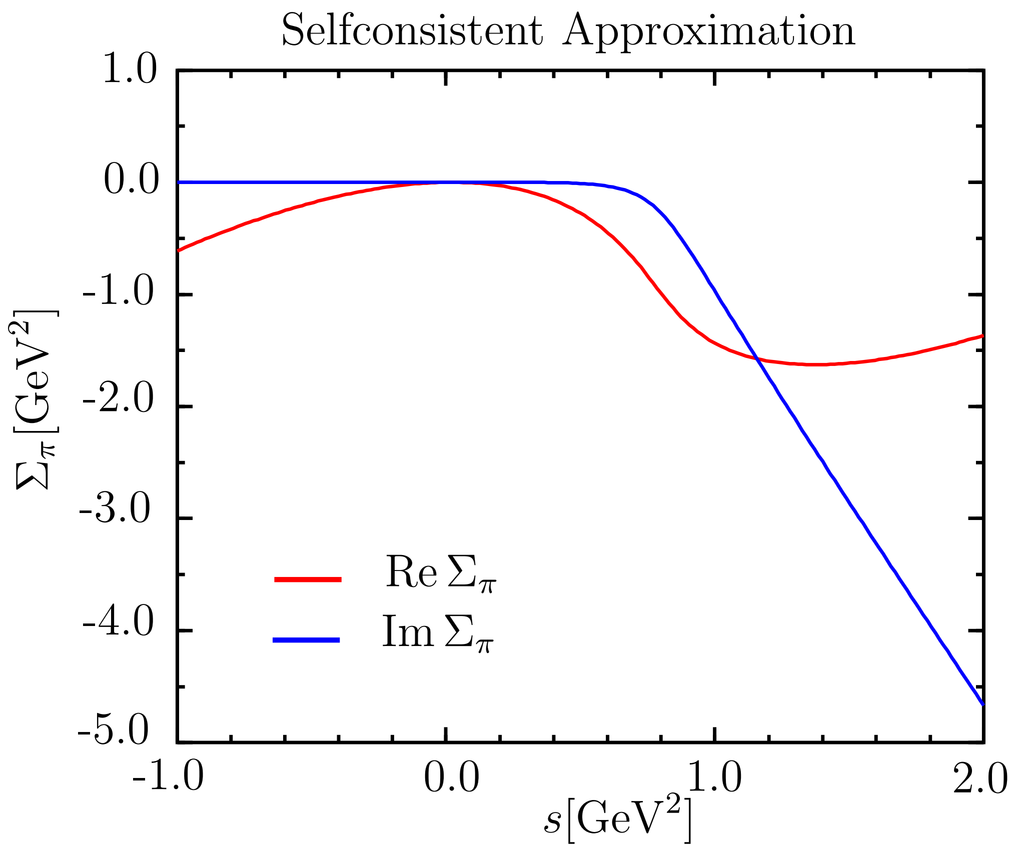
$$G(s) = \frac{1}{\pi} \int_0^\infty dm^2 \Delta(m^2, s) A(m^2) \text{ with } A(s) = -\operatorname{Im} G(s)$$

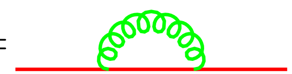
- $\Delta(m^2, s)$: Feynman-propagator ⇒ integral kernels ⇒ can be renormalized using standard techniques
- self consistent finite set of coupled integral equations solvable numerically by iteration
- Tadpole in vacuum absorbed into mass renormalization

Results in vacuum

The π-Self-Energy

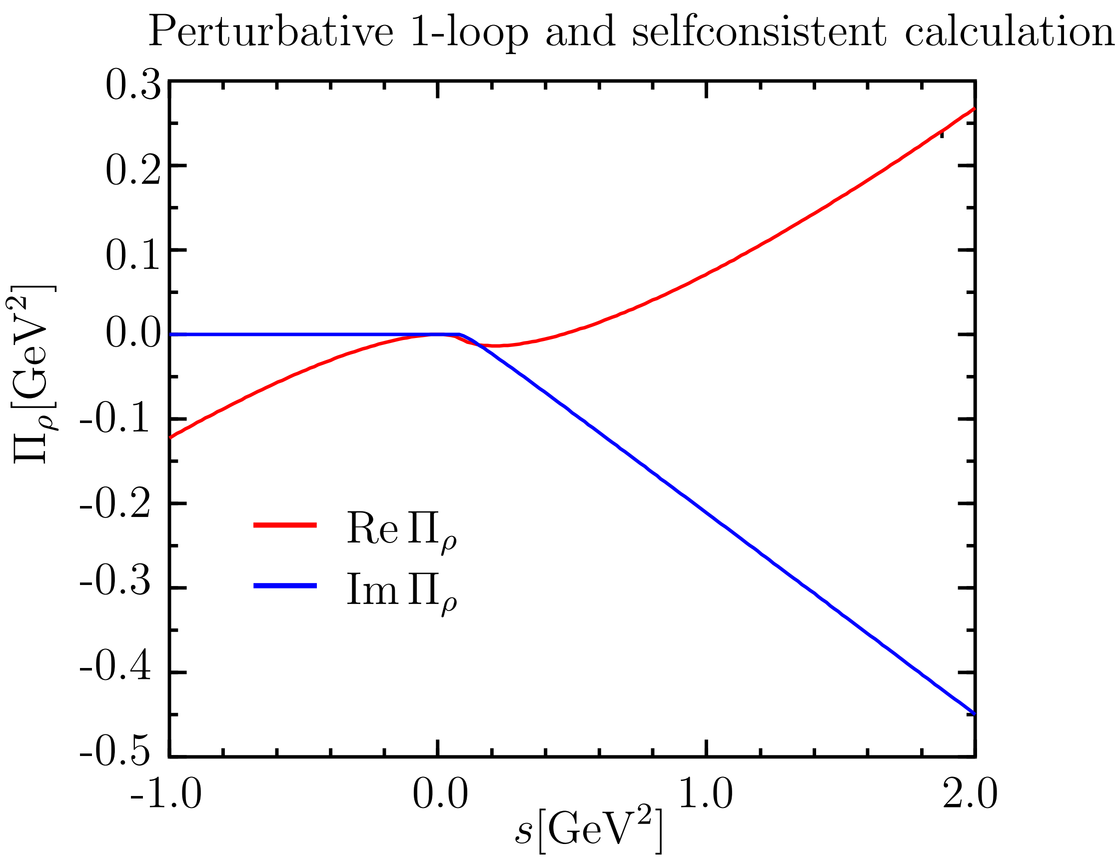


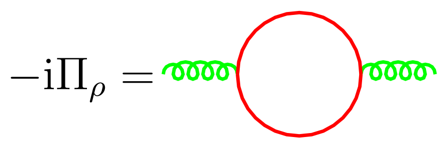


$-\mathrm{i}\Sigma_\pi =$ 

Result in vacuum

The ρ-Self-Energy

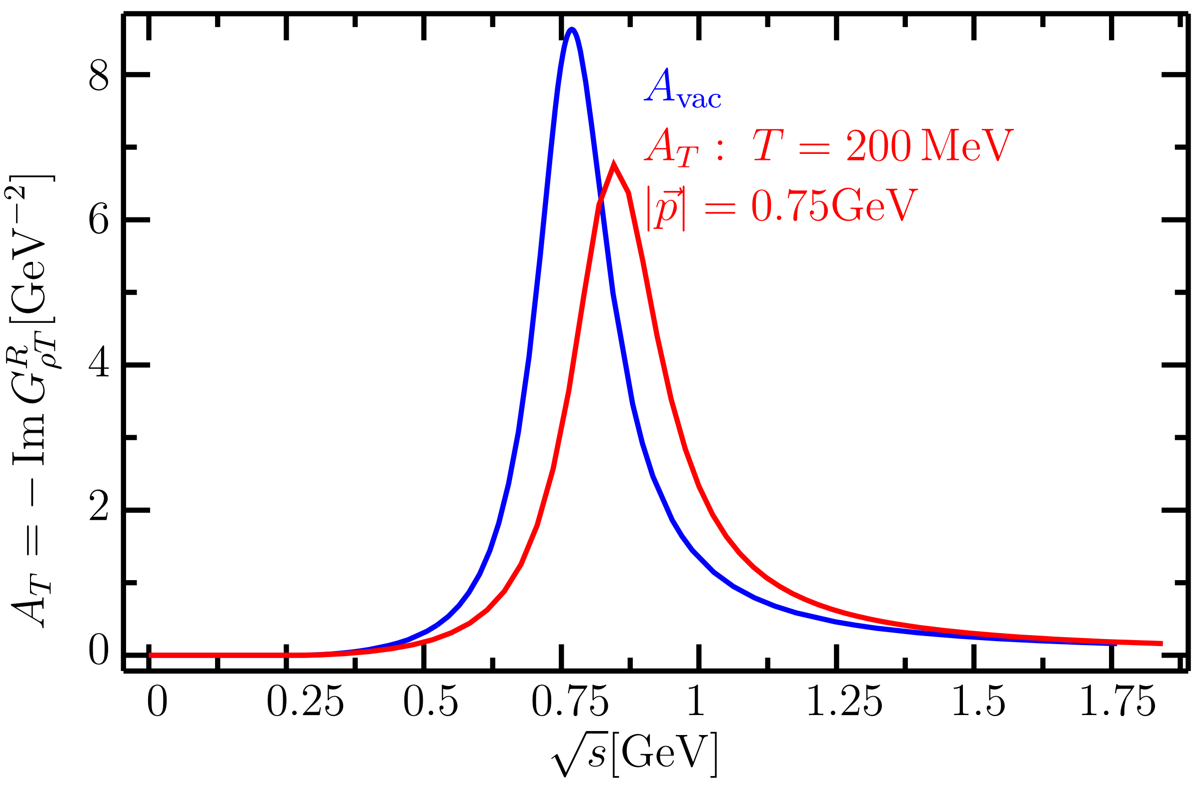




Outlook

Work to do

- Exploit non-abelian part of the ρ-interaction
- Selfconsistent approximation for $T, \mu > 0 \Rightarrow$
  Need to include tadpole contributions $\Rightarrow$
  Renormalization of the vertex
- Gauge invariance?



The 1-loop transverse spectral function of the ρ-meson in vacuum and at finite temperature.