Heavy-Quark Diffusion in the QGP in Heavy-Ion Collisions

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Outline

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5 Summary and Outlook

6 Backup: Static heavy-quark potentials from lattice QCD + Brückner T-matrix
collisions of relativistic (heavy) nuclei
many collisions of partons inside nucleons
creation of many particles $\Rightarrow$ hot and dense fireball
formation of (thermalized) QGP?
how to learn about properties of QGP?
Hydrodynamical radial flow of the bulk

- ideal fluid in local thermal equilibrium ⇒ low viscosity/(entropy density), $\eta/s$
- needs strong interactions
- hydrodynamical model for ultra-relativistic heavy-ion collisions
  - after short formation time ($t_0 \lesssim 1 \text{ fm}/c$)
  - QGP in local thermal equilibrium → hadronization at $T_c \simeq 160 - 190 \text{ MeV}$
  - chemical freeze-out: (inelastic collisions cease) $T_{\text{ch}} \simeq 160 - 175 \text{ MeV}$
  - thermal freeze-out: (also elastic scatterings cease)
Hydrodynamical elliptic flow of the bulk

- particle spectra compatible with collective flow of a (nearly) ideal fluid ⇒ small $\eta/s$
- medium in local thermal equilibrium

\[ v_2 = \langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \rangle \]
Thermal Models for Chemical Freezeout

- particle abundancies compatible with thermalized hadron-resonance gas
- grand-canonical ensemble
  - fix mean energy ⇒ temperature $T_{ch}$ (expect $T_c \simeq T_{ch}$)
  - fix mean conserved “charges” ⇒ chemical potentials $\mu_b$, $\mu_s$, $\mu_q$.

\[
n_i = \frac{g_i}{(2\pi)^3} 4\pi \int_0^{\infty} dp \frac{p^2}{\exp \left( \frac{\sqrt{p^2+m_i^2} - \mu_i}{T_{ch}} \right) \pm 1}
\]

\[
\mu_i = \mu_b B_i + \mu_s S_i + \mu_q Q_i
\]

[ ]

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Heavy Probes in HICs (Theory II)
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Jet Quenching

- comparison to proton-proton collisions: nuclear-modification factor

\[ R_{AA} = \frac{dN_{AA}/d\pt}{N_{\text{coll}}dN_{pp}/d\pt} \]

- \( R_{AA} < 1 \) for large \( \pt \): jets absorbed by medium
- density > \( \rho_{\text{crit}} \) (comparison to lattice QCD)
Constituent-quark-number scaling of $v_2$

- elliptic flow, $v_2$ scales with number of constituent quarks

$$v_2^{(\text{had})}(p_T^{(\text{had})}) = n_q v_2^{(q)}(p_T^{(\text{had})}/n_q)$$

- suggests coalescence of quarks at $T_c$

possible microscopic mechanism hadron-resonance formation at $T_c$ ⇒ resonance-recombination model [Ravagli, HvH, Rapp, PRC 79, 064902 (2009)]

other hint to quark coalescence:

enhanced baryon/meson ratio compared to pp collisions
Heavy quarks in the sQGP

hard production of HQs
described by PDF’s + pQCD (PYTHIA)

c, b quark

HQ rescattering in QGP: Langevin simulation
drag and diffusion coefficients from
microscopic model for HQ interactions in the sQGP

Hadronization to $D, B$ mesons via
quark coalescence + fragmentation

semileptonic decay $\Rightarrow$
“non-photonic” electron observables
The relativistic Boltzmann equation

- describe **heavy-quark scattering** in the QGP by (semi-)classical transport equation
- \( f_Q(t, \vec{x}, \vec{p}) \): phase-space distribution of **heavy quarks**
- equation of motion for HQ-fluid cell at time \( t \) at \((\vec{p}, \vec{x})\):

\[
df_Q = dt \left( \frac{\partial}{\partial t} + \vec{v} \frac{\partial}{\partial \vec{x}} + \vec{F} \cdot \frac{\partial}{\partial \vec{p}} \right) f_Q
\]

  - change of phase-space distribution with time (non-equilibrium)
  - drift of HQ-fluid cell with velocity \( \vec{v} = \vec{p}/E_{\vec{p}}, \quad E_{\vec{p}} = \sqrt{m_Q^2 + \vec{p}^2} \)
  - change of momentum with mean-field force, \( \vec{F} \)

- change must be due to **collisions with surrounding medium**

\[
\frac{d}{dt} f_Q = C[f_Q] = \int d^3\vec{k} \left[ w(\vec{p} + \vec{k}, \vec{k}) f_Q(t, \vec{x}, \vec{p} + \vec{k}) - w(\vec{p}, \vec{k}) f_Q(t, \vec{x}, \vec{p}) \right]
\]

  - \( w(\vec{p}, \vec{k}) \): transition rate for collision of a **heavy quark** with momentum, \( \vec{p} \) with a heat-bath particle with momentum transfer, \( \vec{k} \)
Transition rates

- relation to cross sections of microscopic scattering processes
- e.g., elastic scattering of heavy quark with light quarks

\[ w(\vec{p}, \vec{k}) = \gamma_q \int \frac{d^3 \vec{q}}{(2\pi)^3} f_q(\vec{q}) v_{\text{rel}}(\vec{p}, \vec{q} \rightarrow \vec{p} - \vec{k}, \vec{q} + \vec{k}) \frac{d\sigma}{d\Omega} \]

- \[ \gamma_q = 2 \times 3 = 6: \text{spin-color-degeneracy factor} \]
- \[ v_{\text{rel}} := \sqrt{(p \cdot q)^2 - (m_Q m_q)^2 / (E_Q E_q)}; \text{covariant relative velocity} \]
- in terms of invariant matrix element

\[ C[f_Q] = \frac{1}{2E_Q} \int \frac{d^3 \vec{q}}{(2\pi)^3 2E_q} \int \frac{d^3 \vec{p}'}{(2\pi)^3 2E'_p} \int \frac{d^3 \vec{q}'}{(2\pi)^3 2E'_q} \times \frac{1}{\gamma_Q} \sum_{c,s} |M(\vec{p}', \vec{q}') \rightarrow (\vec{p}, \vec{q})|^2 \times (2\pi)^4 \delta^{(4)}(p + q - p' - q') [f_Q(\vec{p}')f_q(\vec{q}') - f_Q(\vec{p})f_q(\vec{q})] \]

- \[ \vec{p}, \vec{q}, (\vec{p}', \vec{q}') \text{ initial (final) momenta of heavy and light quark} \]
- momentum transfer: \[ \vec{k} = \vec{q}' - \vec{q} = \vec{p} - \vec{p}' \]
- sum over all (“relevant”) scattering processes
The Fokker-Planck Equation

- heavy quarks ↔ light quarks/gluons: momentum transfers small
- \( w(\vec{p} + \vec{k}, \vec{k}) \): peaked around \( \vec{k} = 0 \)
- expansion of collision term around \( \vec{k} = 0 \)

\[
\begin{align*}
  w(\vec{p} + \vec{k}, \vec{k}) f_Q(\vec{p} + \vec{k}) & \simeq w(\vec{p}, \vec{k}) f_Q(\vec{p}) + \vec{k} \cdot \frac{\partial}{\partial \vec{p}} [w(\vec{p}, \vec{k}) f_Q(\vec{p})] \\
  & \quad + \frac{1}{2} k_i k_j \frac{\partial^2}{\partial p_i \partial p_k} [w(\vec{p}, \vec{k}) f_Q(\vec{p})]
\end{align*}
\]

- collision term

\[
C[f_Q] = \int d^3 \vec{k} \left[ k_i \frac{\partial}{\partial p_i} + \frac{1}{2} k_i k_j \frac{\partial^2}{\partial p_i \partial p_j} \right] [w(\vec{p}, \vec{k}) f_Q(\vec{p})].
\]
Boltzmann equation $\Rightarrow$ simplifies to Fokker-Planck equation

$$\partial_t f_Q(t, \vec{x}, \vec{p}) + \frac{\vec{p}}{E_{\vec{p}}} \cdot \frac{\partial}{\partial \vec{x}} f_Q(t, \vec{x}, \vec{p}) = \frac{\partial}{\partial p_i} \left\{ A_i(\vec{p}) f_Q(t, \vec{x}, \vec{p}) \right. \\
+ \frac{\partial}{\partial p_j} \left[ B_{ij}(\vec{p}) f_Q(t, \vec{p}) \right] \right\}$$

with drag and diffusion coefficients

$$A_i(\vec{p}) = \int d^3 \vec{k} \, k_i w(\vec{p}, \vec{k}), \quad B_{ij}(\vec{p}) = \frac{1}{2} \int d^3 \vec{k} \, k_i k_j w(\vec{p}, \vec{k})$$

equilibrated light quarks and gluons: coefficients in heat-bath frame

matter homogeneous and isotropic

$$A_i(\vec{p}) = A(p) p_i, \quad B_{ij}(\vec{p}) = B_0(p) P_{ij}^\perp + B_1(p) P_{ij}^\parallel$$

with $P_{ij}^\parallel(\vec{p}) = \frac{p_i p_j}{\vec{p}^2}$, $P_{ij}^\perp(\vec{p}) = \delta_{ij} - \frac{p_i p_j}{\vec{p}^2}$
Meaning of the Coefficients

- Simplified equation for momentum distribution, \( F_Q(t, \vec{p}) \)
- Integrate Fokker-Planck equation over whole spatial volume:

\[
F_Q(t, \vec{p}) = \int_V d^3\vec{x} f_Q(t, \vec{x}, \vec{p}),
\]

\[
\int_V d^3\vec{x} \text{ div } \vec{x} \left[ \frac{\vec{p}}{E_{\vec{p}}} f(t, \vec{x}, \vec{p}) \right] = \int_{\partial V} d\vec{S} \cdot \left[ \frac{\vec{p}}{E_{\vec{p}}} f(t, \vec{x}, \vec{p}) \right] = 0 \Rightarrow
\]

\[
\frac{\partial}{\partial t} F_Q(t, \vec{p}) = \frac{\partial}{\partial p_i} \left\{ A_i(\vec{p}) F_Q(t, \vec{p}) + \frac{\partial}{\partial p_j} [B_{ij}(\vec{p}) F_Q(t, \vec{p})] \right\}
\]

- most simple case in non-relativistic limit \( A(\vec{p}) = A = \text{const}, \)
  \( B_0(\vec{p}) = B_1(\vec{p}) = B = \text{const} \)

\[
F_Q(t, \vec{p}) = \left\{ \frac{A}{2\pi D} \left[1 - \exp(-2\gamma t)\right] \right\}^{-3/2}
\times \exp \left[ -\frac{A}{2B} \frac{[\vec{p} - \vec{p}_0 \exp(-At)]^2}{1 - \exp(-2\gamma t)} \right]
\]
Meaning of the Coefficients

- **solution**: Gaussian with 
\[
\langle \vec{p}(t) \rangle = \vec{p}_0 \exp(-At), \quad \Delta \vec{p}^2(t) = \langle \vec{p}^2 \rangle - \langle \vec{p} \rangle^2 = \frac{3B}{A} [1 - \exp(-2At)].
\]

- **$A$**: friction/drag coefficient \(\Rightarrow\) dissipation
- **$1/A$**: relaxation time to reach equilibrium
- **$B$**: momentum-diffusion coefficient
- measures size of momentum fluctuations
  (result of random uncorrelated collisions of heavy quarks with medim)
- \(\Rightarrow\) effective description of collisions: white-noise-random force

- **equilibrium limit** \((t \rightarrow \infty)\)
\[
F_Q(t, \vec{p}) \overset{\text{t \rightarrow \infty}}{\approx} \left(\frac{2\pi B}{A}\right)^{3/2} \exp\left(-\frac{A\vec{p}^2}{2B}\right)
\]

- has to be **Maxwell-Boltzmann distribution** \(\Rightarrow\)
\[
B = m_Q AT
\]

- **$T$**: given temperature of the QGP
- Einstein’s **dissipation-fluctuation** relation (1905)
Realization as Langevin process

- **Langevin process**: friction force + Gaussian random force
- in the (local) rest frame of the heat bath

\[ d\vec{x} = \frac{\vec{p}}{E_p} dt, \]
\[ d\vec{p} = -A \vec{p} dt + \hat{C} \vec{w} \sqrt{dt} \]

- \( \vec{w}(t) \): Gaussian-distributed random variable

\[ \langle \vec{w}(t) \rangle = 0, \quad \langle w_j(t) w_k(t') \rangle = \delta(t - t') \]

- \( \hat{C} = \hat{C}^t \): covariance matrix of random force
- stochastic process depends on choice of momentum argument of \( \hat{C} \)

\[ \hat{C} \rightarrow \hat{C}(t, \vec{x}, \vec{p} + \xi d\vec{p}), \quad \xi \in [0, 1] \]

- usual values of \( \xi \)
  - \( \xi = 0 \): pre-point Ito realization
  - \( \xi = 1/2 \): Stratonovich realization
  - \( \xi = 1 \): post-point Ito (Hänggi-Klimontovich) realization
Langevin ↔ Fokker-Planck

- **heavy-quark phase-space distribution**

\[
f_Q(t, \vec{x}, \vec{p}) = \left\langle \delta^{(3)}[\vec{x} - \vec{x}'(t)]\delta^{(3)}[\vec{p} - \vec{p}'(t)] \right\rangle
\]  \hspace{1cm} (1)

- \([\vec{x}'(t), \vec{p}'(t)]:\) trajectories according to stochastic Langevin process

\[
d\vec{x} = \frac{\vec{p}}{E_p} \, dt, \quad d\vec{p} = -A \vec{p} \, dt + \hat{C} \hat{w} \sqrt{dt}
\]  \hspace{1cm} (2)

- perform timestep of Eq. (1) using (2)

\[
\frac{\partial f_Q}{\partial t} + \frac{p_j}{E} \frac{\partial f_Q}{\partial x_j} = \frac{\partial}{\partial p_j} \left[ \left( Ap_j - \xi C_{lk} \frac{\partial C_{jk}}{\partial p_l} \right) f_Q \right] + \frac{1}{2} \frac{\partial^2}{\partial p_j \partial p_k} \left( C_{jl} C_{kl} f_Q \right)
\]

\[
\Rightarrow \quad C_{jk} = \sqrt{2B_0} P_{jk}^\perp + \sqrt{2B_1} P_{jk}^\parallel
\]

- Form of Fokker-Planck equation ok, **but how to chose \(\xi\)?**
Langevin ↔ Fokker-Planck

- Choice of $\xi$: $f_Q \rightarrow$ Maxwell-Boltzmann distribution for $t \rightarrow \infty$:
  \[
  f_Q^{eq}(p) \propto \exp(-\sqrt{p^2 + m^2_Q/T})
  \]

- Langevin process with $B_0 = B_1 = D(E) \Rightarrow C_{jk} = \sqrt{2D(E)}\delta_{jk}$
- MB distribution solution of stationary FP equation $\Rightarrow$
  \[
  A(E)ET - D(E) + (1 - \xi)TD'(E) = 0
  \]
- simples choice: $\xi = 1$ (post-point Ito realization)
- then simple Einstein dissipation-fluctuation relation
  \[
  D = TEA
  \]
- for models for FP coefficients: relation not well satisfied for $B_1$
- $\Rightarrow$ use $\xi = 1$ and $B_1 = TEA$
- numerical check: Langevin simulation has right equilibrium limit
Langevin simulation for heavy-ion collisions

- need to simulate heavy-quark diffusion in sQGP
- “bulk” (light quarks + gluons) described by thermal fireball model
- flowing medium in local thermal equilibrium
- FP coefficients and Langevin process in restframe of the heat bath
- way out: boost to local heat-bath frame with flow velocity $v(t, \vec{x})$
- do time step to “update” momenta
- boost back to “lab frame”
- defines HQ distribution as “freezeout at constant lab time”
- NB: leads to covariant equilibrium distribution

$$dN_Q = \frac{\gamma_Q}{(2\pi)^3} d^3 \vec{x}^{(h_b)} \frac{d^3 \vec{p}}{p_0} p \cdot u(x) \exp \left( -\frac{p \cdot u(x)}{T(x)} \right)$$

- $u(t, \vec{x}) = [1, \vec{v}(t, \vec{x})]/\sqrt{1 - \vec{v}^2(t, \vec{x})}$: velocity-flow field (4-vector)
- $T(x)$: temperature field (4-scalar)
Fire-ball model

- Elliptic fire-ball parameterization fitted to hydrodynamical flow pattern [Kolb ’00]
  \[ V(t) = \pi(z_0 + v_z t)a(t)b(t), \quad a, b: \text{semi-axes of ellipse,} \]
  \[ v_{a,b} = v_\infty[1 - \exp(-\alpha t)] \mp \Delta v[1 - \exp(-\beta t)] \]
  \(a, b\): semi-axes of ellipse

- Isentropic expansion: \( S = \text{const} \) (fixed from \(N_{\text{ch}}\))

- QGP Equation of state:
  \[ s = \frac{S}{V(t)} = \frac{4\pi^2}{90} T^3(16 + 10.5n_f^*), \quad n_f^* = 2.5 \]

- obtain \(T(t) \Rightarrow A(t, p), B_0(t, p)\) and \(B_1 = \text{TEA}\)

- for semicentral collisions \((b = 7 \text{ fm})\): \(T_0 = 340 \text{ MeV},\) QGP lifetime \(\simeq 5 \text{ fm/c}.\)

- simulate FP equation as relativistic Langevin process
Initial conditions

- need initial $p_T$-spectra of charm and bottom quarks
- (modified) PYTHIA to describe exp. D meson spectra, assuming $\delta$-function fragmentation
- exp. non-photonic single-$e^\pm$ spectra: Fix bottom/charm ratio
Elastic pQCD processes

- Lowest-order matrix elements [Cambridge 79]

- Debye-screening mass for $t$-channel gluon exch. $\mu_g = gT$, $\alpha_s = 0.4$

- not sufficient to understand RHIC data on “non-photonic” electrons

[Moore, Teaney PRC 71, volume 71, 064904 (2005)]
Non-perturbative interactions: Resonance Scattering

- General idea: Survival of $D$- and $B$-meson like resonances above $T_c$
- Model based on chiral symmetry (light quarks) HQ-effective theory
- Elastic heavy-light-(anti-)quark scattering

$D$, $D'$, $D_s$- and $B$-meson like resonances in sQGP

- Parameters
  - $m_D = 2$ GeV, $\Gamma_D = 0.4 \ldots 0.75$ GeV
  - $m_B = 5$ GeV, $\Gamma_B = 0.4 \ldots 0.75$ GeV
• total pQCD and resonance cross sections: comparable in size
• BUT pQCD forward peaked ↔ resonance isotropic
• resonance scattering more effective for friction and diffusion
Transport coefficients: pQCD vs. resonance scattering

- three-momentum dependence

![Graphs showing three-momentum dependence with γ and D as functions of p for different values of Γ and α_s for T=200 MeV.]

- resonance contributions factor \(\sim 2 \ldots 3\) higher than pQCD!
Transport coefficients: pQCD vs. resonance scattering

- Temperature dependence

![Graph showing temperature dependence of transport coefficients with pQCD and resonance scattering](image-url)
Spectra and elliptic flow for heavy quarks

\[ \mu_D = gT, \quad \alpha_s = g^2/(4\pi) = 0.4 \]

- **resonances** \( \Rightarrow c\)-quark thermalization without upscaling of cross sections
- **Fireball parametrization** consistent with hydro

\[ 2\pi T D \approx \frac{3}{2\alpha_s^2} \]
Spectra and elliptic flow for heavy quarks

Au-Au $\sqrt{s}=200$ GeV ($b=7$ fm)

**Diagram:***

- Red line: $c$, reso ($\Gamma=0.4-0.75$ GeV)
- Blue line: $c$, pQCD, $\alpha_s=0.4$
- Green line: $b$, reso ($\Gamma=0.4-0.75$ GeV)

**Legend:**

- **LO QCD** [Moore, Teaney '04]
Observables: $p_T$-spectra ($R_{AA}$), $v_2$

- Hadronization: **Coalescence** with light quarks + fragmentation
  $\Leftrightarrow \bar{c}c, \bar{b}b$ conserved

- single electrons from decay of $D$- and $B$-mesons

Without further adjustments: data quite well described

[HvH, V. Greco, R. Rapp, Phys. Rev. C 73, 034913 (2006)]
Observables: $p_T$-spectra ($R_{AA}$), $v_2$

- Hadronization: Fragmentation only
- single electrons from decay of $D$- and $B$-mesons

coalescence brings up both, $R_{AA}$ and $v_2$
- due to additional momentum kick from light quarks
Observables: $p_T$-spectra ($R_{AA}$), $v_2$

- Central Collisions
- single electrons from decay of $D$- and $B$-mesons

Coalescence+Fragmentation

Fragmentation only

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Comparison to newer data

(a) 0−10% central

- Armesto et al. (I)
- van Hees et al. (II)
- 3/(2πT) Moore &
- 12/(2πT) Teaney (III)

Au+Au @ \( s_{NN} = 200 \) GeV

(b) minimum bias

- \( \pi^0 R_{AA}, p_T > 4 \) GeV/c
- \( \pi^0 v_2, p_T > 2 \) GeV/c
- \( e^\pm R_{AA}, e^\pm v^{HF}_2 \)

PHENIX Collaboration
PRL 98 172301 (2007)
Transport properties of the sQGP

- spatial diffusion coefficient: Fokker-Planck \( \Rightarrow D_s = \frac{T}{m_A} = \frac{T^2}{D} \)
- coupling strength in plasma: viscosity/entropy density, \( \eta/s \)

\[
\frac{\eta}{s} \simeq \frac{1}{2} TD_s \quad \text{(AdS/CFT)}, \quad \frac{\eta}{s} \simeq \frac{1}{5} TD_s \quad \text{(wQGP)}
\]
Boltzmann Transport Equations

can be derived from classical mechanics or quantum-many-body theory
(semi-)classical statistical description of interacting many-body systems
equations for single-particle phase-space distribution
collision term: transition probabilities from microscopic cross sections
many-body systems ⇔ microscopic properties of constituents

Fokker-Planck Equations

heavy particles immersed in medium of light particles
momentum transfer in single collision small ⇒
integro-differential Boltzmann equation ⇒ partial differential equation
HQ-medium interactions ⇒ friction/drag coefficient + diffusion coefficients
related by Einstein dissipation-fluctuation relation
Langevin Equations

- stochastic differential equation equivalent to Fokker-Planck equation
- drag/friction force + random forces = uncorrelated Gaussian noise
- depends on realization of stochastic process
- right process $\Rightarrow$ equilibrium limit = relativistic MB distribution
- application to flowing sQGP

Heavy-quark interactions in the sQGP

- elastic scattering with light quarks and gluons: pQCD + screening
- resonance scattering with light (anti-)quarks

Non-photonic single electron observables

- $R_{AA}(p_T)$ and $v_2(p_T)$ of electrons from $D$- and $B$-meson decays
- Langevin simulation $\rightarrow$ coalescence+fragmentation hadronization $\rightarrow$ semi-leptonic decay
- pQCD (with realistic $\alpha_s$) too weak
- with resonance-scattering interactions good description of data
Microscopic model: Static potentials from lattice QCD

- color-singlet free energy from lattice
- use internal energy

\[ U_1(r, T) = F_1(r, T) - T \frac{\partial F_1(r, T)}{\partial T}, \]
\[ V_1(r, T) = U_1(r, T) - U_1(r \to \infty, T) \]

- Casimir scaling for other color channels [Nakamura et al 05; Döring et al 07]

\[ V_3 = \frac{1}{2} V_1, \quad V_6 = -\frac{1}{4} V_1, \quad V_8 = -\frac{1}{8} V_1 \]

[NvH, M. Mannarelli, V. Greco, R. Rapp, PRL 100, 192301 (2008); HvH, M. Mannarelli, R. Rapp, EJC 61, 799 (2009)]
Brueckner many-body approach for elastic $Qq$, $Q\bar{q}$ scattering

\[ T = V + V T \]

\[ \Sigma = \Sigma_{\text{glu}} + T \]

- reduction scheme: 4D Bethe-Salpeter $\rightarrow$ 3D Lippmann-Schwinger
- $S$- and $P$ waves
- same scheme for light quarks (self consistent!)
- Relation to invariant matrix elements

\[ \sum \mid \mathcal{M}(s) \mid^2 \propto \sum_q d_a \left( \mid T_{a,l=0}(s) \mid^2 + 3 \mid T_{a,l=1}(s) \mid^2 \cos \theta_{\text{cm}} \right) \]

[HvH, M. Mannarelli, V. Greco, R. Rapp, PRL 100, 192301 (2008); HvH, M. Mannarelli, R. Rapp, EJC 61, 799 (2009)]
- resonance formation at lower temperatures $T \simeq T_c$
- melting of resonances at higher $T$! $\Rightarrow$ sQGP
- $P$ wave smaller
- resonances near $T_c$: natural connection to quark coalescence

[Ravagli, Rapp 07; Ravagli, HvH, Rapp 08]

- model-independent assessment of elastic $Qq$, $Q\bar{q}$ scattering
- problems: uncertainties in extracting potential from lQCD
- in-medium potential $U$ vs. $F$?
Transport coefficients

- From non-pert. interactions reach $A_{\text{non-pert}} \approx 1/(7 \text{ fm/c}) \approx 4A_{\text{pQCD}}$
- $A$ decreases with higher temperature
- Higher density (over)compensated by melting of resonances!
- Spatial diffusion coefficient

$$D_s = \frac{T}{mA}$$

Increases with temperature
Non-photonic electrons at RHIC

- same model for bottom
- quark coalescence + fragmentation $\rightarrow D/B \rightarrow e + X$

- coalescence crucial for description of data
- increases both, $R_{AA}$ and $v_2$ $\leftrightarrow$ “momentum kick” from light quarks!
- “resonance formation” towards $T_c$ $\Rightarrow$ coalescence natural [Ravagli, Rapp 07]