Implementation of Hagedorn States into UrQMD

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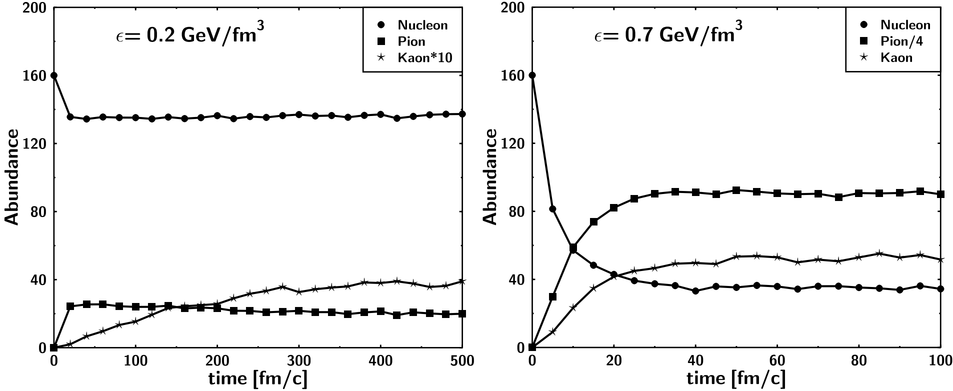
Setup of UrQMD box calculations

- cubic box with cyclic boundary conditions
- inital particles: 80 n + 80 p in $V = 1000$ fm^3
- uniform distribution in configuraton and momentum space
- fixed $\rho_B = 0.16$ fm^{-3} and $\rho_S = 0$ fm^{-3}
- runs done for $\epsilon = 0.2$ GeV/fm^3 and $\epsilon = 0.7$ GeV/fm^3
- multiplicities of kaons, pions and nucleons are averaged over 50 events

Problem - Chem. equilibration in UrQMD(box) too long



M. Belkacem et al., PRC 58 (1998), 1727

Successfull Application: C. Greiner, J. Noronha-Hostler et al.

- at SPS energies strong increase of anitprotons and antihyperons is explained by 'clustering' of mesons

$$n_1\pi + n_2K \leftrightarrow \bar{Y} + p \qquad (1)$$

- giving an chemical equilibration time of $t_{eq} \approx 1 - 3$ fm/c
- for RHIC energies $t_{eq} \sim 10$ fm/c for antibaryons
- quick chemical equilibration mechanism is provided by HS:

$$\left(n_1\pi + n_2K + n_3\bar{K} \leftrightarrow\right)\ HS \leftrightarrow \bar{B} + B + X \qquad (2)$$

- dynamical description of (2) described by set of coupled rate equations
- assuming HS and pions start in equilibrium $B\bar{B}$-pairs chem. equilibrate in $t_{eq} \approx 5$ fm/c

Intention

Full integration of Hagedorn States (HS) into UrQMD

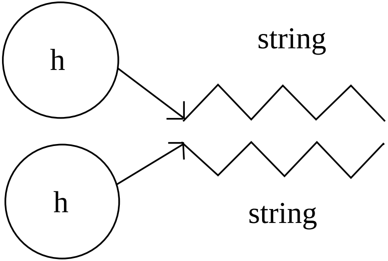
- particle multiplicities
- chemical equilibration times
- $\frac{\eta}{s}$
- $\ldots$

UrQMD

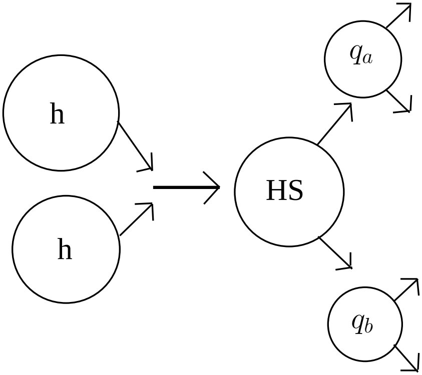
- microscopic Hadron-String Transport model simulating p+p, p+N and A+A collisions in the energy range from Bevalac and SIS up to AGS, SPS and RHIC
- detailed balance is enforced for following processes: meson-baryon, meson-meson, resonance-nucleon and resonance-resonance interaction
- for high $\sqrt{s}$ beside string production also HS production should be possible

Aspired changes

strings in UrQMD now



our project: strings + HS



History

- in 1965 Rolf Hagedorn posutlated the so called "Statistical Bootstrap Model" [1]
- highly excited lumps of matter are not essentially different from observed hadronic resonances at lower excitation
- fireballs contain all known and unknown particles, among them the 'resonances', which are regarded as small fireballs too
- fireballs and their constituents are the same and they all should be counted by the same mass spectrum
- nesting fireballs into each other leads to a mathematical self-consistency condition on the hadron mass spectrum

[1] Nuovo Cim.Suppl. 3 (1965) 147-186

Thermodynamical Approach

- the original approach is based on derivation of the partition function
- in high energy collisions a created system decays in about 3 fm/c
- assumption of establishment of equilibrium in a shorter time than typical time scales must be made since a temperature will be introduced
- strong forward-backward motions can be kinematically separated from the isotropic thermal motion
- number of different kinds of hadrons is infinite

Thermodynamical Approach

- usual partition function of a non-interacting gas reads

$$Z\left(T,V\right)=\sum_i \exp\left(\frac{E_i}{T}\right)\equiv\int_0^\infty \mathrm{d}E\,\sigma\left(E,V\right)\exp\left(-\frac{E}{T}\right) \quad (3)$$

- σ is a continous density of states in the system
- partition function of a relativistic gas of massive particles with quantum statistics reads

$$Z\left(T,V\right)=\exp\left(\frac{VT}{2\pi^2}\sum_{n=1}^{\infty}\frac{(\mp 1)^{n+1}}{n^2}\sum_i m_i^2 K_2\left(\frac{nm_i}{T}\right)\right) \quad (4)$$

Thermodynamical Approach

- introduction of continuous particle spectrum ρ

$$Z\left(T,V\right)=\exp\left(\frac{VT}{2\pi^2}\sum_{n=1}^{\infty}\frac{\left(\mp 1\right)^{n+1}}{n^2}\int_0^{\infty}\mathrm{d}m\,\rho\left(m,n\right)m^2K_2\left(\frac{nm}{T}\right)\right) \tag{5}$$

- bootrap condition requires

$$\lim_{m\to\infty}\frac{\log\left(\rho\left(m\right)\right)}{\log\left(\sigma\left(m\right)\right)}=1 \tag{6}$$

- σ is inverse Laplace transformation of Z and hence a function of ρ

$$\sigma\left(m\right)=F\left[\rho\left(m\right)\right] \tag{7}$$

Thermodynamical Approach

- solution of bootstrap condition obtained by iteration $\rho^{i+1} = \sigma^{i}$

$$\rho\left(m\right) \sim \sigma\left(m\right) \sim \exp\left(\frac{m}{T}\right) \tag{8}$$

- make solution more unique employ "weak bootstrap condition"

$$\lim_{m\to\infty} \frac{cm^{a}\rho\left(m\right)}{\sigma\left(m\right)} = 1 \tag{9}$$

- the weak non-logarithmic bootstrap condition leads to

$$\rho\left(m\right) \sim \sigma\left(m\right) \sim cm^{a}\exp\left(\frac{m}{T}\right) \tag{10}$$

- (9) only fullfilled if $a = -2.5$

Phase-Space Approach: S. Frautschi, 1971

- partition function ansatz starts with $n = 1$ where $n = 2$ required for compound systems
- use of temperature requires a thermally equilibrated system
- to circumvent this drawbacks work with phase space directly

$$\rho_{out}^{nc}(m) = \sum_{n=2}^{\infty} \left(\frac{V}{(2\pi)^3} \right)^{n-1} \frac{1}{n!} \prod_{i=1}^{n} \int \mathrm{d}m_i \, \rho_{in}(m_i) \times \int \mathrm{d}^3 p_i \, \delta\left(\sum_{i=1}^{n} E_i - m \right) \delta^{(3)}\left(\sum_{i=1}^{n} \vec{p}_i \right) \quad (11)$$

- nc denotes non-covariant measure $\mathrm{d}^3 p$
- covariant counterpart $\frac{\mathrm{d}^3 p}{2E}$ would be more justified

Low Mass Input

- (11) known from microcanonical ensemble by conserving energy and momentum
- each state of motion can be occupied by particles with different masses considered by ρ_{in}
- ρ_{out} and ρ_{in} should asymptotically converge as required by "strong bootstrap condition"

$$\rho_{out}(m) \sim \rho_{in}(m) \quad \text{for} \quad m \to \infty \tag{12}$$

- this condition requires $a < -2.5$ in ρ_{in}
- to bound a from below a strenghtened assumption is made: $\rho_{out} = \rho_{in}$ for all masses above some threshold
- to describe low mass region more accurate a low-mass-input must be taken into account

$$\rho_{in}(m) = \rho_{out}(m) + \rho(\text{low-mass-input}) \tag{13}$$

Charged Spectra

- charged spectra with B,S and Q are described by

$$\rho^{out}_{B,S,Q}(m) = \sum_{n=2}^{\infty} \left(\frac{V}{(2\pi)^3}\right)^{n-1} \frac{1}{n!} \prod_{i=1}^{n} \int \mathrm{d}m_i \sum_{B_i,S_i,Q_i} \rho^{in}_{B_i,S_i,Q_i}(m_i)$$
$$\times \int \mathrm{d}^3 p_i \, \delta\left(\sum_{i=1}^{n} E_i - m\right) \delta^{(3)}\left(\sum_{i=1}^{n} \vec{p}_i\right)$$
$$\times \, \delta\left(\sum_i B_i - B\right) \delta\left(\sum_i S_i - S\right) \delta\left(\sum_i Q_i - Q\right) \quad (14)$$

Toy Model

- the iteration character is mathematically represented by

$$f^{j+1}\left(x\right) = \int^{x} \mathrm{d}x' f^{j}\left(x'\right) \tag{15}$$

- first toy model is to build up the particle density spectrum solely of π^0
- only $n = 2$ term is regarded and $\rho_{low} = \delta\left(m_i - m_0\right)$

$$\begin{aligned} \rho_{out}^{j+1}\left(m\right) = \frac{1}{2}\frac{V}{\left(2\pi\right)^3} \int^{m} \mathrm{d}\, m_1 \int^{m-m1} \mathrm{d}m_2 \\ \times \left[\rho_{out}^{j}\left(m_1\right) + \delta\left(m_1 - m_0\right)\right]\left[\rho_{out}^{j}\left(m_2\right) + \delta\left(m_2 - m_0\right)\right] \\ \times \frac{4\pi}{4m^3}\left(m_1^2 - m_2^2 + m^2\right)\left(m_2^2 - m_1^2 + m^2\right) p_{cm}\left(m, m_1, m_2\right) \end{aligned} \tag{16}$$

Toy Model

- for j=0 we start with two δ-functions containing m_0

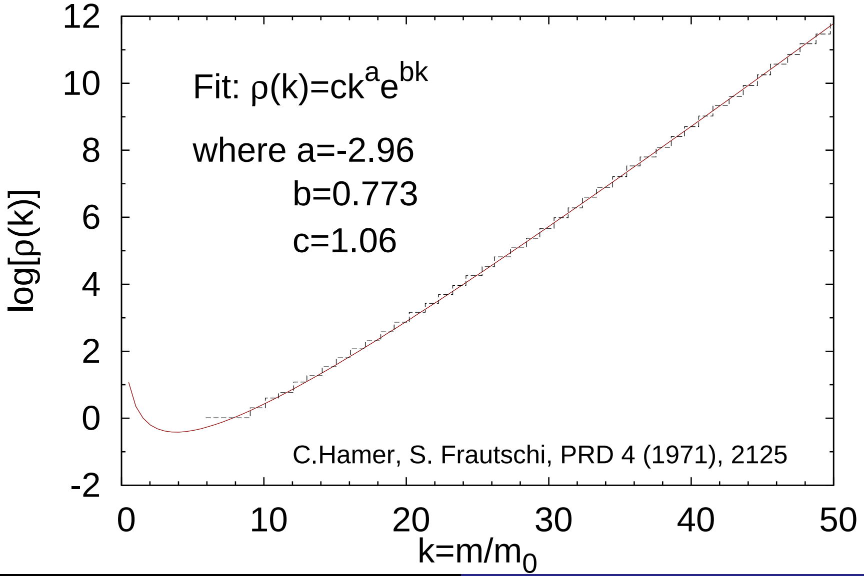
$$\rho^1_{out}(m) = \frac{1}{2}\frac{V}{(2\pi)^3}\frac{4\pi}{8} m \sqrt{m^2 - (2m_0)^2} \qquad (17)$$

- look for upper mass limit k for which ρ^1_{out} contains at least one particle

$$\int\limits_{2m_0}^{km_0} \mathrm{d}m \rho^1_{out}(m) \geq 1 \qquad (18)$$

- if k is found add low-mass-input and put it back into (16)
- procedure is repeated where the mass intervall before each iteration is increased by m_0

Toy Model Result



Cross Section and Decay Width

- cross section for formation of a resonance q out of two particles q_1 and q_2

$$\sigma\left(q_1+q_2 \to q\right) = \frac{2\pi m_1 m_2}{m p_{cm}\left(m, m_1, m_2\right)} \rho\left(m\right) \left|\mathcal{M}_{q_1+q_2\to q}\right|^2 \tag{19}$$

- partial decay width of a resonance q into two particles q_1 and q_2

$$\Gamma\left(q \to q_1+q_2\right) = \frac{p_{cm}}{\pi m} \int \mathrm{d}m_1 \frac{m_1 \rho\left(m_1, q_1\right)}{2E_1} \int \mathrm{d}m_2 \frac{m_2 \rho\left(m_2, q_2\right)}{2E_1} \times \left|\mathcal{M}_{q\to q_1+q_2}\right|^2 \tag{20}$$

- detailed balance requires $\left|\mathcal{M}_{q_1+q_2\to q}\right|^2 = \left|\mathcal{M}_{q\to q_1+q_2}\right|^2$

$$\sigma\left(q_1+q_2 \to q\right) = \frac{2\pi^2 \rho\left(m, q\right)}{p_{cm}^2\left(m, m_1, m_2\right)} \Gamma\left(q \to q_1+q_2\right) \tag{21}$$

- $\sigma\left(q_1+q_2 \rightarrow q\right)$ and $\Gamma\left(q \rightarrow q_1+q_2\right)$ are both not known
- for finial state in continuum geometrical cross section is used

$$\sigma\left(q_1+q_2 \rightarrow q\right)=\left\langle I^1 I_z^1 I^2 I_z^2 \| I I_z\right\rangle \pi R^2 \quad \text{with } R \approx r_0\left(\frac{m}{m_d}\right)^{\frac{1}{3}} \tag{22}$$

- now the partial decay width reads

$$\Gamma\left(q \rightarrow q_1+q_2\right)=\frac{\left\langle I^1 I_z^1 I^2 I_z^2 \| I I_z\right\rangle R^2 p_{cm}^2}{2 \pi \rho(m, q)} \times \int \mathrm{d} m_1 \frac{m_1 \rho\left(m_1, q_1\right)}{2 E_1} \int \mathrm{d} m_2 \frac{m_2 \rho\left(m_2, q_2\right)}{2 E_1} \tag{23}$$

- three different decay modes possible (i) q_1 and q_2 both hadrons, (ii) q_1 hadron q_2 HS, (iii) q_1 and q_2 both HS

- decay mode (i) (both hadrons represented by δ-functions)

$$\Gamma^{(i)}(m,q) = \frac{\langle I^1 I^1_z I^2 I^2_z \| I I_z \rangle R^2(m)\, p^2_{cm}(m)}{2\pi\rho(m,q)} \tag{24}$$

- decay mode (ii) (hadron+HS)

$$\Gamma^{(ii)}(m,q) = \frac{\langle I^1 I^1_z I^2 I^2_z \| I I_z \rangle m R^2(m)}{2\pi\rho(m,q)} \times \int_0^{p_{cm}} \mathrm{d}p\, p^3 \frac{\rho\left(\sqrt{m^2+m_1^2-2mE_1}, q_1\right)}{E_1\sqrt{m^2+m_1^2-2mE_1}} \tag{25}$$

- decay mode (iii) (both HS)

$$\Gamma^{(iii)}(m,q) = \frac{\langle I^1 I^1_z I^2 I^2_z \| I I_z \rangle R^2(m)}{2\pi\rho(m,q)} \times \int^{m} \mathrm{d}m_1 \int^{m-m_1} \mathrm{d}m_2\, p_{cm}(m,m_1,m_2)\, \rho(m_1,q_1)\, \rho(m_2,q_2) \tag{26}$$

- in total decay width expression sums run over all quantum numbers allowed $(q = q_1 + q_2)$

$$\Gamma\left(m,q\right) = \sum_{q_1,q_2} \Gamma^{(i)} + \sum_{q_1,q_2} \Gamma^{(ii)} + \sum_{q_1,q_2} \Gamma^{(iii)} \tag{27}$$

- a moving resonance will live an aveeage time $\langle\tau\rangle$ where γ is the resonance Lorentz factor

$$\langle\tau\rangle = \frac{\gamma}{\Gamma\left(m,q\right)} \tag{28}$$

Conclusions and Outlook

- chemical equilibration times in UrQMD $t_{eq.} \gg 3$ fm/c
- violation of detailed balance by strings and some hadronic decays $(\omega \rightarrow 3\pi)$
- creation and propagation of HS in UrQMD in binary collisions with $\sqrt{s} \leq 6$ GeV using geometrical cross section $\left(\sigma \sim R^2\right)$
- deployment of Statistical Bootstrap Model according to phase-space formulation to get ρ
- no decay of HS in UrQMD realized because $\rho_{B,S,Q}$ for each binary collison of particles q_1 and q_2 is not known yet

- calculation of particle densities $\rho_{B,S,Q}$ out of the phase-space approach
- examination of chem. equilibration times in UrQMD with HS