Thermalization of Heavy Quarks in the QGP

Hendrik van Hees

Texas A&M University

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Collaborators: V. Greco, R. Rapp
Outline

Motivation

Chiral Heavy-Quark Model

The Fokker-Planck Equation

Friction and Diffusion Coefficients

Relativistic Langevin Process
Motivation

- $p_T$ spectra and $v_2$ of D mesons
- Single-electron $v_2$ measurements from PHENIX, STAR ’04
- Coalescence model describes data under assumption of flowing thermalized $c$ quarks
Motivation

▶ importance of dissociation and regeneration in

\[ c + \bar{c} \leftrightarrow J/\psi + X \]

▶ in-medium spectral properties of charmonia in QGP (Grandchamp, Rapp ’02)

▶ importance of thermalization of heavy quarks
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\[ c + \bar{c} \leftrightarrow J/\psi + X \]

- in-medium spectral properties of charmonia in QGP (Grandchamp, Rapp ’02)
- importance of thermalization of heavy quarks
- Possible mechanism: Survival of “D-mesonic resonances” above \( T_c \)
- suggestive from lattice QCD (Umeda et al ’02, Datta et al ’03)
Free Lagrangian: Particle Content

Chiral symmetry $\text{SU}_V(2) \otimes \text{SU}_A(2)$ in light-quark sector of QCD

$$\mathcal{L}_D^{(0)} = \sum_{i=1}^{2} \left[ (\partial_\mu \Phi_i^\dagger)(\partial^\mu \Phi_i) \right. - \left. m_D^2 \Phi_i^\dagger \Phi_i \right] + \text{massive (pseudo-)vectors } D^*$$

$\Phi_i$: two doublets: pseudo-scalar $\sim \left( \begin{array}{c} D^0 \\ D^- \end{array} \right)$ and scalar

$\Phi_i^*$: two doublets: vector $\sim \left( \begin{array}{c} D^{0*} \\ D^{-*} \end{array} \right)$ and pseudo-vector

$$\mathcal{L}_{qc}^{(0)} = \bar{q} i \gamma^\mu q + \bar{c} (i \gamma^\mu - m_c) c$$

$q$: light-quark doublet $\sim \left( \begin{array}{c} u \\ d \end{array} \right)$

c: singlet
Chiral Symmetry

Infinitesimal version:

\[ q \rightarrow (1 + i\delta \vec{\phi}_V \vec{t} + i\delta \vec{\phi}_A \tau_5)q, \quad c \rightarrow c. \]

Light quarks massless in chiral limit!

\[ \Phi_1 \rightarrow \Phi_1 + i\delta \vec{\phi}_V \vec{t}\Phi_1 + i\delta \vec{\phi}_A \tau_5 \Phi_2, \]
\[ \Phi_2 \rightarrow \Phi_2 + i\delta \vec{\phi}_V \vec{t}\Phi_2 + i\delta \vec{\phi}_A \tau_5 \Phi_1. \]

Mesons must have chiral partners

In the vacuum: chiral symmetry spontaneously broken
In QGP: chiral symmetry restored
Interactions

Interactions determined by chiral symmetry
Strong interactions also preserve parity
For transversality of vector mesons: use heavy-quark effective theory vertices

\[ L_{\text{int}} = - G_S \left( \frac{1 + \gamma}{2} \Phi_1 c_v + \frac{1 + \gamma}{2} i \gamma^5 \Phi_2 c_v + h.c. \right) \]
\[ \quad \quad - G_V \left( \frac{1 + \gamma}{2} \gamma^\mu \Phi_{1\mu} c_v + \frac{1 + \gamma}{2} i \gamma^\mu \gamma^5 \Phi_{2\mu} c_v + h.c. \right) \]

\( v \): four momentum of heavy quark in HQET: spin symmetry
\[ \Rightarrow G_S = G_V \]
Dressing the $D$ Mesons

Dressing the $D$ mesons with self-energies

\[ \rho_k^D(k^2 = 0) = 0, \quad \Pi^D(k^2 = 0) = 0 \]

or dipole form-factor cutoff:

\[ F_{\text{dip}} = \frac{2\Lambda^2}{k^2 + \frac{m^2}{c^2}} \]

Bare mass and coupling adjusted such that $m_D = 2$ GeV, $\Gamma_D = (0.3 \ldots 0.8)$ GeV (from in-medium Bethe-Salpeter calculations)
Dressing the $D$ Mesons

Dressing the $D$ mesons with self-energies

\[ D, D', D_s \]
\[ k \]
\[ q \]
\[ c \]
\[ D, D', D_s \]
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Divergencies: wave-function + mass renormalization:

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Dressing the $D$ mesons with self-energies

\[ D, D', D_s \xrightarrow{\text{q}} D, D', D_s \]

\[ k \xrightarrow{\text{c}} k \]

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\[ F_{\text{dip}} = \left( \frac{2\Lambda^2}{2\Lambda^2 + k_{\text{cm}}^2} \right)^2, \quad k_{\text{cm}} = \frac{s - m_c^2}{2\sqrt{s}} \]
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Resonance Scattering

heavy-light-(anti-)quark scattering

\[
\begin{align*}
\bar{q} & \rightarrow c & D, D', D_s \\
D, D', D_s & \rightarrow \bar{q} & c \\
\end{align*}
\]
Contributions from pQCD

Lowest-order matrix elements (Combridge ’79)

In-medium Debye-screening mass for $t$-channel gluon exchange:

\[ \mu_g = g T, \quad \alpha_s = 0.3, 0.4, 0.5 \]
Cross sections

- total pQCD and resonance cross sections: comparable in size
- BUT pQCD forward peaked ↔ resonance isotropic
- resonance scattering more effective for friction and diffusion
The Fokker-Planck Equation

Heavy particle (c quarks) in a heat bath of light particles (QGP)

\[
\frac{\partial f(t, \vec{p})}{\partial t} = \frac{\partial}{\partial p_i} \left[ A_i(t, \vec{p}) + \frac{\partial}{\partial p_j} B_{ij}(t, \vec{p}) \right] f(t, \vec{p})
\]

Assumption: Relevant scattering processes are soft
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Assumption: Relevant scattering processes are soft
\(A_i\) and \(B_{ij}\) given by averages over initial momenta \(\vec{q}\) of light particles and summation over final states (Svetitsky '88):

\[
\langle X(\vec{p}') \rangle = \frac{1}{\gamma_c 2E_p} \int \frac{d^3\vec{q}}{(2\pi)^3 2E_q} \int \frac{d^3\vec{q}'}{(2\pi)^3 2E_q'} \int \frac{d^3\vec{p}'}{(2\pi)^3 2E_p'} \sum |\mathcal{M}|^2 (2\pi)^4 \delta^{(4)}(\vec{p} + \vec{q} - \vec{p}' - \vec{q}') \hat{f}(\vec{q}) X(\vec{p}')
\]
Friction and Diffusion Coefficients

For $t$, $\vec{p}$-independent coefficients:

$$\frac{\partial f}{\partial t} = \gamma \frac{\partial}{\partial \vec{p}}(\vec{p} f) + D \frac{\partial^2}{\partial \vec{p}^2} f$$
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Solution for $c$ quark with given momentum $\vec{p}_0$ at $t = 0$:

$$G(t, \vec{p} | \vec{p}_0) = \left\{ \frac{\gamma}{2\pi D [1 - \exp(-2\gamma t)]} \right\}^{3/2} \exp \left\{ -\frac{\gamma}{2D} \left[ \vec{p} - \vec{p}_0 \exp(-\gamma t) \right]^2 \right\}$$

- $\langle \vec{p} \rangle = \vec{p}_0 \exp(-\gamma t) \Rightarrow \gamma = \text{friction coefficient (dissipation)}$
- $\Delta \vec{p}^2 = 3D/\gamma [1 - \exp(-2\gamma t)] \Rightarrow D = \text{diffusion (fluctuation)}$
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- $\langle \vec{p} \rangle = \vec{p}_0 \exp(-\gamma t) \Rightarrow \gamma =$ friction coefficient (dissipation)
- $\Delta \vec{p}^2 = 3D / \gamma [1 - \exp(-2\gamma t)] \Rightarrow D =$ diffusion (fluctuation)
- $t \rightarrow \infty$: temperature $T^* = D / \gamma m_c$
- consistency condition $T^* \overset{!}{=} T$ (heat bath-temperature)
- fulfilled within $\sim 15\%$ for relevant temperature region
The Coefficients: pQCD vs. resonance scattering

- only weakly $p$-dependent
- resonance contributions factor $\sim 2\ldots3$ higher than pQCD!
The Coefficients: pQCD vs. resonance scattering

- temperature dependence $\Rightarrow$ need to treat Fokker-Planck equation with time-dependent coefficients
- Solvable with method of characteristics
Time evolution of the fire ball

- Simple fire-ball parameterization:

\[ V(\tau) = \pi (z_0 + v_z \tau)(r_0 + \frac{1}{2} a_\perp \tau^2)^2 \]
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- Adiabatic expansion: \( S = \text{const} = 10^4 \text{@RHIC} \)

- Equation of state:
  \[ s = \frac{4\pi^2}{90} T^3 (16 + 10.5 n_f^*), \quad n_f^* = 2.5 \]

- obtain \( \Rightarrow T(\tau) \)
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- Obtain \( \Rightarrow T(\tau) \)

- Initial condition from PYTHIA

\[ \frac{d^2N}{dp_T^2} := f(p_T, t = 0) \propto \frac{(p_T + A)^2}{(1 + p_T/B)^\alpha} \]
Evolved $p_T$ spectra

- initially $\sqrt{\langle p_T^2 \rangle} = 1.66$ GeV
- with pQCD: not much change in spectrum
- with resonance contributions: $p_T^{(\text{max})} \sim 0.66$ GeV
- nearly thermal: $T \sim 290$ MeV
Evolution of $p_T$ spectra
Relativistic Langevin Process

- aim: include (asymmetric) flow + $p$-dep. FP coefficients
- simulate FP equation as relativistic Langevin process
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- simulate FP equation as relativistic Langevin process
- friction $\hat{=} \text{deterministic drag force}$, diffusion $\hat{=} \text{stochastic force}$
- in (local) restframe of the heat bath

$$\delta \vec{x} = \frac{\vec{p}}{E} \delta t$$

$$\delta \vec{p} = -\gamma(t, \vec{p} + \delta \vec{p}) \vec{p} \delta t + \delta W(t, \vec{p} + \delta \vec{p})$$

$$P(\delta \vec{W}) \propto \exp \left[ - \frac{\delta \vec{W}^2}{4D(t, \vec{p} + \delta \vec{p}) \delta t E^2 / m^2} \right]$$
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- Hänggi-Klimontovich realization $\Leftrightarrow$ rel. Maxwell distribution as equilibrium limit with unchanged FP coefficients
Observables: $p_T$-spectra, $v_2$, $(R_{AA})$

- use elliptic flow parametrization for fireball, based on hydro [Kolb et al]
- boost “labframe” $\leftrightarrow$ local heat-bath-rest frame (see also [Moore, Teaney 2004])

![Graphs showing $dN/dp_T$ and $R_{AA}$ versus $p_T$]

- $dN/d^2p_T$ [GeV$^2$]
- $R_{AA}$
- PYTHIA initial
- pQCD (elastic)
- res. scattering
Observables: $p_T$-spectra, $v_2$, $(R_{AA})$
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- $D$-meson $v_2$ via coalescence
- $e^\pm$ from subsequent decay
Conclusions

- Assumption: survival of resonances in the QGP
- possible mechanism for strong interactions beyond $T_c$
- Equilibration of heavy quarks in QGP
- Observables via Langevin approach and coalescence