T-Matrix Approach to Heavy-Quark Diffusion and Quarkonia

Hendrik van Hees

Justus-Liebig Universität Gießen

January 4, 2011
Outline

1. Heavy-quark interactions in the sQGP
   - Heavy quarks in heavy-ion collisions
   - Heavy-quark diffusion: The Langevin Equation
   - Elastic pQCD heavy-quark scattering

2. Microscopic model for non-perturbative HQ interactions
   - Static heavy-quark potentials from lattice QCD
   - T-matrix approach

3. Non-photonic electrons at RHIC

4. T-matrix approach to Quarkonium-Bound-State Problem

5. Summary and Outlook
**Hard production of HQs**
described by PDF’s + pQCD (PYTHIA)

**c, b quark**

HQ rescattering in QGP: Langevin simulation
drag and diffusion coefficients from
microscopic model for HQ interactions in the sQGP

**Hadronization to D, B mesons via**
quark coalescence + fragmentation

V. Greco, C. M. Ko, R. Rapp, PLB 595, 202 (2004)

**Semileptonic decay** ⇒

“non-photonic” electron observables

\[ R_{AA}^{e^+e^-}(p_T), \nu_e^{e^+e^-}(p_T) \]
Relativistic Langevin process

- **Langevin process**: friction force + Gaussian random force
- in the (local) rest frame of the heat bath

\[ \frac{d\vec{x}}{dt} = \frac{\vec{p}}{E_p} dt, \]
\[ \frac{d\vec{p}}{dt} = -A \vec{p} dt + \sqrt{2} dt \left[ \sqrt{B_0 P_{\perp}} + \sqrt{B_1 P_{\parallel}} \right] \vec{w} \]

- \( \vec{w} \): normal-distributed random variable
- \( A \): friction (drag) coefficient
- \( B_{0,1} \): diffusion coefficients
- dependent on realization of stochastic process
Local-Equilibrium Limit

- to guarantee correct (local) equilibrium limit:
  - use Hänggi-Klimontovich calculus, i.e., use $B_{0/1}(t, \vec{p} + d\vec{p})$
  - Einstein dissipation-fluctuation relation $B_0 = B_1 = E_p T A$

- to implement flow of the medium
  - use Lorentz boost to change into local “heat-bath frame”
  - use update rule in heat-bath frame
  - boost back into “lab frame”

- Realizes Milekhin-freeze-out distribution for $t \to \infty$

\[
\frac{dN}{d^3x d^3p} = \frac{g}{(2\pi)^3} \frac{p \cdot u(x)}{E} \exp \left[ -\frac{p \cdot u(x)}{T(x)} \right].
\]

- $E = p^0 = \sqrt{m^2 + p^2}$
- $u(x)$: four-velocity-flow field of the medium
- $T(x)$: temperature field of the medium
- thermal fireball adjusted such as to lead to correct light-meson and baryon observables
- using the same coalescence/fragmentation model for hadronization
- bulk $v_2$ sensitive to freeze-out description (see next talk by P. Gossiaux)
Elastic pQCD processes

- Lowest-order matrix elements [Combridge 79]

- Debye-screening mass for $t$-channel gluon exch. $\mu_g = gT$, $\alpha_s = 0.4$

- not sufficient to understand RHIC data on “non-photonic” electrons
Microscopic model: Static potentials from lattice QCD

- color-singlet free energy from lattice
- use internal energy

\[
U_1(r, T) = F_1(r, T) - T \frac{\partial F_1(r, T)}{\partial T},
\]

\[
V_1(r, T) = U_1(r, T) - U_1(r \to \infty, T)
\]

- Casimir scaling for other color channels [Nakamura et al. 05; Döring et al. 07]

\[
V_3 = \frac{1}{2} V_1, \quad V_6 = -\frac{1}{4} V_1, \quad V_8 = -\frac{1}{8} V_1
\]
Brueckner many-body approach for elastic $Qq$, $Q\bar{q}$ scattering

\[ T = V + T \]

\[ V = \Sigma_{\text{glu}} + T \Sigma \]

reduction scheme: 4D Bethe-Salpeter $\rightarrow$ 3D Lippmann-Schwinger

$S$- and $P$ waves

same scheme for light quarks (self consistent!)

Relation to invariant matrix elements

\[ \sum |M(s)|^2 \propto \sum_q d_a \left( |T_{a,l=0}(s)|^2 + 3|T_{a,l=1}(s)|^2 \cos \theta_{\text{cm}} \right) \]
Resonance formation: T-matrix calculation

- use static heavy-quark potentials from lQCD
- resonance formation at lower temperatures $T \approx T_c$
- melting of resonances at higher $T! \Rightarrow sQGP$
- model-independent assessment of elastic $Qq, Q\bar{q}$ scattering
- problems: uncertainties in extracting potential from lQCD in-medium potential $V$ vs. $F$?
Transport coefficients

- from non-pert. interactions reach $A_{\text{non-pert}} \approx 1/(7 \text{ fm}/c) \approx 4A_{\text{pQCD}}$
- $A$ decreases with higher temperature
- higher density (over)compensated by melting of resonances!
- spatial diffusion coefficient

\[ D_s = \frac{T}{mA} \]

increases with temperature
Time evolution of the fire ball

- **Elliptic fire-ball** parameterization fitted to hydrodynamical flow pattern [Kolb ’00]

\[ V(t) = \pi(z_0 + v_z t)a(t)b(t), \quad a, b: \text{semi-axes of ellipse}, \]

\[ v_{a,b} = v_\infty[1 - \exp(-\alpha t)] \pm \Delta v[1 - \exp(-\beta t)] \]

- **Isentropic expansion**: \( S = \text{const} \) (fixed from \( N_{\text{ch}} \))

- **QGP Equation of state**:

\[ s = \frac{S}{V(t)} = \frac{4\pi^2}{90} T^3(16 + 10.5n_f^*) , \quad n_f^* = 2.5 \]

- obtain \( T(t) \Rightarrow A(t,p), B_0(t,p) \) and \( B_1 = TEA \)

- for semicentral collisions \( (b = 7 \text{ fm}) \): \( T_0 = 340 \text{ MeV} \), QGP lifetime \( \simeq 5 \text{ fm}/c \).

- simulate FP equation as **relativistic Langevin process**
Initial conditions

- need initial $p_T$-spectra of charm and bottom quarks
- (modified) PYTHIA to describe exp. D meson spectra, assuming $\delta$-function fragmentation
- exp. non-photonic single-$e^\pm$ spectra: Fix bottom/charm ratio

![Graph showing $1/(2\pi p_T) dN/dp_T$ vs. $p_T$ for $d+Au$ at $\sqrt{s_{NN}}=200$ GeV and $e^\pm$ production in STAR (prelim) and STAR (pp).]
Non-photonic electrons at RHIC

- same model for bottom
- quark \textit{coalescence} + \textit{fragmentation} $\rightarrow D/B \rightarrow e + X$

- coalescence crucial for description of data
- increases both, $R_{AA}$ and $v_2 \Leftrightarrow \text{“momentum kick” from light quarks!}$
- “resonance formation” towards $T_c \Rightarrow \text{coalescence natural}$ [Ravagli, Rapp 07]
Transport properties of the sQGP

- spatial diffusion coefficient: \( \text{Fokker-Planck} \Rightarrow D_s = \frac{T}{m_A} = \frac{T^2}{D} \)
- measure for coupling strength in plasma: \( \frac{\eta}{s} \)

\[
\frac{\eta}{s} \approx \frac{1}{2} T D_s \quad (\text{AdS/CFT}), \quad \frac{\eta}{s} \approx \frac{1}{5} T D_s \quad (w\text{QGP})
\]

[Lacey, Taranenko (2006)]
T-matrix approach for quarkonium-bound-state problem

- T-matrix Brückner approach for heavy quarkonia as for HQ diffusion
- consistency between HQ diffusion and $\bar{Q}Q$ suppression!

\[ T_c = V + V T \]

\[ T = \sum \text{glu} + \sum \bar{Q} T q k q' \]

4D Bethe-Salpeter equation $\rightarrow$ 3D Lippmann-Schwinger equation

relativistic interaction $\rightarrow$ static heavy-quark potential (lQCD)

\[ T_\alpha(E; q', q) = V_\alpha(q', q) + \frac{2}{\pi} \int_0^\infty dk \frac{k^2}{k} V_\alpha(q', k) G_{Q\bar{Q}}(E; k) T_\alpha(E; k, q) \times \{1 - n_F[\omega_1(k)] - n_F[\omega_2(k)]\} \]

- $q$, $q'$, $k$ relative 3-momentum of initial, final, intermediate $\bar{Q}Q$ state

[F. Riek, R. Rapp, PRC 82, 035201 (2010)]
The potential

- **non-perturbative static gluon propagator**
  \[
  D_{00}(\vec{k}) = \frac{1}{(\vec{k}^2 + \mu_D^2)} + \frac{m_G^2}{(\vec{k}^2 + \tilde{m}_D^2)^2}
  \]

- **finite-T HQ color-singlet-free energy** from Polyakov loops
  \[
  \exp[-F_1(r, T)/T] = \left\langle \text{Tr}[\Omega(x)\Omega^\dagger(y)]/N_c \right\rangle
  \]
  \[
  = \exp \left[ \frac{g^2}{2N_cT^2} \left\langle A_{0,\alpha}(x)A_{0,\alpha}(y) - A_{0,\alpha}^2(x) \right\rangle \right] + O(g^6)
  \]

- identify \( \left\langle A_{0,\alpha}(x)A_{0,\alpha}(y) \right\rangle = D_{00}(x - y) \)

- **color-singlet free energy**
  \[
  F_1(r, T) = -\frac{4}{3}\alpha_s \left\{ \frac{\exp(-m_D r)}{r} + \frac{m_G^2}{2\tilde{m}_D} \left[ \exp(-\tilde{m}_D r) - 1 \right] + m_D \right\}
  \]

- **in vacuo** \( m_D, \tilde{m}_D \rightarrow 0 \)

\[
F_1(r) = -\frac{4}{3}\alpha_s \frac{\alpha_s m_G^2}{r} + \sigma r, \quad \sigma = \frac{2\alpha_s m_G^2}{3}
\]

Heavy quarkonia

- fit parameters, $\alpha_s(T)$, $m_D(T)$, $\tilde{m}_D(T)$, $\tilde{m}_G(T)$ to lQCD
- calculate internal energy $U(r, T) = F(r, T) - T \frac{\partial}{\partial T} F(r, T)$
- solve Lippmann-Schwinger equation $\Rightarrow$ adjust $m_Q$ to get $s$-wave charmonia/bottomonia masses in vacuum

in the following

- **potential 1**: $N_f = 2 + 1$ [O. Kaczmarek]
- **potential 2**: $N_f = 3$ [P. Petreczky]
- **BbS**: Blancenblecler-Sugar reduction scheme
- **Th**: Thompson reduction scheme

- vacuum-mass splittings
  - uncertainty for charmonia 50-100 MeV
  - uncertainty for bottomonia 30-70 MeV
  - overall uncertainty $\simeq 10\%$

- melting temperatures with $U$ and $F$
  - $s$-wave ($\eta_c$, $J/\psi$): $2\text{-}2.5T_c$, $\gtrsim 1.3T_c$
  - $\Upsilon$: $> 2T_c$, $\gtrsim 1.7T_c, 1T_c$, $\gtrsim 2T_c$, $1T_c, 1T_c$
  - $p$-wave ($\chi_c$): $\gtrsim 1.2T_c, 1T_c$, $\chi_b$: $\gtrsim 1.7T_c, 1.2T_c$, all $\gtrsim 1T_c$

Quarkonium-spectral functions in the vacuum
In-medium charmonium-spectral functions (s states)

Potential 1
BbS-Scheme

Potential 2
BbS-Scheme

Potential 1
Th-Scheme

Potential 2
Th-Scheme

\( \frac{\sigma}{[\text{GeV}^2]} \)

\( E_{\text{cm}} [\text{GeV}] \)

\( \tau [\text{fm}] \)

\( T = 0 \ T_C \)  \( T = 1.2 \ T_C \)  \( T = 1.35 \ T_C \)

\( T = 1.5 \ T_C \)  \( T = 1.75 \ T_C \)  \( T = 2 \ T_C \)
In-medium charmonium-spectral functions (p states)
Summary and Outlook

Summary
- **Heavy quarks in the sQGP**
- **non-perturbative interactions**
  - mechanism for **strong coupling**: resonance formation at $T \gtrsim T_c$
  - IQCD potentials parameter free
  - res. melt at higher temperatures $\Leftrightarrow$ consistency betw. $R_{AA}$ and $v_2$
  - same model also used for quarkonia in medium
- also provides “natural” mechanism for quark coalescence
- problems
  - potential approach at finite $T$: $F$, $V$ or combination?

Outlook
- use more realistic **bulk-medium description** ($\rightarrow$ following talks by P. Gossiaux and M. He)
- include **inelastic heavy-quark processes** (gluo-radiative processes)
- take into account D/B-meson rescattering in the hadronic phase
- other **heavy-quark observables** like charmonium suppression/regeneration ($\rightarrow$ talk by X. Zhao on Thursday)