Thermalization of Heavy Quarks through Resonance Exchange

Hendrik van Hees

Texas A&M University

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Outline

Motivation

- in-medium spectral properties of charmonia in QGP
- p_T spectra of D mesons, v_2
- importance of dissociation and regeneration in

$$c + \bar{c} \leftrightarrow J/\psi + X$$

- Indications for thermalization of heavy quarks
- Survival of "D-mesonic resonances" above T_c (indicated by recent lattice QCD calculations)

The Particle Content

Chiral symmetry $\mathrm{SU}_V(2) \otimes \mathrm{SU}_A(2)$ in the light-quark sector of QCD

$$\mathscr{L}_D^{(0)} = \sum_{i=1}^{2} [(\partial_\mu D_i^\dagger)(\partial^\mu D_i) - m_D^2 D_i^\dagger D_i] + \text{usual massive vector Lagrangian for } D^*$$

D_i: two doublets: pseudo-scalar $\sim \binom{\overline{D^0}}{D-}, \binom{D^0}{D+}$ and scalar

D_i^*: two doublets: vector $\binom{\overline{D^{0*}}}{D-*}, \binom{D^{0*}}{D+*}$ and pseudo-vector

$$\mathscr{L}_{qc}^{(0)} = \bar{q} \mathrm{i} \partial\!\!\!/ q + \bar{c}(\mathrm{i} \partial\!\!\!/ - m_c) c$$

q: light-quark doublet $\sim \binom{u}{d}$

c: singlet

Chiral Symmetry

Infinitesimal version:

$$q \longrightarrow (1 + \mathrm{i}\delta\vec{\phi}_V\vec{t} + \mathrm{i}\delta\vec{\phi}_A\vec{t}\gamma_5)q, \quad c \longrightarrow c.$$

Light quarks massless in chiral limit!

$$D_1 \longrightarrow D + \mathrm{i}\delta\vec{\phi}_V\vec{t}D_1 + \mathrm{i}\delta\vec{\phi}_A\vec{t}D_2,$$
$$D_2 \longrightarrow D_2 + \mathrm{i}\delta\vec{\phi}_V\vec{t}D_2 + \mathrm{i}\delta\vec{\phi}_A\vec{t}D_1.$$

Mesons must have chiral partners
In the vacuum: chiral symmetry spontaneously broken
In QGP: chiral symmetry restored

Interactions

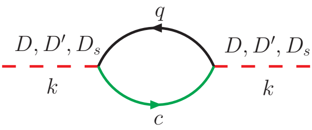
Interactions determined by chiral symmetry
Strong interactions also preserve parity
For transversality of vector mesons: use heavy-quark effective theory vertices

$$\begin{aligned}\mathscr{L}_{\text{int}} = & - G_S \left(\bar{q} \frac{1+\not{v}}{2} D_1 c_v + \bar{q} \frac{1+\not{v}}{2} \mathrm{i}\gamma^5 D_2 c_v + h.c. \right) \\ & - G_V \left(\bar{q} \frac{1+\not{v}}{2} \gamma^\mu D_{1\mu}^* c_v + \bar{q} \frac{1+\not{v}}{2} \mathrm{i}\gamma^\mu \gamma^5 D_{2\mu}^* c_v + h.c. \right)\end{aligned}$$

v: four momentum of heavy quark in HQET: spin symmetry $\Rightarrow G_S = G_V$

Dressing the D Mesons

Dressing the D mesons with self-energies



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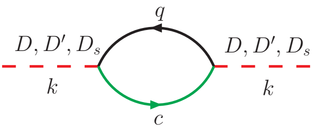

Divergencies: wave-function + mass renormalization:

$$\Pi_D(k^2=0)=0, \quad \partial_{k^2}\Pi_D(k^2=0)=0.$$

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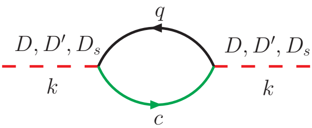

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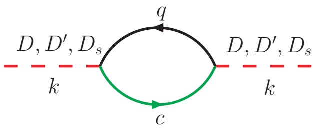

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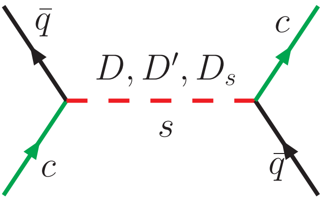
Mass and coupling adjusted such that

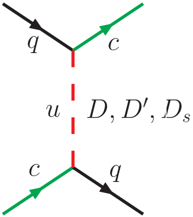
$$m_D = 2\text{ GeV}, \quad \Gamma_D = (0.3\ldots 0.8)\text{GeV}$$

(from in-medium Bethe-Salpeter calculations)

Elastic scattering cross sections

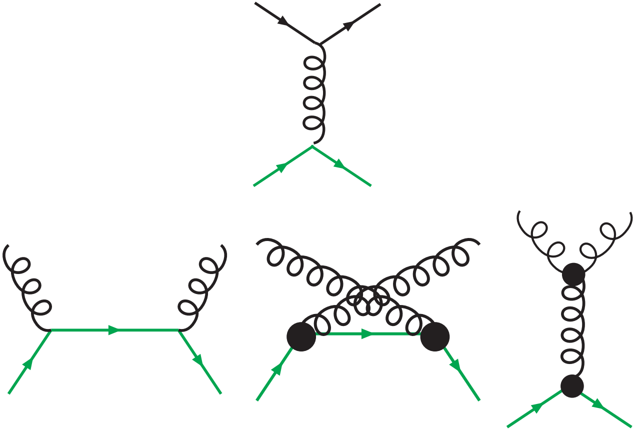
Resonant heavy-light-(anti-)quark scattering





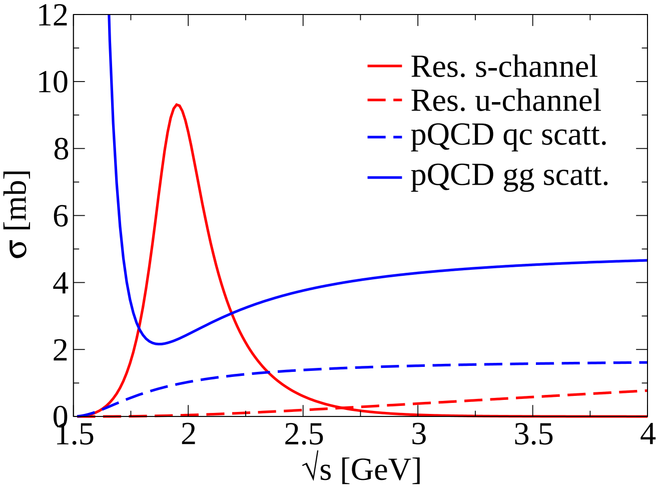
Contributions from pQCD

Usual matrix elements (Combridge et al) for $qc \to qc$-, $gc \to gc$-scattering:



In-medium Debye-screening mass for t-channel gluon exchange: $\mu_g = gT$, $\alpha_s = 0.3, 0.4, 0.5$

Cross sections



The Fokker-Planck Equation

heavy particle (c quarks) in a heat bath of light particles (QGP)

$$\frac{\partial f(t,\vec{p})}{\partial t} = \frac{\partial}{\partial p_i}\left[A_i(t,\vec{p}) + \frac{\partial}{\partial p_j} B_{ij}(t,\vec{p})\right] f(t,\vec{p})$$

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A_i and B_{ij} given by averages over initial momenta $\vec{q}$ of light particles and summation over final states:

$$\langle F(\vec{p}\,')\rangle = \frac{1}{\gamma_c}\frac{1}{2E_p}\int \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3 2E_q}\int \frac{\mathrm{d}^3\vec{q}\,'}{(2\pi)^3 2E_{q'}}\int \frac{\mathrm{d}^3\vec{p}\,'}{(2\pi)^3 2E_{p'}} \sum |\mathcal{M}|^2 (2\pi)^4 \delta^{(4)}(p+q-p'-q')\hat{f}(\vec{q})F(\vec{p}\,')$$

Meaning of the Fokker-Planck coefficients

For t, $\vec{p}$-independent coefficients:

$$\frac{\partial f}{\partial t} = \gamma \frac{\partial}{\partial \vec{p}}(\vec{p} f) + D \frac{\partial^2}{\partial \vec{p}^2} f.$$

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Solution for c quark with given momentum $\vec{p}_0$ at $t = 0$:

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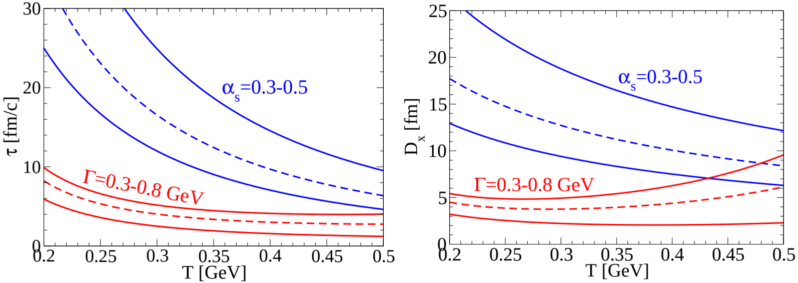
- temperature $T^* = \gamma m_c / D$ (dissipation-fluctuation theorem)
- consistency condition $T^* \stackrel{!}{=} T$ (heat bath-temperature)
- fulfilled within 14% for relevant temperature region

The Coefficients

Coefficients not much $\vec{p}$ dependent
Temperature dependence



$\tau = 1/\gamma$: relaxation time scale
$D_x = 2dD/(m\gamma^2)$: spatial diffusion coefficient

$$|\Delta\vec{x}|^2 = D_x t$$

Equilibration

Simple fire-ball parameterization:

$$V(\tau) = \pi(z_0 + v_z\tau)(r_0 + \frac{1}{2}a_\perp\tau^2)^2$$

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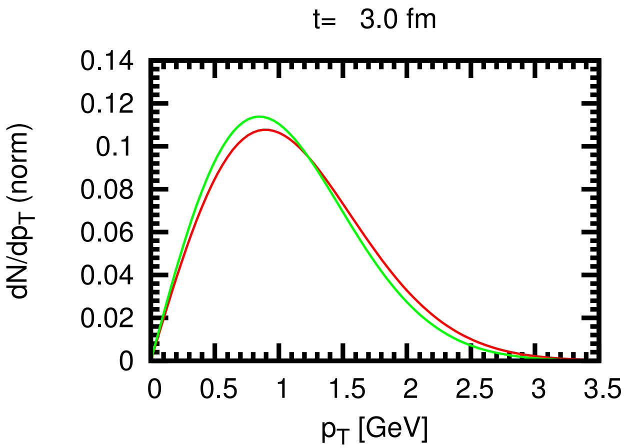
Solve Fokker-Planck equation with time dependent coefficients
initial condition:

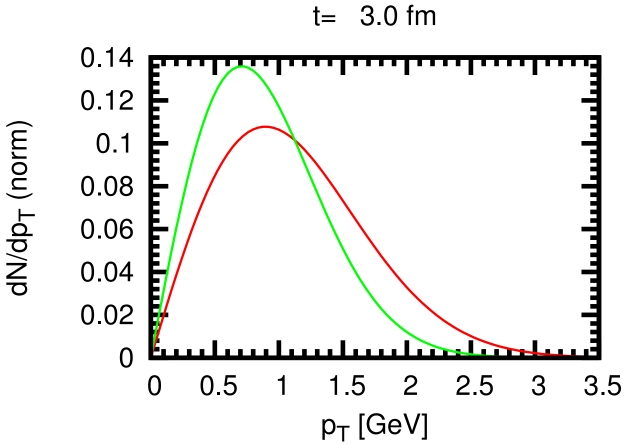
$$\frac{\mathrm{d}N}{\mathrm{d}p_t} := f(\vec{p}, t = 0) \propto \exp(-b\vec{p}^2)$$

describes *pp*-collision data.

Evolution of p_t spectra: pQCD



Evolution of p_t spectra: resonances + pQCD



Conclusions and Outlook

- survival of resonances in the QGP
- possible mechanism for strong interactions beyond T_c
- Equilibration of heavy quarks in QGP
- application to secondary production of $c\bar{c}$ pairs
- need to include flow in p_T-spectra calculation