Dynamic enhancement of fluctuations at the QCD phase transition

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Transport meeting, June 14th 2012

HGS-HIRe for FAIR
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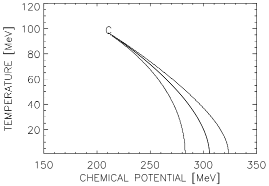
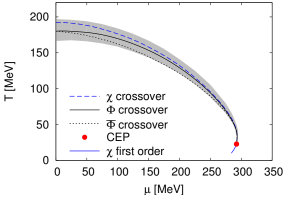
Effective models of QCD

Sigma model



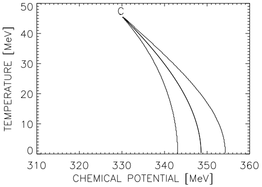
(Scavenius, Mocsy, Mishustin and Rischke, PRC **64** (2001))

Polyakov-quark-meson model



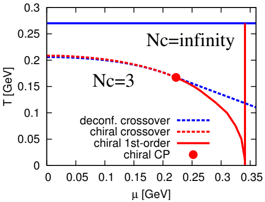
(Herbst, Pawlowski, Schaefer, PLB **696** (2011))

Nambu-Jona-Lasinio model



(Scavenius, Mocsy, Mishustin and Rischke, PRC **64** (2001))

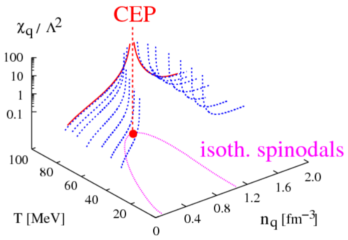
Polyakov-NJL model

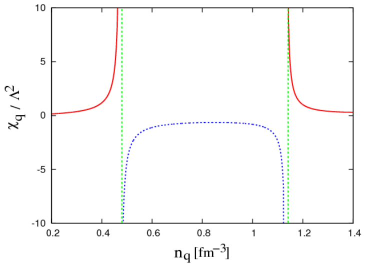


(C. Sasaki, APPS.3:659-668 (2010))

Signals for a first order phase transition

Out of equilibrium: Fluctuations at the first order transition can be as strong as at the critical point

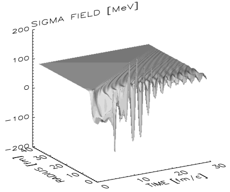




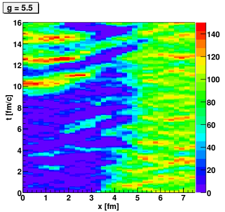
(Sasaki, Friman and Redlich, J. Phys. G **35** (2008))

Goal: study phase transition within fully dynamical model

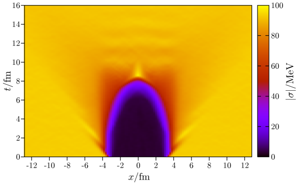
Chiral fluid dynamics with a Polyakov loop



(I. N. Mishustin and O. Scavenius, Phys. Rev. Lett. **83** (1999))

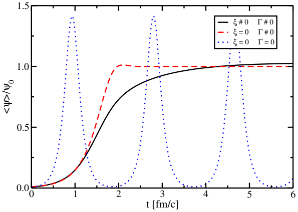


(K. Paech, H. Stöcker and A. Dumitru, Phys. Rev. C **68** (2003))



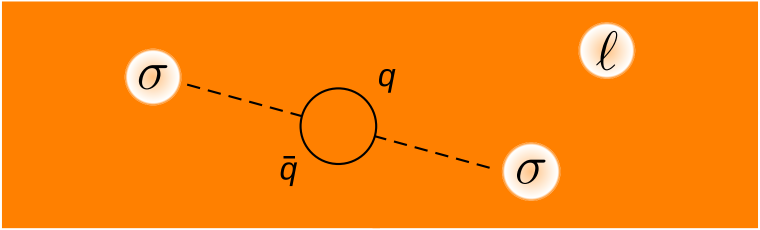
(M. Nahrgang, C. H., S. Leupold, I. N. Mishustin and

M. Bleicher, arXiv:1105.1962v2)



(Fraga, Krein, Mizher, PRD **76** (2007))

Chiral fluid dynamics with a Polyakov loop



- quarks: heat bath in local thermal equilibrium, interacting with:
- σ: mesonic field, propagated via Langevin equation
- ℓ: Polyakov loop, coupled to heat bath
- dynamical, self-consistent and energy-conserving
- nonequilibrium effects

(I. N. Mishustin and O. Scavenius, Phys. Rev. Lett. **83** (1999),

K. Paech, H. Stöcker and A. Dumitru, Phys. Rev. C **68** (2003),

M. Nahrgang, S. Leupold, C. H. and M. Bleicher, Phys. Rev. C **84** (2011),

In preparation: C. H., M. Nahrgang, I. N. Mishustin and M. Bleicher)

The Polyakov loop extended linear-sigma-model

The Lagrangian

$$\mathcal{L} = \bar{q}\left[\mathrm{i}\left(\gamma^\mu\partial_\mu - \mathrm{i}g_{QCD}\gamma^0 A_0\right) - g\sigma\right]q + {}^1\!/_2\left(\partial_\mu\sigma\right)^2 - U(\sigma) - \mathcal{U}(\ell,\bar{\ell})$$

with the mesonic potential

$$U(\sigma) = \frac{\lambda^2}{4}\left(\sigma^2 - \nu^2\right)^2 - h_q\sigma - U_0$$

and the Polyakov loop potential

$$\frac{\mathcal{U}}{T^4}(\ell,\bar{\ell}) = -\frac{b_2(T)}{4}\left(|\ell|^2 + |\bar{\ell}|^2\right) - \frac{b_3}{6}\left(\ell^3 + \bar{\ell}^3\right) + \frac{b_4}{16}\left(|\ell|^2 + |\bar{\ell}|^2\right)^2$$

(C. Ratti, M. A. Thaler, W. Weise, Phys. Rev. D **73** (2006), B.-J. Schaefer, J. M. Pawlowski and J. Wambach, Phys. Rev. D **76** (2007))

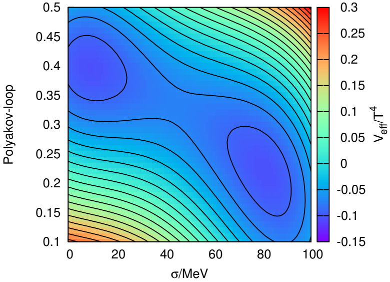
Thermodynamics

grand canonical potential at $\mu_B = 0$, $\ell = \bar{\ell}$, mean-field

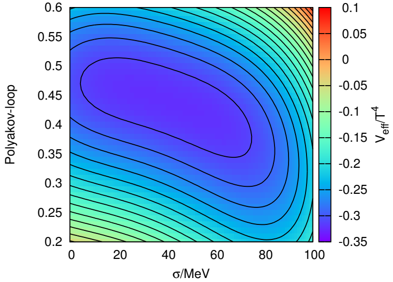
$$\Omega_{\bar{q}q} = -4N_f T \int \frac{\mathrm{d}^3 p}{(2\pi)^3} \ln\left[1 + 3\ell \mathrm{e}^{-\beta E} + 3\ell \mathrm{e}^{-2\beta E} + \mathrm{e}^{-3\beta E}\right]$$

effective potential

$$V_{eff}(\sigma, \ell, T) = U(\sigma) + \mathcal{U}(\ell) + \Omega_{\bar{q}q}(\sigma, \ell, T)$$

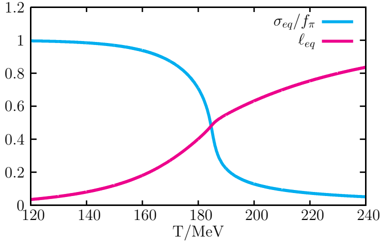


first order transition,
$g = 4.7,\ T_c = 172.9$ MeV

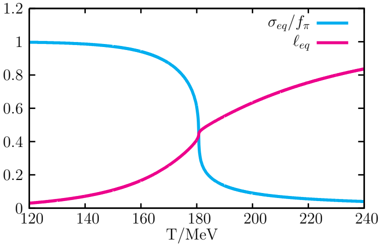


critical point,
$g = 3.52,\ T_c = 180.5$ MeV

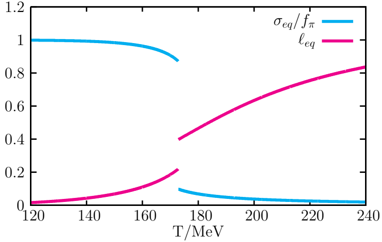
Thermodynamics



crossover



critical point

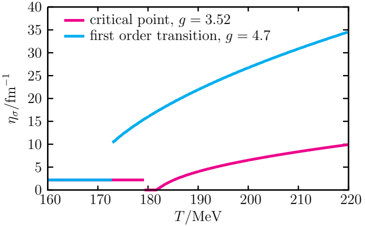


first order transition

order of the transition at $\mu = 0$
tuned via coupling g
$g = 3.2$ (physical): crossover
$g = 3.52$: critical point
$g = 4.7$: first order

The equations of motion

$$\partial_\mu \partial^\mu \sigma + \eta_\sigma(T)\partial_t \sigma + \frac{\partial V_{eff}}{\partial \sigma} = \xi_\sigma$$



Dissipation-fluctuation relation

$$\langle \xi_\sigma(t)\xi_\sigma(t')\rangle = \frac{1}{V}\delta(t-t')m_\sigma \eta_\sigma \coth\left(\frac{m_\sigma}{2T}\right)$$

(M. Nahrgang, S. Leupold, C. H. and M. Bleicher, Phys. Rev. C **84** (2011))

Assume similar structure for Polyakov loop equation of motion

$$\frac{2N_c}{g^2_{QCD}}\partial_\mu \partial^\mu \ell T^2 + \eta_\ell \partial_t \ell + \frac{\partial V_{eff}}{\partial \ell} = \xi_\ell$$

$$\langle \xi_\ell(t)\xi_\ell(t')\rangle = \frac{1}{V}\delta(t-t')2\eta_\ell T$$

(cf. A. Dumitru and R. D. Pisarski, Nucl. Phys. A **698** (2002))

Propagation of the quark fluid

Ideal relativistic fluid dynamics

$$\partial_\mu \left(T_q^{\mu\nu} + T_\sigma^{\mu\nu} + T_\ell^{\mu\nu} \right) = 0$$

Equation of state $p = p(e)$ from

$$\begin{aligned} e(\sigma, \ell, T) &= T \frac{\partial p(\sigma, \ell, T)}{\partial T} - p(\sigma, \ell, T) \\ p(\sigma, \ell, T) &= -\Omega_{\bar{q}q}(\sigma, \ell, T) \end{aligned}$$

Propagation of the quark fluid: Source terms

For the quarks: $T^{\mu\nu}$ of ideal fluid

$$\partial_\mu T_q^{\mu\nu} = -\partial_\mu \left(T_\sigma^{\mu\nu} + T_\ell^{\mu\nu} \right) = S_\sigma^\nu + S_\ell^\nu$$

Source terms

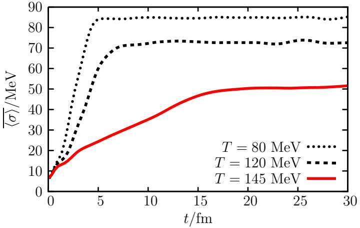
$$\begin{aligned} S_\sigma^\nu &= \left(-\frac{\partial \Omega_{q\bar{q}}}{\partial \sigma} - \eta_\sigma \partial_t \sigma \right) \partial^\nu \sigma \\ S_\ell^\nu &= \left(-\frac{\partial \Omega_{q\bar{q}}}{\partial \ell} - \frac{2N_c}{g_\mathrm{s}^2} \eta_\ell \partial_t \ell T^2 \right) \partial^\nu \ell \end{aligned}$$

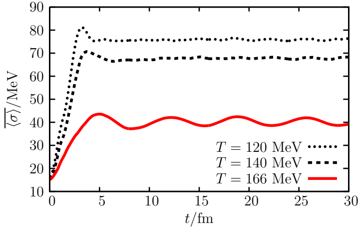
- account for energy-momentum transfer only due to damping
- transfer of stochastic energy is estimated numerically

Box: Relaxation to equilibrium

- both transition scenarios
- initialize a cubic volume with $T > T_c$
- initialize σ, ℓ with their equilibrium values
- quench to $T < T_c$
- initialize energy density and pressure of quark fluid
- let system evolve and relax

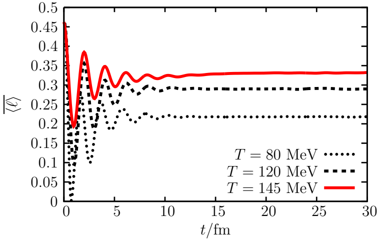
Box: Relaxation to equilibrium

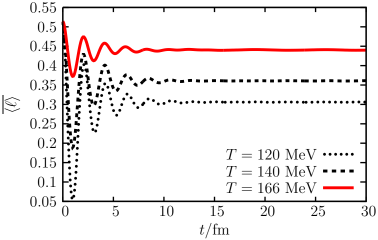




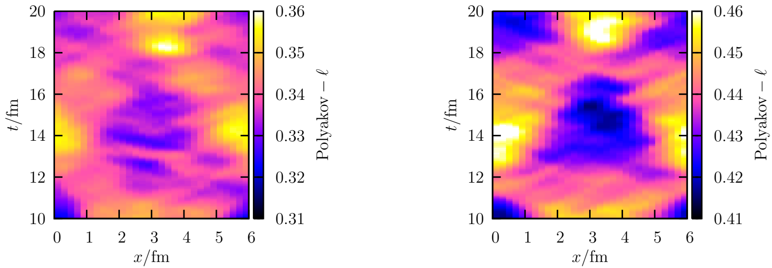
first order phase transition

critical point





Box: Fluctuations at the critical point

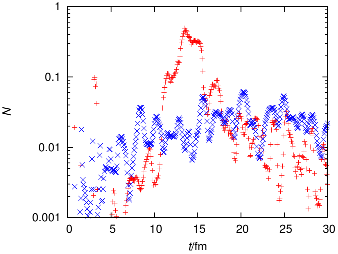


first order phase transition critical point

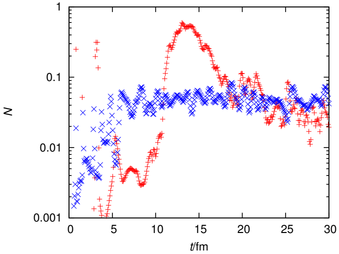
Fluctuations at the critical point are

- stronger
- correlated over long times
- critical slowing down

Box: Fourier analysis of Polyakov loop fluctuations



$0 \leq |k| < 100$ MeV

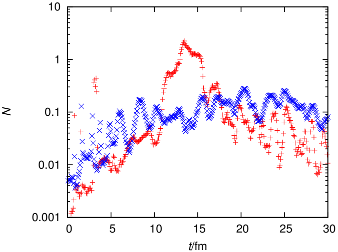


$100 \leq |k| < 200$ MeV

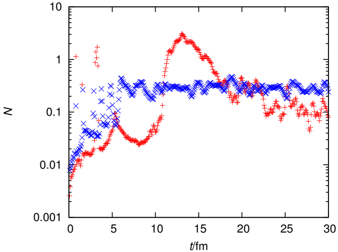
Intensity of Polyakov loop fluctuations:

$$N = \int_{\Delta k} \mathrm{d}^3 k\, N_k = \int_{\Delta k} \mathrm{d}^3 k \frac{a_k^\dagger a_k}{(2\pi)^3 2\omega_k} = \int_{\Delta k} \mathrm{d}^3 k T^2 \frac{\omega_k^2 |\ell_k|^2 + |\dot{\ell}_k|^2}{(2\pi)^3 2\omega_k}$$

Box: Fourier analysis of sigma fluctuations



$0 \leq |k| < 100$ MeV

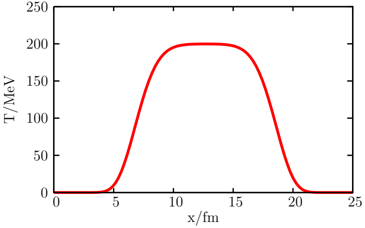


$100 \leq |k| < 200$ MeV

Intensity of sigma fluctuations:

$$N = \int_{\Delta k} \mathrm{d}^3 k \; N_k = \int_{\Delta k} \mathrm{d}^3 k \frac{a_k^\dagger a_k}{(2\pi)^3 2\omega_k} = \int_{\Delta k} \mathrm{d}^3 k \frac{\omega_k^2 |\sigma_k|^2 + |\dot{\sigma}_k|^2}{(2\pi)^3 2\omega_k}$$

The expanding plasma: Initial conditions



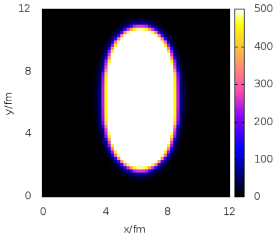
Temperature:
Woods-Saxon distribution

thermal distribution for fields:

$$\sigma = \sigma_{eq}(T) + \delta\sigma(T)$$
$$\ell = \ell_{eq}(T) + \delta\ell(T)$$

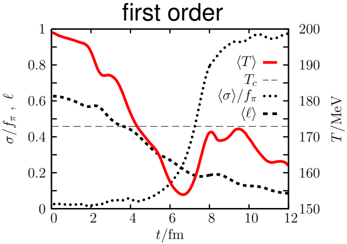
energy density and pressure of fluid:

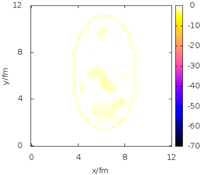
$$e = e(\sigma, \ell, T)$$
$$p = p(\sigma, \ell, T)$$



e/MeVfm^{-3}

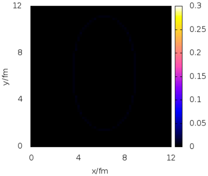
The expanding plasma: First order transition





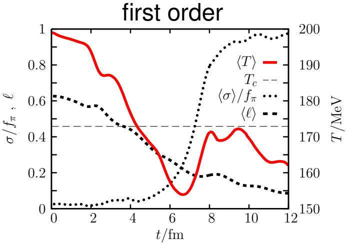
$\sigma - \sigma_{\mathrm{eq}}$

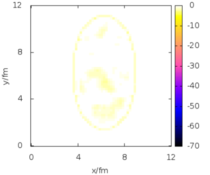
- formation of supercooled phase
- decay after $\sim$ 2 fm
- reheating of the quark fluid



$\ell - \ell_{\mathrm{eq}}$

The expanding plasma: First order transition





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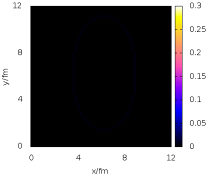
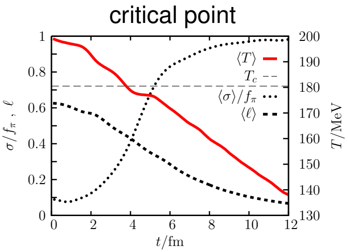
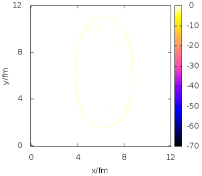


$\ell - \ell_{\mathrm{eq}}$

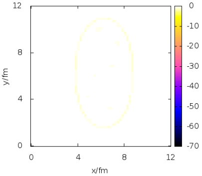
The expanding plasma: Critical point





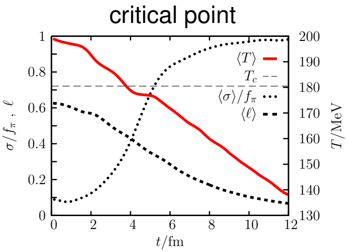
$\sigma - \sigma_{\mathbf{eq}}$

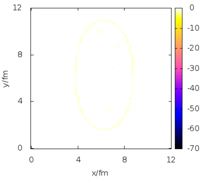
- smooth transition
- saddle point in $\langle T \rangle$ near T_c
- slowing down



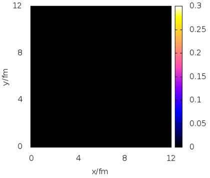
$\ell - \ell_{\mathbf{eq}}$

The expanding plasma: Critical point





$\sigma - \sigma_{\mathrm{eq}}$



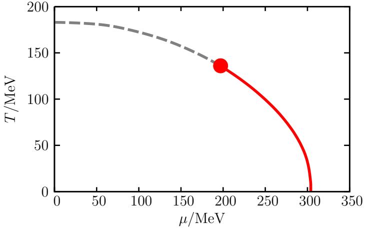
$\ell - \ell_{\mathrm{eq}}$

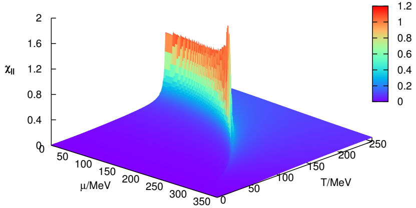
- smooth transition
- saddle point in $\langle T \rangle$ near T_c
- slowing down

Finite density

grand canonical potential at $\mu_B > 0$, $\ell = \bar{\ell}$, mean-field

$$\Omega_{\bar{q}q} = -2N_f T \int \frac{d^3p}{(2\pi)^3} \ln\left[1 + 3\ell e^{-\beta(E-\mu)} + 3\ell e^{-2\beta(E-\mu)} + e^{-3\beta(E-\mu)}\right] + \ln\left[1 + 3\ell e^{-\beta(E+\mu)} + 3\ell e^{-2\beta(E+\mu)} + e^{-3\beta(E+\mu)}\right]$$

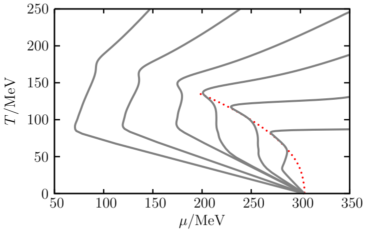




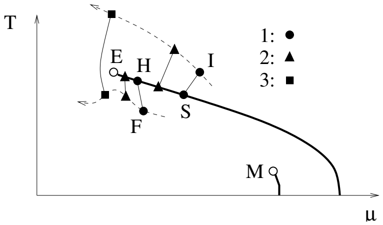
Finite density: Isentropes

Isentropic trajectories in T-μ-plane are determined by

$$\frac{S}{A} = 3\frac{e(T,\mu) + p(T,\mu) - \mu n(T,\mu)}{Tn(T,\mu)}$$

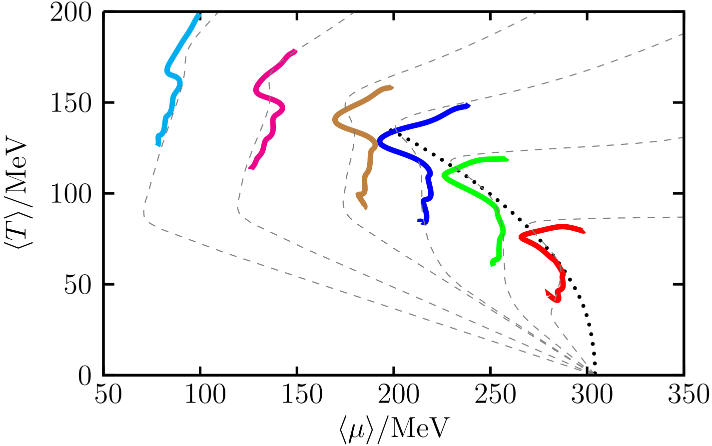


$S/A = 24, 16, 12, 10, 8, 6$



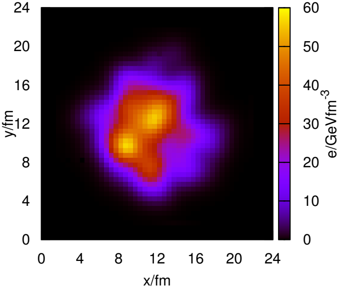
(Stephanov, Rajagopal and Shuryak, PRL **81** (1998))

Finite density: Isentropes



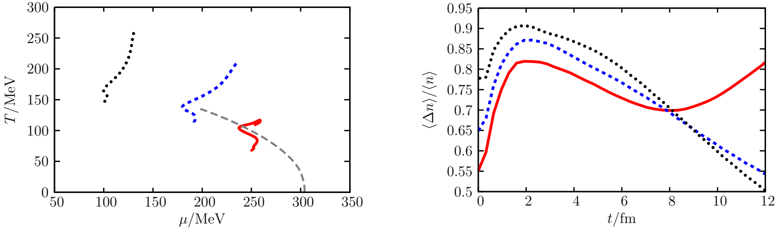
Towards a more realistic description of HIC

- realistic initial conditions
- extracted from UrQMD transport model



- Study evolution of initial fluctuations through the expansion
- Study density inhomogeneities
- Event-by-event fluctuations of characteristic quantities
  - order parameters σ, ℓ
  - baryon density

Finite density: Baryon density fluctuations



- Root mean squared fluctuations $\langle \Delta n \rangle = \sqrt{\langle (n - \langle n \rangle)^2 \rangle}$
- Enhancement of density fluctuations along first order transition line
- Density inhomogeneities are pronounced after crossing the phase transition

Summary

- Chiral fluid dynamics with Polyakov loop
- at zero baryochemical potential:
  - supercooling and reheating
  - critical slowing down
  - out of equilibrium: large fluctuations at first order transition
  - in equilibrium: enhanced fluctuations of soft modes at critical point
- at finite baryochemical potential:
  - trajectories follow isentropes
  - no reheating
  - enhanced density fluctuations at first order transition

Thanks ...

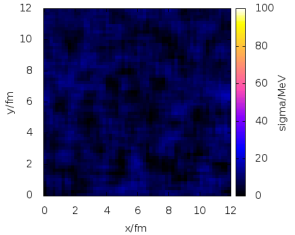
... to the audience and especially to:

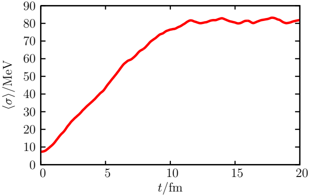
Marcus Bleicher
Carsten Greiner
Igor Mishustin
Stefan Schramm
Marlene Nahrgang
Jan Steinheimer
Thomas Lang

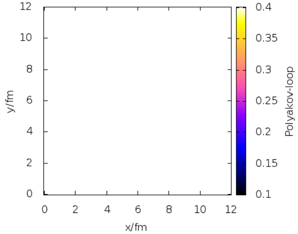
Domain formation at the first order phase transition

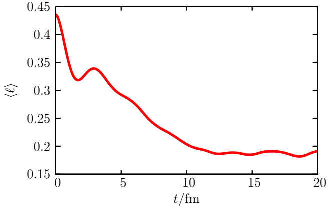
- first order phase transition scenario
- initialize a cubic volume with $T > T_c$
- initialize σ, ℓ with their equilibrium values
- quench to $T < T_c$
- let system evolve, decay through barrier

Domain formation at the first order phase transition









Domain formation at the first order phase transition