Heavy-Quark Kinetics in the Quark-Gluon Plasma

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with

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Outline

1. Heavy-quark interactions in the sQGP
   - Heavy-quark observables in heavy-ion collisions
   - Heavy-quark diffusion: The Fokker-Planck Equation
   - Elastic pQCD heavy-quark scattering
   - Non-perturbative interactions: effective resonance model

2. Non-photonic electrons at RHIC

3. Microscopic model for non-pert. HQ interactions
   - Static heavy-quark potentials from lattice QCD
   - T-matrix approach

4. Resonance-Recombination Model

5. Summary and Outlook
Theory of strong interactions: Quantum Chromo Dynamics, QCD

At high enough densities/temperatures: hadrons dissolve into a Quark-Gluon Plasma (QGP)

hope to create QGP in Heavy-Ion Collisions at RHIC (and LHC)

RHIC: collide gold nuclei with energy of 200 GeV per nucleon:
Evidence for QGP from heavy-ion observables

- particle $p_T$ spectra show hydrodynamical behavior
- collective flow of matter in local thermal equilibrium
- nuclear modification factor $\Rightarrow$ degree of thermalization

$$R_{AA}(p_T) = \frac{dN_{AA}/dp_T}{N_{coll}dN_{pp}/dp_T}$$

- no QGP $\Rightarrow R_{AA} = 1$; observed: $R_{AA} < 1$ (suppression) at high $p_T$
- in non-central collisions: anisotropic collective flow

- initially reaction zone of elliptic shape
- pressure gradients: $\langle |p_x| \rangle > \langle |p_y| \rangle$
- measure of flow anisotropy:

$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle \Rightarrow \langle \cos(2\phi_p) \rangle$$
Heavy Quarks in Heavy-Ion collisions

- Hard production of HQs described by PDF’s + pQCD (PYTHIA)
  - c, b quark

HQ rescattering in sQGP: Langevin simulation, drag and diffusion coefficients from microscopic model for HQ interactions in the sQGP

Hadronization to \(D, B\) mesons via quark coalescence + fragmentation

\(K^\pm\) semileptonic decay \(\Rightarrow\) “non-photonic” electron observables
Heavy-Quark diffusion

- **Fokker Planck Equation**

\[
\frac{\partial f(t, \vec{p})}{\partial t} = \frac{\partial}{\partial p_i} \left[ p_i A(t, p) + \frac{\partial}{\partial p_j} B_{ij}(t, \vec{p}) \right] f(t, \vec{p})
\]

- **drag (friction) and diffusion coefficients**

\[
p_i A(t, \vec{p}) = \langle p_i - p_i' \rangle
\]

\[
B_{ij}(t, \vec{p}) = \frac{1}{2} \langle (p_i - p_i')(p_j - p_j') \rangle
\]

\[
= B_0(t, p) \left( \delta_{ij} - \frac{p_i p_j}{p^2} \right) + B_1(t, p) \frac{p_i p_j}{p^2}
\]

- **transport coefficients defined via \( \mathcal{M} \)**

\[
\langle X(\vec{p}') \rangle = \frac{1}{\gamma_c} \frac{1}{2E_p} \int \frac{d^3\vec{q}}{(2\pi)^3 2E_q} \int \frac{d^3\vec{q}'}{(2\pi)^3 2E_{q'}} \int \frac{d^3\vec{p}'}{(2\pi)^3 2E_{p'}} \sum |\mathcal{M}|^2 (2\pi)^4 \delta^{(4)}(p + q - p' - q') \hat{f}(\vec{q}) X(\vec{p}')
\]

- **correct equil. lim. \( \Rightarrow \) Einstein relation:**

\[
B_1(t, p) = T(t) E_p A(t, p)
\]
Meaning of Fokker-Planck coefficients

- non-relativistic equation with constant $A = \gamma$ and $B_0 = D_1 = D$

$$\frac{\partial f}{\partial t} = \gamma \frac{\partial}{\partial \vec{p}}(\vec{p}f) + D \frac{\partial^2 f}{\partial \vec{p}^2}$$

- Green’s function:

$$G(t, \vec{p}; \vec{p}_0) = \left\{ \frac{\gamma}{2\pi D [1 - \exp(-2\gamma t)]} \right\}^{3/2} \times \exp\left\{ -\frac{\gamma}{2D} \frac{[\vec{p} - \vec{p}_0 \exp(-\gamma t)]^2}{1 - \exp(-2\gamma t)} \right\}$$

- Gaussian with

$$\langle \vec{p}(t) \rangle = \vec{p}_0 \exp(-\gamma t),$$
$$\langle \vec{p}^2(t) \rangle - \langle \vec{p}(t) \rangle^2 = \frac{3D}{\gamma} [1 - \exp(-2\gamma t)] \approx 6Dt$$

- $\gamma$: friction (drag) coefficient; $D$: diffusion coefficient
- equilibrium limit for $t \to \infty$: $D = mT\gamma$
  (Einstein’s dissipation-fluctuation relation)
Relativistic Langevin process

- Fokker-Planck equation equivalent to *stochastic differential equation*
- *Langevin process*: friction force + Gaussian random force
- in the (local) rest frame of the heat bath

\[
\begin{align*}
\mathrm{d}\vec{x} &= \frac{\vec{p}}{E_p} \mathrm{d}t, \\
\mathrm{d}\vec{p} &= -A \vec{p} \mathrm{d}t + \sqrt{2} \mathrm{d}t \left[ \sqrt{B_0} P_\perp + \sqrt{B_1} P_\parallel \right] \vec{w} 
\end{align*}
\]

- \( \vec{w} \): normal-distributed random variable
- dependent on realization of stochastic process
- to guarantee correct equilibrium limit: Use Hänggi-Klimontovich calculus, i.e., use \( B_{0/1}(t, \vec{p} + \mathrm{d}\vec{p}) \)
- for constant coefficients: Einstein dissipation-fluctuation relation \( B_0 = B_1 = E_p T A \).
- to implement flow of the medium
  - use Lorentz boost to change into local “heat-bath frame”
  - use update rule in heat-bath frame
  - boost back into “lab frame”
Elastic pQCD processes

- Lowest-order matrix elements [Combridge 79]

- Debye-screening mass for $t$-channel gluon exch. $\mu_g = gT$, $\alpha_s = 0.4$
- not sufficient to understand RHIC data on “non-photonic” electrons
General idea: Survival of $D$- and $B$-meson like resonances above $T_c$

Chiral symmetry $SU_V(2) \otimes SU_A(2)$ in light-quark sector of QCD

$$\mathcal{L}_D^{(0)} = \sum_{i=1}^{2} \left[ (\partial_\mu \Phi_i^\dagger)(\partial^\mu \Phi_i) - m_D^2 \Phi_i^\dagger \Phi_i \right] + \text{massive (pseudo-)vectors } D^*$$

$\Phi_i$: two doublets: pseudo-scalar $\sim \left( \frac{D_0}{D^-} \right)$ and scalar

$\Phi_i^*$: two doublets: vector $\sim \left( \frac{D_0^*}{D_-^*} \right)$ and pseudo-vector

$$\mathcal{L}_{qc}^{(0)} = \bar{q} i \slashed{\partial} q + \bar{c}(i \slashed{\partial} - m_c) c$$

$q$: light-quark doublet $\sim \left( \begin{array}{c} u \\ d \end{array} \right)$

$c$: singlet
Interactions

- Interactions determined by chiral symmetry
- For transversality of vector mesons:
  heavy-quark effective theory vertices

\[
\mathcal{L}_{\text{int}} = -G_S \left( \frac{1 + \psi}{2} \bar{q} \Phi_1 c_v + \frac{1 + \psi}{2} i \gamma^5 \Phi_2 c_v + h.c. \right) \\
- G_V \left( \frac{1 + \psi}{2} \bar{q} \gamma^\mu \Phi_1^{*\mu} c_v + \frac{1 + \psi}{2} i \gamma^\mu \gamma^5 \Phi_2^{*\mu} c_v + h.c. \right)
\]

- \( v \): four velocity of heavy quark
- In HQET: spin symmetry \( \Rightarrow G_S = G_V \)
Resonance Scattering

- elastic heavy-light-(anti-)quark scattering

- $D$- and $B$-meson like resonances in sQGP

- parameters
  - $m_D = 2$ GeV, $\Gamma_D = 0.4 \ldots 0.75$ GeV
  - $m_B = 5$ GeV, $\Gamma_B = 0.4 \ldots 0.75$ GeV
Cross sections

- total pQCD and resonance cross sections: comparable in size
- BUT pQCD forward peaked $\leftrightarrow$ resonance isotropic
- resonance scattering more effective for friction and diffusion
Transport coefficients: pQCD vs. resonance scattering

- three-momentum dependence

- resonance contributions factor $\sim 2 \ldots 3$ higher than pQCD!

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Transport coefficients: pQCD vs. resonance scattering

- Temperature dependence

\begin{align*}
\gamma &\quad \text{[1/fm]} \\
D &\quad \text{[GeV/fm]}
\end{align*}

\begin{align*}
\text{resonances: } \Gamma &= 0.4 \text{ GeV} \\
pQCD: \quad \alpha_s &= 0.4 \\
total
\end{align*}
Time evolution of the fire ball

- Elliptic fire-ball parameterization fitted to hydrodynamical flow pattern [Kolb '00]

\[ V(t) = \pi (z_0 + v_z t) a(t) b(t), \quad a, b: \text{half-axes of ellipse}, \]

\[ v_{a,b} = v_\infty [1 - \exp(-\alpha t)] \pm \Delta v [1 - \exp(-\beta t)] \]

- Isentropic expansion: \( S = \text{const (fixed from } N_{\text{ch}}) \)

- QGP Equation of state:

\[ s = \frac{S}{V(t)} = \frac{4\pi^2}{90} T^3 (16 + 10.5 n_f^*), \quad n_f^* = 2.5 \]

- obtain \( T(t) \Rightarrow A(t, p), B_0(t, p) \) and \( B_1 = T EA \)

- for semicentral collisions \((b = 7 \text{ fm})\): \( T_0 = 340 \text{ MeV}, \)
QGP lifetime \( \sim 5 \text{ fm}/c. \)

- simulate FP equation as relativistic Langevin process
Initial conditions

- need initial $p_T$-spectra of charm and bottom quarks
- (modified) PYTHIA to describe exp. D meson spectra, assuming δ-function fragmentation
- exp. non-photonic single-$e^\pm$ spectra: Fix bottom/charm ratio

\[ \frac{1}{2\pi p_T} dN/dp_T \ dy \ \text{[a.u.]} \]

$\sigma_{bb}/\sigma_{cc} = 4.9 \times 10^{-3}$
Spectra and elliptic flow for heavy quarks

- $\mu_D = gT$, $\alpha_s = g^2/(4\pi) = 0.4$
- Resonances $\Rightarrow$ $c$-quark thermalization without upscaling of cross sections
- Fireball parametrization consistent with hydro

$\mu_D = 1.5T$ fixed

Spatial diff. coefficient:
\[ D = D_s = \frac{T}{mA} \]

\[ 2\pi T D \sim \frac{3}{2\alpha_s^2} \]
Spectra and elliptic flow for heavy quarks

- c, reso ($\Gamma = 0.4 - 0.75$ GeV)
- c, pQCD, $\alpha_s = 0.4$
- b, reso ($\Gamma = 0.4 - 0.75$ GeV)

Au-Au $\sqrt{s} = 200$ GeV (b=7 fm)

LO QCD [Moore, Teaney '04]
Observables: $p_T$-spectra ($R_{AA}$), $v_2$

- **Hadronization:** Coalescence with light quarks + fragmentation
  \[ \leftrightarrow c\bar{c}, b\bar{b} \text{ conserved} \]
- Single electrons from decay of $D$- and $B$-mesons

Without further adjustments: data quite well described

[HvH, V. Greco, R. Rapp, Phys. Rev. C 73, 034913 (2006)]
Observables: $p_T$-spectra ($R_{AA}$), $v_2$

- Hadronization: Fragmentation only
- single electrons from decay of $D$- and $B$-mesons

![Graphs showing $R_{AA}$ and $v_2$ as functions of $p_T$]
Observables: $p_T$-spectra ($R_{AA}$), $v_2$

- Central Collisions
- Single electrons from decay of $D$- and $B$-mesons

Coalescence + Fragmentation

Fragmentation only

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Comparison to newer data

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color-singlet free energy from lattice
use internal energy

\[
U_1(r, T) = F_1(r, T) - T \frac{\partial F_1(r, T)}{\partial T},
\]
\[
V_1(r, T) = U_1(r, T) - U_1(r \to \infty, T)
\]

Casimir scaling for other color channels \[\text{[Nakamura et al 05; Döring et al 07]}\]

\[
V_3 = \frac{1}{2} V_1, \quad V_6 = -\frac{1}{4} V_1, \quad V_8 = -\frac{1}{8} V_1
\]
T-matrix

- Brueckner many-body approach for elastic $Qq, Q\bar{q}$ scattering

\[
T = V c + V T
\]

\[
\Sigma = \Sigma_{\text{glu}} + T
\]

- reduction scheme: 4D Bethe-Salpeter $\rightarrow$ 3D Lipmann-Schwinger
- $S$- and $P$ waves
- same scheme for light quarks (self consistent!)
- Relation to invariant matrix elements

\[
\sum S |M(s)|^2 \propto \sum Q \left( |T_{a,l=0}(s)|^2 + 3|T_{a,l=1}(s)|^2 \cos \theta_{\text{cm}} \right)
\]
- resonance formation at lower temperatures $T \approx T_c$
- melting of resonances at higher $T! \Rightarrow sQGP$
- $P$ wave smaller
- resonances near $T_c$: natural connection to quark coalescence

[Ravagli, Rapp 07]

- model-independent assessment of elastic $Qq, Q\bar{q}$ scattering
- problems: uncertainties in extracting potential from IQCD in-medium potential $V$ vs. $F$?
Transport coefficients

- from non-pert. interactions reach $A_{\text{non-pert}} \simeq 1/(7 \text{ fm}/c) \simeq 4A_{pQCD}$
- $A$ decreases with higher temperature
- higher density (over)compensated by melting of resonances!
- spatial diffusion coefficient

$$D_s = \frac{T}{mA}$$

increases with temperature
Non-photonic electrons at RHIC

- same model for bottom
- quark coalescence + fragmentation \( \rightarrow D/B \rightarrow e + X \)

**coalescence crucial for explanation of data**
- increases both, \( R_{AA} \) and \( v_2 \) \( \Leftrightarrow \) “momentum kick” from light quarks!
- “resonance formation” towards \( T_{\text{c}} \) - coalescence natural [Ravagli, Rapp 07]
Properties of the sQGP

- measure for coupling strength in plasma: $\eta/s$
- relation to spatial diffusion coefficient

$$\frac{\eta}{s} \simeq \frac{1}{2} TD_s \quad \text{(AdS/CFT)}, \quad \frac{\eta}{s} \simeq \frac{1}{5} TD_s \quad \text{(wQGP)}$$
successes of quark-coalescence models in HI phenomenology
  - high baryon/meson ratio in heavy-ion compared to $pp$ collisions compared
  - Constituent-quark number scaling of $v_2$

$$v_{2,\text{had}}(p_T) = n_q v_{2,q}(p_T/n_q)$$

- experiment: CQNS better for KE$_T$ than $p_t$
- problems with “naive” coalescence models
  - violates conservation laws (energy, momentum!)
  - violates 2nd theorem of thermodynamics (entropy)
- Resonance structures close to $T_c$
  - transport process with $q\bar{q}(qq) \leftrightarrow R$
Resonance-Recombination Model

\[
\frac{\partial}{\partial t} f_M(t, p) = -\frac{\Gamma}{\gamma_p} f_M(t, p) + g(p) \Rightarrow f_M^{(eq)}(p) = \frac{\gamma_p}{\Gamma} g(p)
\]

\[
g(p) = \int \frac{d^3 p_1 d^3 p_2}{(2\pi)^6} \int d^3 x f_q(x, p_1) f_{\bar{q}}(x, p_2) \sigma(s) v_{rel} \delta^3(p - p_1 - p_2)
\]

\[
\sigma(s) = g_\sigma \frac{4\pi}{k_{cm}^2} \frac{(\Gamma m)^2}{(s - m^2)^2 + (\Gamma m)^2}
\]
Meson spectra

- $q\bar{q}$ input: Langevin simulation
- meson output: resonance-recombination model

![Graphs showing meson spectra for $\phi$ and $J/\psi$ at $T=180$ MeV.](image)
Constituent-quark number scaling

- usual coalescence models: factorization ansatz

\[ f_q(p, x, \varphi) = f_q(p, x)[1 + 2v_2^q(p_T) \cos(2\varphi)] \]

- CQNS usually not robust with more realistic parametrizations of \( v_2 \)
- here: \( q \) input from Langevin simulation
Summary

- **Heavy quarks in the sQGP**
- **non-perturbative interactions**
  - mechanism for **strong coupling**: resonance formation at $T \gtrsim T_c$
  - **lQCD potentials** parameter free
  - res. melt at higher temperatures $\Leftrightarrow$ consistency betw. $R_{AA}$ and $v_2$

- also provides “natural” mechanism for quark coalescence
- **resonance-recombination model**
- problems
  - extraction of $V$ from lattice data
  - potential approach at finite $T$: $F$, $V$ or combination?

Outlook

- include **inelastic heavy-quark processes** (gluon-radiation processes)
- other **heavy-quark observables** like charmonium suppression/regeneration