Heavy-Quark Kinetics
in the Quark-Gluon Plasma

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Heavy Quarks in Heavy-Ion collisions

- **Hard production of HQs** described by PDF's + pQCD (PYTHIA)
- **c, b quark**

- **HQ rescattering in QGP**: Langevin simulation
drag and diffusion coefficients from microscopic model for HQ interactions in the sQGP

- **Hadronization** to $D, B$ mesons via quark coalescence + fragmentation


- **Semileptonic decay** $\Rightarrow$
  - "non-photonic" electron observables
  - $R_{AA}^{e^+e^-}(p_T)$, $v_2^{e^+e^-}(p_T)$

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Relativistic Langevin process

- **Langevin process**: friction force + Gaussian random force
- in the (local) rest frame of the heat bath

\[
d\vec{x} = \frac{\vec{p}}{E_p} dt,
\]

\[
d\vec{p} = -A \vec{p} dt + \sqrt{2} dt \left[ \sqrt{B_0 P_\perp} + \sqrt{B_1 P_\parallel} \right] \vec{w}
\]

- \( \vec{w} \): normal-distributed random variable
- \( A \): friction (drag) coefficient
- \( B_{0,1} \): diffusion coefficients
- dependent on realization of stochastic process
- to guarantee correct equilibrium limit: Use Hänggi-Klimontovich calculus, i.e., use \( B_{0/1}(t, \vec{p} + d\vec{p}) \)
- Einstein dissipation-fluctuation relation \( B_0 = B_1 = E_p TA \).
- to implement flow of the medium
  - use Lorentz boost to change into local “heat-bath frame”
  - use update rule in heat-bath frame
  - boost back into “lab frame”
Elastic pQCD processes

- Lowest-order matrix elements [Combridge 79]

- Debye-screening mass for $t$-channel gluon exch. $\mu_g = gT$, $\alpha_s = 0.4$
- not sufficient to understand RHIC data on “non-photonic” electrons
Non-perturbative interactions: Resonance Scattering

- General idea: Survival of $D$- and $B$-meson like resonances above $T_c$
- elastic heavy-light-(anti-)quark scattering

![Diagram of resonance scattering]

- $D$- and $B$-meson like resonances in sQGP

- parameters
  - $m_D = 2$ GeV, $\Gamma_D = 0.4 \ldots 0.75$ GeV
  - $m_B = 5$ GeV, $\Gamma_B = 0.4 \ldots 0.75$ GeV
Cross sections

- total pQCD and resonance cross sections: comparable in size
- BUT pQCD forward peaked ↔ resonance isotropic
- resonance scattering more effective for friction and diffusion
Time evolution of the fire ball

- **Elliptic fire-ball parameterization**
  fitted to hydrodynamical flow pattern [Kolb ’00]

  \[
  V(t) = \pi (z_0 + v_z t) a(t) b(t), \quad a, b: \text{semi-axes of ellipse},
  \]

  \[
  v_{a,b} = v_\infty [1 - \exp(-\alpha t)] \mp \Delta v [1 - \exp(-\beta t)]
  \]

- **Isentropic expansion**: \( S = \text{const} \) (fixed from \( N_{\text{ch}} \))
- **QGP Equation of state**:

  \[
  s = \frac{S}{V(t)} = \frac{4\pi^2}{90} T^3 (16 + 10.5n_f^*) , \quad n_f^* = 2.5
  \]

  obtain \( T(t) \Rightarrow A(t,p), B_0(t,p) \) and \( B_1 = TEA \)

  for semicentral collisions (\( b = 7 \text{ fm} \)): \( T_0 = 340 \text{ MeV} \)

  QGP lifetime \( \simeq 5 \text{ fm}/c. \)

- simulate FP equation as **relativistic Langevin process**
Initial conditions

- need initial $p_T$-spectra of charm and bottom quarks
- (modified) PYTHIA to describe exp. D meson spectra, assuming $\delta$-function fragmentation
- exp. non-photonic single-$e^\pm$ spectra: Fix bottom/charm ratio
Spectra and elliptic flow for heavy quarks

- $\mu_D = gT$, $\alpha_s = g^2/(4\pi) = 0.4$
- resonances $\Rightarrow$ $c$-quark thermalization without upscaling of cross sections
- Fireball parametrization consistent with hydro
Comparison to single-electron spectra @ RHIC

(a) 0−10% central

Armesto et al. (I)
van Hees et al. (II)
Moore &
Teaney (III)

(b) minimum bias

π^0 R_{AA}, p_T > 4 GeV/c
π^0 v_2, p_T > 2 GeV/c
e^± R_{AA}, e^± v_2^{HF}

PHENIX Collaboration
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Microscopic model: Static potentials from lattice QCD

- color-singlet free energy from lattice
- use internal energy

\[ U_1(r,T) = F_1(r,T) - T \frac{\partial F_1(r,T)}{\partial T}, \]
\[ V_1(r,T) = U_1(r,T) - U_1(r \to \infty, T) \]

- Casimir scaling for other color channels [Nakamura et al 05; Döring et al 07]

\[ V_3 = \frac{1}{2} V_1, \quad V_6 = -\frac{1}{4} V_1, \quad V_8 = -\frac{1}{8} V_1 \]
Brueckner many-body approach for elastic \( Qq, Q\bar{q} \) scattering

\[
T = T_c q, \bar{q} + V
\]

\[
\Sigma = \Sigma_{\text{glu}} + T
\]

reduction scheme: 4D Bethe-Salpeter \( \rightarrow \) 3D Lipmann-Schwinger

\( S \)- and \( P \) waves

same scheme for light quarks (self consistent!)

Relation to invariant matrix elements

\[
\sum_q |M(s)|^2 \propto \sum_q d_a \left( |T_{a,l=0}(s)|^2 + 3 |T_{a,l=1}(s)|^2 \cos \theta_{\text{cm}} \right)
\]
Microscopic justification for resonances: T-matrix calculation

- use static heavy-quark potentials from lQCD
- resonance formation at lower temperatures $T \approx T_c$
- melting of resonances at higher $T$! $\Rightarrow$ sQGP
- model-independent assessment of elastic $Qq, Q\bar{q}$ scattering
- problems: uncertainties in extracting potential from lQCD in-medium potential $V$ vs. $F$?
Transport coefficients

- from non-pert. interactions reach \( A_{\text{non-pert}} \approx 1/(7 \text{ fm}/c) \approx 4 A_{\text{pQCD}} \)
- \( A \) decreases with higher temperature
- higher density (over)compensated by melting of resonances!
- spatial diffusion coefficient

\[
D_s = \frac{T}{mA}
\]

increases with temperature
Non-photonic electrons at RHIC

- same model for bottom
- quark coalescence + fragmentation $\to D/B \to e + X$

- coalescence crucial for description of data
- increases both, $R_{AA}$ and $v_2$ $\leftrightarrow$ “momentum kick” from light quarks!
- “resonance formation” towards $T_c$ $\Rightarrow$ coalescence natural [Ravagli, Rapp 07]
Transport properties of the sQGP

- spatial diffusion coefficient: Fokker-Planck \( \Rightarrow D_s = \frac{T}{mA} = \frac{T^2}{D} \)
- measure for coupling strength in plasma: \( \frac{\eta}{s} \)

\[
\frac{\eta}{s} \simeq \frac{1}{2} T D_s \quad (\text{AdS/CFT}) , \quad \frac{\eta}{s} \simeq \frac{1}{5} T D_s \quad (\text{wQGP})
\]

![Graph showing charm quark data and theoretical predictions for various models](image)
Summary and Outlook

Summary

- **Heavy quarks in the sQGP**
- **non-perturbative interactions**
  - mechanism for strong coupling: resonance formation at $T \gtrsim T_c$
  - IQCD potentials parameter free
  - res. melt at higher temperatures $\Leftrightarrow$ consistency betw. $R_{AA}$ and $v_2$!
- also provides “natural” mechanism for quark coalescence
- **problems**
  - potential approach at finite $T$: $F$, $V$ or combination?

Outlook

- use more realistic **bulk-medium description** (real hydro)
- include **inelastic heavy-quark processes** (gluo-radiative processes)
- take into account D/B-meson rescattering in the hadronic phase
- other **heavy-quark observables** like charmonium suppression/regeneration