Heavy-Quark Transport in the QGP

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1. Heavy-quark transport in the sQGP
   - Heavy quarks in heavy-ion collisions
   - Heavy-quark diffusion: The Fokker-Planck Equation
   - Relativistic Langevin simulations

2. Microscopic model for non-perturbative HQ interactions
   - Static heavy-quark potentials from lattice QCD
   - Elastic pQCD heavy-quark scattering
   - T-matrix approach

3. Non-photonic electrons at RHIC
   - Transport properties of the sQGP

4. Summary and Outlook
Motivation

- Fast equilibration of hot and dense matter in heavy-ion collisions: collective flow (nearly ideal hydrodynamics) $\Rightarrow$ sQGP
- Heavy quarks as calibrated probe of QGP properties
  - produced only in early hard collisions: well-defined initial conditions
  - not fully equilibrated due to large masses
  - heavy-quark diffusion $\Rightarrow$ probes for QGP-transport properties
- Langevin simulation
- drag and diffusion coefficients
  - $T$-matrix approach with static lattice-QCD heavy-quark potentials
  - resonance formation close to $T_c$
  - mechanism for non-perturbative strong interactions
Heavy Quarks in Heavy-Ion collisions

- Hard production of HQs described by PDF’s + pQCD (PYTHIA)
- $c,b$ quark

HQ rescattering in QGP: Langevin simulation
drag and diffusion coefficients from microscopic model for HQ interactions in the sQGP

Hadronization to $D,B$ mesons via quark coalescence + fragmentation

$K$ semileptonic decay $\Rightarrow$

"non-photonic" electron observables

$R_{AA}^{e^+e^-}(p_T), \nu_2^{e^+e^-}(p_T)$
The Fokker-Planck Equation

- Fokker-Planck equation

\[ \frac{\partial}{\partial t} F_Q(t, \vec{p}) = \frac{\partial}{\partial p_i} \left\{ A_i(\vec{p}) F_Q(t, \vec{p}) + \frac{\partial}{\partial p_j} [B_{ij}(\vec{p}) F_Q(t, \vec{p})] \right\} \]

- transition rates

\[ w(\vec{p}, \vec{k}) = \gamma_q \int \frac{d^3 \vec{q}}{(2\pi)^3} f_q(\vec{q}) v_{\text{rel}}(\vec{p}, \vec{q} \rightarrow \vec{p} - \vec{k}, \vec{q} + \vec{k}) \frac{d\sigma}{d\Omega} \]

- with drag and diffusion coefficients

\[ A_i(\vec{p}) = \int d^3 \vec{k} \ k_i w(\vec{p}, \vec{k}), \quad B_{ij}(\vec{p}) = \frac{1}{2} \int d^3 \vec{k} \ k_i k_j w(\vec{p}, \vec{k}) \]

- equilibrated light quarks and gluons: coefficients in heat-bath frame
- matter homogeneous and isotropic

\[ A_i(\vec{p}) = A(p)p_i, \quad B_{ij}(\vec{p}) = B_0(p) P^\perp_{ij} + B_1(p) P^\parallel_{ij} \]

with

\[ P^\parallel_{ij}(\vec{p}) = \frac{p_i p_j}{\vec{p}^2}, \quad P^\perp_{ij}(\vec{p}) = \delta_{ij} - \frac{p_i p_j}{\vec{p}^2} \]
Relativistic Langevin process

- **Langevin process**: friction force + Gaussian random force
- in the (local) rest frame of the heat bath

\[
d\vec{x} = \frac{\vec{p}}{E_p} dt,
\]

\[
d\vec{p} = -A\vec{p} dt + \sqrt{2dt}[\sqrt{B_0 P_\perp} + \sqrt{B_1 P_\parallel}] \vec{w}
\]

- \(\vec{w}\): normal-distributed random variable
- \(A\): friction (drag) coefficient
- \(B_{0,1}\): diffusion coefficients
- dependent on realization of stochastic process
- to guarantee correct equilibrium limit: Use Hänggi-Klimontovich calculus, i.e., use \(B_{0/1}(t, \vec{p} + d\vec{p})\)
- Einstein dissipation-fluctuation relation \(B_0 = B_1 = E_p TA\).
- to implement flow of the medium: Lorentz boost between heat-bath and lab frame
- still ambiguities in “freeze-out description”

Elastic pQCD processes

- Lowest-order matrix elements [Combridge 79]

Debye-screening mass for $t$-channel gluon exch. $\mu_g = gT$, $\alpha_s = 0.4$

not sufficient to understand RHIC data on “non-photonic” electrons
Static heavy-quark potentials from lattice QCD

- lattice QCD at finite temperature: calculate free energy $F = U - TS$
- calculate difference between $F_{\bar{Q}Q}$ with $Q$ and $\bar{Q}$ at distance, $r$ and $F$
- average Hamiltonian for static $\bar{Q}Q$: $\langle H \rangle_T = U = -T^2 \partial(F/T)/\partial T$
- long-distance limit: $2M_D(T) \simeq 2m_c + U(\infty, T)$
- can be reinterpreted as medium-modified heavy-quark mass
- short-distance: enhancement of $U$ over $T = 0$ Cornell potential
- “right” potential $V = xU + (1 - x)F$?
The potential fit to lattice data

- non-perturbative static gluon propagator
  \[ D_{00}(\vec{k}) = \frac{1}{(\vec{k}^2 + \mu_D^2)} + \frac{m_G^2}{(\vec{k}^2 + \tilde{m}_D^2)^2} \]

- finite-T HQ color-singlet-free energy from Polyakov loops
  \[ \exp[-F_1(r, T)/T] = \left\langle \text{Tr}[\Omega(x)\Omega^\dagger(y)]/N_c \right\rangle \]
  \[ = \exp \left[ \frac{g^2}{2N_cT^2} \left\langle A_{0,\alpha}(x)A_{0,\alpha}(y) - A_{0,\alpha}(x)^2 \right\rangle \right] + \mathcal{O}(g^6) \]

- identify \( \left\langle A_{0,\alpha}(x)A_{0,\alpha}(y) \right\rangle = D_{00}(x - y) \)

- color-singlet free energy
  \[ F_1(r, T) = -\frac{4}{3} \alpha_s \left\{ \frac{\exp(-m_D r)}{r} + \frac{m_G^2}{2\tilde{m}_D} \left[ \exp(-\tilde{m}_D r) - 1 \right] + m_D \right\} \]

- in vacuo \( m_D, \tilde{m}_D \to 0 \)
  \[ F_1(r) = -\frac{4}{3} \frac{\alpha_s}{r} + \sigma r, \quad \sigma = \frac{2\alpha_s m_G^2}{3} \]

[F. Riek, R. Rapp, PRC 82, 035201 (2010)]
T-matrix approach for qQ scattering

- **T-matrix Brückner approach** for heavy quarkonia as for HQ diffusion
- consistency between HQ diffusion and $\bar{Q}Q$ suppression!

\[
T(q, \bar{q}) = V + VT
\]

\[
\Sigma = \Sigma_{\text{glu}} + T
\]

- 4D Bethe-Salpeter equation $\rightarrow$ 3D Lippmann-Schwinger equation
- relativistic interaction $\rightarrow$ static heavy-quark potential (lQCD)

\[
T_\alpha(E; q', q) = V_\alpha(q', q) + \frac{2}{\pi} \int_0^\infty dk \ k^2 V_\alpha(q', k) G_{Q\bar{Q}}(E; k) T_\alpha(E; k, q) \\
\times \left\{ 1 - n_F[\omega_1(k)] - n_F[\omega_2(k)] \right\}
\]

- $q$, $q'$, $k$ relative 3-momentum of initial, final, interm. $qQ$ or $\bar{Q}Q$ state

[F. Riek, R. Rapp, PRC 82, 035201 (2010)]
**T-matrix results**

- **Resonance formation** at lower temperatures $T \approx T_c$
- Melting of resonances at higher $T$
- Model-independent assessment of elastic $Qq, Q\bar{q}$ scattering!
Transport coefficients

- From non-pert. interactions reach $A_{\text{non-pert}} \approx 1/(7 \text{ fm}/c) \approx 4A_{\text{pQCD}}$
- Results for free-energy potential, $F$ considerably smaller
Bulk evolution and initial conditions

- bulk evolution as elliptic thermal fireball
- isentropic expansion with QGP Equation of State
- initial $p_T$-spectra of charm and bottom quarks
  - (modified) PYTHIA to describe exp. D meson spectra, assuming $\delta$-function fragmentation
  - exp. non-photonic single-$e^\pm$ spectra: Fix bottom/charm ratio
Spectra and elliptic flow for $b$-quarks

Au-Au $\sqrt{s}=200$ GeV (b=7 fm), $b$-quarks

$R_{AA}$ vs $p_T$ (GeV)

$V_2$ (%) vs $p_T$ (GeV)
Non-photonic electrons at RHIC

- quark coalescence + fragmentation $\rightarrow D/B \rightarrow e + X$

- coalescence crucial for description of data
- increases both, $R_{AA}$ and $v_2 \iff$ “momentum kick” from light quarks!
- “resonance formation” towards $T_c$ $\Rightarrow$ coalescence natural

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Transport properties of the sQGP

- spatial diffusion coefficient: Fokker-Planck \( \Rightarrow D_s = \frac{T}{m_A} = \frac{T^2}{D} \)
- measure for coupling strength in plasma: \( \frac{\eta}{s} \)

\[
\frac{\eta}{s} \simeq \frac{1}{2} T D_s \quad \text{(AdS/CFT)}, \quad \frac{\eta}{s} \simeq \frac{1}{5} T D_s \quad \text{(wQGP)}
\]

![Graph showing transport properties](image)

[Lacey, Taranenko (2006)]
Heavy quarks in the sQGP

- non-perturbative interactions
  - mechanism for strong coupling: resonance formation at $T \gtrsim T_c$
  - lattice-QCD potentials parameter free
  - resonances melt at higher temperatures
    \iff consistency betw. $R_{AA}$ and $v_2$!

also provides “natural” mechanism for quark coalescence


- potential approach at finite $T$: $F$, $V$ or combination?

- Non-photonic electron observables
  - described by model independent lQCD-based potentials
  - resonance formation provides strong coupling of HQs to plasma
  - $\Rightarrow$ transport properties of sQGP (small $\eta/s$)

Outlook

- include inelastic heavy-quark processes (gluo-radiative processes)
- other heavy-quark observables like charmonium suppression/regeneration