Heavy Quarks in the QGP

Hendrik van Hees

Justus-Liebig Universität Gießen

March 03, 2008
1. Heavy-quark interactions in the sQGP
   - Heavy-quark observables in heavy-ion collisions
   - Heavy-quark diffusion: The Fokker-Planck Equation
   - Elastic pQCD heavy-quark scattering
   - Non-perturbative interactions: effective resonance model

2. Non-photonic electrons at RHIC

3. Microscopic model for non-perturbative HQ interactions
   - Static heavy-quark potentials from lattice QCD
   - T-matrix approach

4. Other Approaches
   - Radiative energy loss
   - Collisional dissociation/fragmentation in the QGP

5. Summary and Outlook
Heavy-Ion collisions in a Nutshell

- Theory of strong interactions: Quantum Chromo Dynamics, QCD
- At high enough densities/temperatures: hadrons dissolve into a Quark-Gluon Plasma (QGP)
- hope to create QGP in Heavy-Ion Collisions at RHIC (and LHC)
- RHIC: collide gold nuclei with energy of 200 GeV per nucleon:

![Diagram showing initial state, pre-equilibrium, QGP and hydrodynamic expansion, hadronization, and hadronic phase and freeze-out.]
Evidence for QGP from heavy-ion observables

- particle $p_T$ spectra show hydrodynamical behavior
- collective flow of matter in local thermal equilibrium
- nuclear modification factor $\Rightarrow$ degree of thermalization

$$R_{AA}(p_T) = \frac{dN_{AA}/dp_T}{N_{\text{coll}}dN_{pp}/dp_T}$$

- no QGP $\Rightarrow R_{AA} = 1$; observed: $R_{AA} < 1$ (suppression) at high $p_T$
- in non-central collisions: anisotropic collective flow

- initially reaction zone of elliptic shape
- pressure gradients: $\langle |p_x| \rangle > \langle |p_y| \rangle$
- measure of flow anisotropy:

$$v_2 = \left\langle \frac{p_x^2 - p_y^2}{p_x^2 + p_y^2} \right\rangle = \langle \cos(2\phi_p) \rangle$$
Hard production of HQs described by PDF's + pQCD (PYTHIA)

$c, b$ quark

HQ rescattering in QGP: Langevin simulation
drag and diffusion coefficients from microscopic model for HQ interactions in the sQGP

Hadronization to $D, B$ mesons via quark coalescence + fragmentation
V. Greco, C. M. Ko, R. Rapp, PLB 595, 202 (2004)

Semileptonic decay $\Rightarrow$ “non-photonic” electron observables
Relativistic Langevin process

- **Langevin process**: friction force + Gaussian random force
- in the (local) rest frame of the heat bath

\[
\begin{align*}
\text{d}\vec{x} &= \frac{\vec{p}}{E_p} \text{d}t, \\
\text{d}\vec{p} &= -A\vec{p} \text{d}t + \sqrt{2} \text{d}t [\sqrt{B_0 P_\perp} + \sqrt{B_1 P_\parallel}] \vec{w}
\end{align*}
\]

- \( \vec{w} \): normal-distributed random variable
- \( A \): friction (drag) coefficient
- \( B_{0,1} \): diffusion coefficients
- dependent on realization of stochastic process
- to guarantee correct equilibrium limit: Use Hänggi-Klimontovich calculus, i.e., use \( B_{0/1}(t, \vec{p} + \text{d}\vec{p}) \)
- Einstein dissipation-fluctuation relation \( B_0 = B_1 = E_p T A \).
- to implement flow of the medium
  - use Lorentz boost to change into local “heat-bath frame”
  - use update rule in heat-bath frame
  - boost back into “lab frame”
Elastic pQCD processes

- Lowest-order matrix elements [Combridge 79]

- Debye-screening mass for $t$-channel gluon exch. $\mu_g = gT, \alpha_s = 0.4$

- not sufficient to understand RHIC data on “non-photonic” electrons
Non-perturbative interactions: Resonance Scattering

- General idea: Survival of $D$- and $B$-meson like resonances above $T_c$
- elastic heavy-light-(anti-)quark scattering

\[
\begin{align*}
\bar{q} & \quad c \quad D, D', D_s \\
q & \quad c \quad u \quad D, D', D_s \\
D, D', D_s & \quad q \quad D, D', D_s \\
k & \quad c \quad k \quad D, D', D_s
\end{align*}
\]

- $D$- and $B$-meson like resonances in sQGP

- parameters
  - $m_D = 2$ GeV, $\Gamma_D = 0.4 \ldots 0.75$ GeV
  - $m_B = 5$ GeV, $\Gamma_B = 0.4 \ldots 0.75$ GeV
total pQCD and resonance cross sections: comparable in size

BUT pQCD forward peaked ↔ resonance isotropic

resonance scattering more effective for friction and diffusion
Transport coefficients: pQCD vs. resonance scattering

- three-momentum dependence

![Graphs showing the comparison between pQCD and resonance scattering with different resonance contributions and pQCD values.]

- resonance contributions factor $\sim 2 \ldots 3$ higher than pQCD!
Transport coefficients: pQCD vs. resonance scattering

- Temperature dependence

![Graphs showing temperature dependence of transport coefficients](image-url)
**Time evolution of the fire ball**

- **Elliptic fire-ball** parameterization fitted to hydrodynamical flow pattern [Kolb ’00]

\[
V(t) = \pi(z_0 + v_z t)a(t)b(t), \quad a, b: \text{semi-axes of ellipse},
\]

\[
v_{a,b} = v_{\infty}[1 - \exp(-\alpha t)] \mp \Delta v[1 - \exp(-\beta t)]
\]

- **Isentropic expansion:** \( S = \text{const} \) (fixed from \( N_{ch} \))
- **QGP Equation of state:**

\[
s = \frac{S}{V(t)} = \frac{4\pi^2}{90}T^3(16 + 10.5n_f^*), \quad n_f^* = 2.5
\]

- obtain \( T(t) \Rightarrow A(t, p), B_0(t, p) \) and \( B_1 = TEA \)
- for semicentral collisions (\( b = 7 \text{ fm} \)): \( T_0 = 340 \text{ MeV} \), QGP lifetime \( \simeq 5 \text{ fm}/c. \)
- simulate FP equation as relativistic Langevin process
Initial conditions

- need initial $p_T$-spectra of charm and bottom quarks
- (modified) PYTHIA to describe exp. D meson spectra, assuming $\delta$-function fragmentation
- exp. non-photonic single-$e^\pm$ spectra: Fix bottom/charm ratio

\[
\frac{1}{2\pi p_T} dN/dp_T \text{ [a.u.]} 
\]

\[
\sigma_{bb}/\sigma_{cc} = 4.9 \times 10^{-3}
\]
Spectra and elliptic flow for heavy quarks

\[ \mu_D = gT, \quad \alpha_s = g^2/(4\pi) = 0.4 \]

- resonances ⇒ c-quark thermalization without upscaling of cross sections
- Fireball parametrization consistent with hydro

- \[ \mu_D = 1.5T \text{ fixed} \]
- spatial diff. coefficient:
  \[ D = D_s = \frac{T}{mA} \]
- \[ 2\pi TD \sim \frac{3}{2\alpha_s^2} \]

\[ \text{Au-Au } \sqrt{s}=200 \text{ GeV (b=7 fm)} \]

\[ R_{AA} \]

\[ p_T \text{ [GeV]} \]

\[ 0 \quad 0.5 \quad 1 \quad 1.5 \quad 2 \quad 2.5 \quad 3 \quad 3.5 \quad 4 \quad 4.5 \quad 5 \]

\[ c, \text{ reso (} \Gamma=0.4-0.75 \text{ GeV)} \]

\[ c, \text{ pQCD, } \alpha_s=0.4 \]

\[ b, \text{ reso (} \Gamma=0.4-0.75 \text{ GeV)} \]
Spectra and elliptic flow for heavy quarks

- $c$, reso ($\Gamma = 0.4-0.75$ GeV)
- $c$, pQCD, $\alpha_s = 0.4$
- $b$, reso ($\Gamma = 0.4-0.75$ GeV)

Au-Au $\sqrt{s} = 200$ GeV (b=7 fm)

LO QCD

[Moore, Teaney ’04]
Observables: $p_T$-spectra ($R_{AA}$), $v_2$

- Hadronization: **Coalescence** with light quarks + fragmentation $\leftrightarrow c\bar{c}, b\bar{b}$ conserved
- single electrons from decay of $D$- and $B$-mesons

Without further adjustments: data quite well described 
[HvH, V. Greco, R. Rapp, Phys. Rev. C 73, 034913 (2006)]
Observables: $p_T$-spectra ($R_{AA}$), $v_2$

- Hadronization: Fragmentation only
- single electrons from decay of $D$- and $B$-mesons

![Graphs showing $R_{AA}$ and $v_2$ as functions of $p_T$ for Au-Au collisions at $\sqrt{s}=200$ GeV with $b=7$ fm.](image)
Observables: $p_T$-spectra ($R_{AA}$), $v_2$

- Central Collisions
- single electrons from decay of $D$- and $B$-mesons

Coalescence+Fragmentation

Fragmentation only

![Graph showing $R_{AA}$ vs. $p_T$ for central Au-Au collisions at $\sqrt{s}=200$ GeV. The graph compares data from PHENIX prel (0-10%) and STAR (0-5%) with theoretical models such as c+b reso and c+b pQCD.](image)
Comparison to newer data

(a) 0–10% central
- Armesto et al. (I)
- van Hees et al. (II)

(b) minimum bias
- \( p_T > 4 \text{ GeV/c} \)
- \( p_T > 2 \text{ GeV/c} \)
- \( e^\pm, p_T > H F \)

PHENIX Collaboration
PRL 98 172301 (2007)
Microscopic model: Static potentials from lattice QCD

- color-singlet free energy from lattice
- use internal energy

\[ U_1(r, T) = F_1(r, T) - T \frac{\partial F_1(r, T)}{\partial T}, \]

\[ V_1(r, T) = U_1(r, T) - U_1(r \to \infty, T) \]

- Casimir scaling for other color channels [Nakamura et al 05; Döring et al 07]

\[ V_3 = \frac{1}{2} V_1, \quad V_6 = -\frac{1}{4} V_1, \quad V_8 = -\frac{1}{8} V_1 \]
T-matrix

- Brueckner many-body approach for elastic $Qq, Q\bar{q}$ scattering

\[
T = V + V \Sigma + \Sigma_{\text{glu}} + T
\]

- reduction scheme: 4D Bethe-Salpeter $\rightarrow$ 3D Lipmann-Schwinger
- $S$- and $P$ waves
- same scheme for light quarks (self consistent!)
- Relation to invariant matrix elements

\[
\sum |\mathcal{M}(s)|^2 \propto \sum_q d_a \left( |T_{a,l=0}(s)|^2 + 3 |T_{a,l=1}(s)|^2 \cos \theta_{\text{cm}} \right)
\]
- Resonance formation at lower temperatures $T \approx T_c$
- Melting of resonances at higher $T$! $\Rightarrow$ sQGP
- $P$ wave smaller
- Resonances near $T_c$: natural connection to quark coalescence

[Ravagli, Rapp 07; Ravagli, HvH, Rapp 08]

- Model-independent assessment of elastic $Qq$, $Q\bar{q}$ scattering
- Problems: uncertainties in extracting potential from lQCD in-medium potential $V$ vs. $F$?
from non-pert. interactions reach \( A_{\text{non-pert}} \simeq 1/(7 \text{ fm}/c) \simeq 4A_{\text{pQCD}} \)

- A decreases with higher temperature
- higher density (over)compensated by melting of resonances!
- spatial diffusion coefficient

\[
D_s = \frac{T}{mA}
\]

increases with temperature
Non-photonic electrons at RHIC

- same model for bottom
- quark coalescence + fragmentation $\rightarrow D/B \rightarrow e + X$

- coalescence crucial for description of data
- increases both, $R_{AA}$ and $v_2$ ⇔ “momentum kick” from light quarks!
- “resonance formation” towards $T_c \Rightarrow$ coalescence natural [Ravagli, Rapp 07]
Radiative energy loss

- **Gluo-bremsstrahlung energy-loss calculations**
  - medium modelled by static scattering centers
  - energy loss through *gluo bremsstrahlung*: \( \Delta E = \frac{\alpha_s}{2} \hat{q}L^2 \)
  - perturbative estimate for RHIC conditions: \( \hat{q} \approx 1 \text{ GeV}^2/\text{fm} \)

![Graph showing mass and scale uncertainties](image)

- Need \( \hat{q} = 14 \text{ GeV}^2/\text{fm} \); \( v_2 \): only through almond-shape geometry
- without drag \( \Rightarrow \) no heavy-quark collective flow:
  - no consistent description of \( R_{AA} \) and \( v_2 \)!

[Armesto, Cacciari et al. (2006)]
Collisional dissociation/fragmentation in the QGP

- in-medium dissociation of $D/B$ mesons $\leftrightarrow$ in-medium fragmentation of $c/b$ quarks
- medium modification of quark-wave functions in QGP
- dissociation by collision with QGP particles
- in-medium fragmentation $c/b \rightarrow D/B$

$B$ mesons stronger bound than $D$ mesons
smaller $B$ formation times $\Leftrightarrow$ stronger suppression for $B$ than for $D$!
could be distinguished from HQ elastic-scattering processes by separate measurement of $D$ and $B$ only!

[Adil, Vitev (2007)]
Transport properties of the sQGP

- spatial diffusion coefficient: Fokker-Planck \[ \Rightarrow D_s = \frac{T}{mA} = \frac{T^2}{D} \]
- measure for coupling strength in plasma: \( \eta/s \)

\[
\frac{\eta}{s} \simeq \frac{1}{2} TD_s \quad \text{(AdS/CFT)}, \quad \frac{\eta}{s} \simeq \frac{1}{5} TD_s \quad \text{(wQGP)}
\]

[Lacey, Taranenko (2006)]

[Hendrik van Hees (JLU Gießen) | Heavy Quarks in the QGP | March 03, 2008]
Summary and Outlook

Summary

- **Heavy quarks in the sQGP**
- non-perturbative interactions
  - mechanism for strong coupling: resonance formation at $T \gtrsim T_c$
  - IQCD potentials parameter free
  - res. melt at higher temperatures $\Leftrightarrow$ consistency betw. $R_{AA}$ and $v_2$!
- also provides “natural” mechanism for quark coalescence
- resonance-recombination model
- problems
  - extraction of $V$ from lattice data
  - potential approach at finite $T$: $F$, $V$ or combination?

Outlook

- include inelastic heavy-quark processes (gluo-radiative processes)
- other heavy-quark observables like charmonium suppression/regeneration