Dirty details of an elastic energy loss Monte Carlo model

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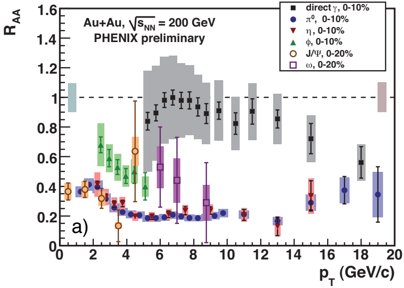
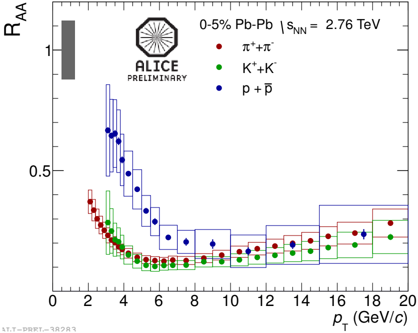
Outline

Suppression of high-energy hadrons

Nuclear modification factor:

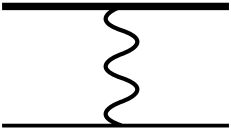
$$R^h_{AA}(\mathbf{b}) = \frac{\frac{d^2N^{AA\to h+X}}{dp_T dy}}{\langle T_{AA}(\mathbf{b})\rangle \frac{d^2\sigma^{pp\to h+X}}{dp_T dy}}$$



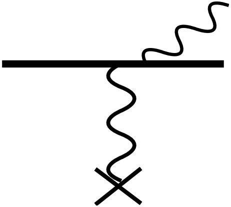


PHENIX figure: J. Phys. G 35, 104045 (2008), ALICE figure: arXiv:1210.6995

Energy loss mechanisms



Elastic: No coherence effects (formation time $\tau_f \approx 0$). All scatterings independent $\rightarrow$ Easy to implement in Monte Carlo.

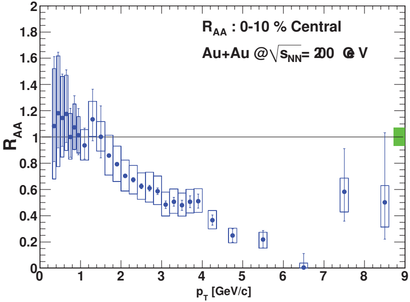


Radiative: Nonzero formation time ($\tau_f \sim \frac{\omega}{k_T^2}$). Successive scatterings not necessarily independent $\rightarrow$ Complicates MC implementation.

Heavy flavor suppression

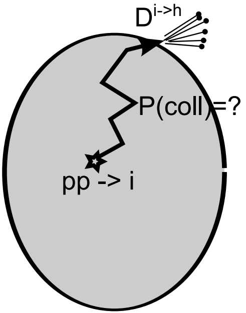
- Radiative energy loss of heavy flavor with mass M suppressed in the "dead cone" $\theta < \frac{M}{E}$.
- Heavy flavors still strongly suppressed $\Rightarrow$ collisional energy loss stronger than expected?

PHYSICAL REVIEW C **84**, 044905 (2011)



PHENIX open heavy-flavor electron R_{AA}.

The Monte Carlo elastic energy loss model



Propagate the high-energy parton through the medium in small time steps Δt.
At each step, calculate the probability to collide with a particle from the medium.

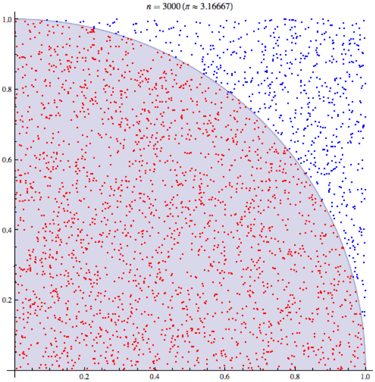
Poisson probability:

$$\boxed{P(\text{1 or more collisions in } \Delta t) = 1 - e^{-\Gamma \Delta t}}$$

where $\Gamma = \Gamma(E_1, T)$ is the scattering rate for the hard parton.

Random sampling

Rejection method:

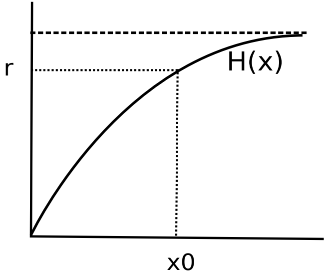


Source: Wikipedia.

Random sampling

The integral function method:
$H(x)$ = the integral function of probablity distribution $p(x)$ of the variable x.

Pick a random number r from the interval $[H(x_{min}), H(x_{max})]$. The sampled value x_0 of the random variable is found by solving the equation $H(x_0) - r = 0$.



Advantage over simple rejection method: No random numbers wasted.
Disadvantage: Calculation of H(x) possibly challenging.

The Monte Carlo simulation: Initialization

Sample the hard parton i with transverse momentum p_T and rapidity y from

$$\frac{d\sigma^{pp\to i+X}}{dp_T^2 dy} = \int dy_1 dy_2 \sum_{(lm)} \frac{d\sigma^{pp\to lm+X}}{dp_T^2 dy_1 dy_2} \cdot [\delta_{li}\delta(y-y_1) + \delta_{mi}\delta(y-y_2)] \frac{1}{1+\delta_{lm}},$$

with $\frac{d\sigma^{pp\to lm+X}}{dp_T^2 dy_1 dy_2} = \sum_{ab} x_1 f_a(x_1, Q^2) x_2 f_b(x_2, Q^2) \frac{d\sigma^{ab\to lm}}{d\hat{t}}$.

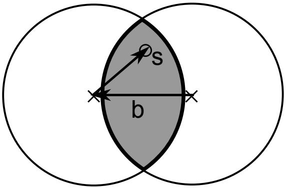
- CTEQ6L1[1] parton distribution functions used, nuclear effects ignored

Integral function of $\frac{d\sigma^{pp\to i+X}}{dp_T^2 dy}$ is calculated over $p_{Tmin} \le p_T \le p_{Tmax}$, with p_{Tmax} increasing in 1 GeV steps up to the limit value $\frac{\hat{s}}{2}$. The corresponding p_T in each momentum bin is taken to be $p_{Tmax} - 0.5$.

[1] D. Stump *et al.*, JHEP 0310, 046 (2003).

Initial position of the hard parton

Nuclear overlap function $T_{AA}(\mathbf{b})$:



$$T_{AA}(\mathbf{b}) = \int d^2\mathbf{s}\, T_A(\mathbf{s}) T_A(\mathbf{b}+\mathbf{s}),$$

$$T_A(\mathbf{s}) = \int_{-\infty}^{\infty} dz\, n_A(\sqrt{\mathbf{s}^2 + z^2}).$$

Nuclear density is given by Woods-Saxon distribution

$$n_A(r) = n_0 (1 + e^{\frac{r - R_A}{d}})^{-1}.$$

Starting position (x_0, y_0) is sampled using the rejection method with tabulated values of $T_A(\mathbf{s})T_A(\mathbf{b}+\mathbf{s})$ on a fine grid with spacing $\sim O(0.01)$ fm.

Scattering rate

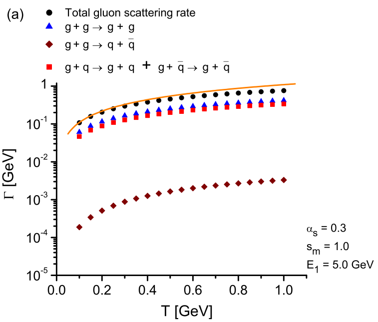
- Scattering rate for a process $ij \to kl$ in the local rest frame of the fluid:

$$\Gamma_{ij\to kl}(E_1,T) = \frac{1}{16\pi^2 E_1^2}\int_{\frac{m^2}{2E_1}}^{\infty} dE_2 f_j(E_2,T)\int_{2m^2}^{4E_1E_2} d\hat{s}[\hat{s}\sigma_{ij\to kl}(\hat{s})].$$

- The regularisation of the cross section: $\sigma_{ij\to kl}(\hat{s}) = \frac{1}{16\pi\hat{s}^2}\int_{-\hat{s}+m^2}^{-m^2} d\hat{t}\,|M|^2_{ij\to kl}$, $m = s_m g_s T = s_m\sqrt{4\pi\alpha_s}T$.
- Kinematic limits (massless particles!): $\hat{s} \geq 2m^2$, $|\cos\theta_{12}| = |1 - \frac{\hat{s}}{2E_1E_2}| \leq 1$.
- Temperature T obtained from the hydrodynamical model.

Free parameters of the model: α_s, s_m.

Scattering rates of gluon and light quark



Producing the thermal particle

The energy E_2 of the plasma particle is sampled using rejection method from Γ, rewritten as:

$$\Gamma_X \sim \int_{\frac{m^2}{2E_1}}^{\infty} dE_2\, f(E_2, T)(H(\frac{4E_1E_2}{m^2}) - H(2)),$$

$H(x) =$ the integral function of $\sigma_X(\hat{s})$, $x = \frac{\hat{s}}{m^2} = \frac{2E_1}{m^2}E_2(1 - \cos\theta_{12})$.
When E_2 is known, $\cos\theta_{12} = 1 - \frac{x_0 m^2}{2E_1E_2}$ can be found using the integral function method.

Scattering angle sampling

The cross section of the process determines the distribution of scattering angle θ_{13}.

- The scattering angle is determined in the CMS frame of the collision.
- The method is very similar to one used with finding the collision angle. The integral function is now $\Sigma_X(\hat{t}) = \int d\hat{t}\, |M|^2_X$ and $\cos\theta_{13} = \frac{2\hat{t}_0}{\hat{s}} + 1$.

The Monte Carlo simulation

- The simulation ends when the temperature of the medium is low enough (≈ 160 MeV).
- Repeat the simulation for several partons, convolute the resulting medium-modified distribution of hard partons with the fragmentation functions[2]:

$$\frac{dN^{AA\to h+X}}{dP_T dy} = \sum_i \int dp_T dy \frac{dN^{AA\to i+X}}{dp_T dy} \int_0^1 dz D_{i\to h}(z, \mu_F^2)\delta(P_T - zp_T).$$

[2] B. A. Kniehl, G. Kramer and B. Potter, Nucl. Phys. B 582, (2000) 514.

Averages versus random sampling

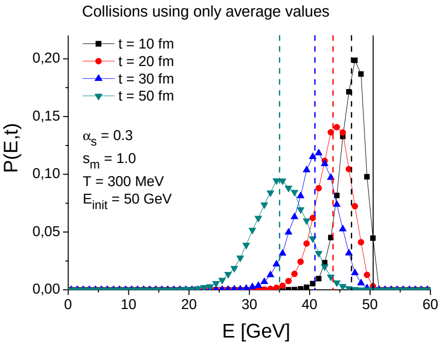
50 GeV quark traveling through a constant-temperature gluon plasma

J. Auvinen and T. Renk, Phys. Rev. C **85**, 037901

Average collision parameters

- $\langle E_2 \rangle = 3T$, $\langle \cos\theta_{12} \rangle = -\frac{1}{3}$
  $\Rightarrow \langle \hat{s} \rangle = 8E_1T$
- $\langle \hat{t} \rangle = \dfrac{\int_{-\hat{s}+m^2}^{-m^2} d\hat{t}\,\hat{t}\frac{d\sigma}{d\hat{t}}}{\int_{-\hat{s}+m^2}^{-m^2} d\hat{t}\frac{d\sigma}{d\hat{t}}}$

Having same parameter values in all collisions produces Gaussian probabilities.



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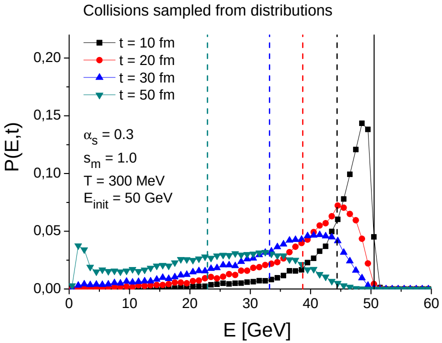
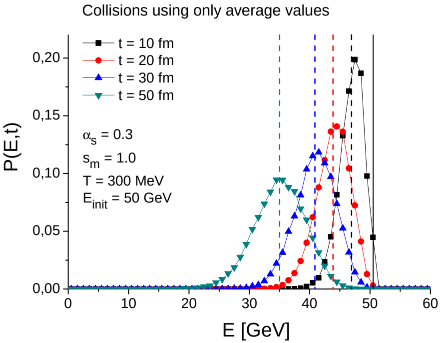
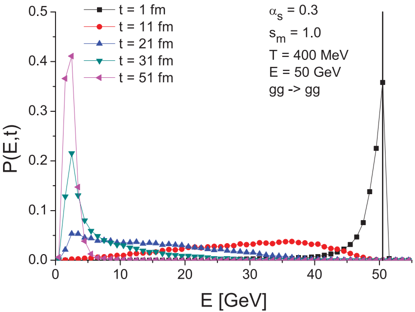


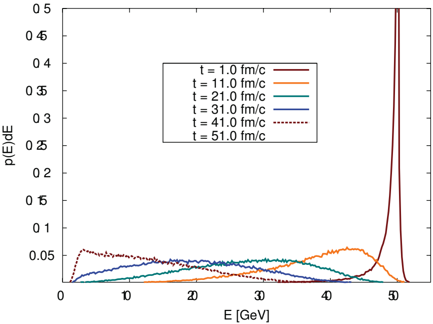


The average values do not give accurate description of the energy loss probability.

Comparing with other models

50-GeV gluon in gluon medium: Qualitative agreement with BAMPS[3], but energy loss about factor 2 stronger.





[3] O. Fochler, Zhe Xu, C. Greiner, Phys. Rev. C 82, 024907 (2010).

Comparing with other models

Zapp et al.[4]: A different choice of regularisation scheme can produce a factor 1.5-2 difference!

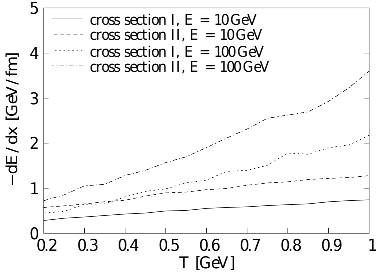


Figure 4. The average parton energy loss dE / dx of a quark of energy E, undergoing multiple elastic collisions over a path length L = 1 fm in a thermal medium of temperature T. Elastic collisions are described by the infra-red regulated partonic cross sections of equation (13) (case I) and equation (14) (case II).

Case I:

$$\sigma = \int_0^{\hat{t}_{max}} d|\hat{t}| \, \frac{\pi\alpha_s(|\hat{t}|+\mu_D^2)}{\hat{s}^2} C_R \frac{\hat{s}^2+(\hat{s}-|\hat{t}|)^2}{(|\hat{t}|+\mu_D^2)^2}$$

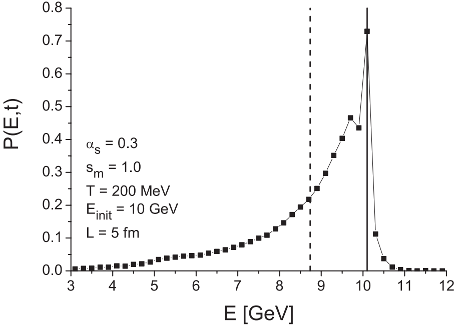
Case II:

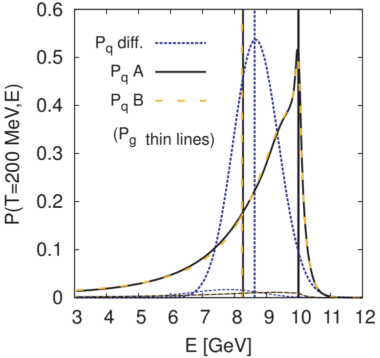
$$\sigma = \int_{\mu_D^2}^{\hat{t}_{max}} d|\hat{t}| \, \frac{\pi\alpha_s(|\hat{t}|)}{\hat{s}^2} C_R \frac{\hat{s}^2+(\hat{s}-|\hat{t}|)^2}{|\hat{t}|^2}$$

[4] K. Zapp, G. Ingelman, J. Rathsman, J. Stachel and U. A. Wiedemann, Eur. Phys. J. C **60**, 617 (2009).

Comparing with other models

10-GeV quark in quark-gluon plasma: Results quite similar with Schenke *et al.*[5]; anomalous 9.75-10.00 GeV bin (ignorance of soft scatterings).

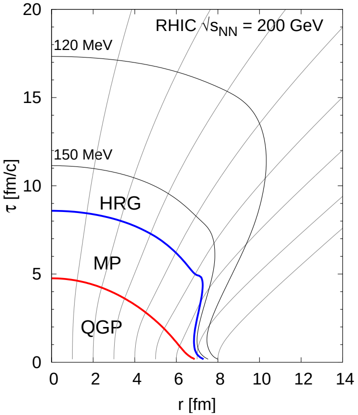




[5] B. Schenke, C. Gale, G.-Y. Qin, Phys. Rev. C 79, 054908 (2009).

Central collisions: Estimating the elastic contribution to energy loss

Hydrodynamical background [6]:



- Initial conditions from the EKRT model Eskola, Kajantie, Ruuskanen and Tuominen, Nucl. Phys. B570 (2000) 379-389.
- Longitudinal boost-invariance, azimuthal symmetry ⇒ (1+1)-dimensional evolution equations.
- Bag model equation of state.

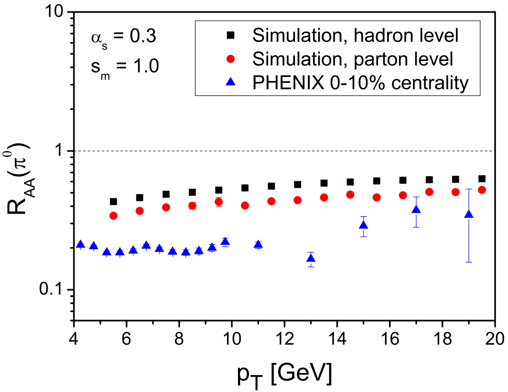
[6] K. J. Eskola, H. Honkanen, H. Niemi, P. V. Ruuskanen and S. S. Rasanen, Phys. Rev. C 72, 044904 (2005).

Mixed phase
(Central collisions, (1+1)-D hydro)

- In the mixed phase, temperature stays constant ($T = T_C$) while energy density ϵ keeps decreasing.
- The scattering rate Γ depends on T but not ϵ $\rightarrow$ How to implement the effect of mixed phase?
- Effective temperature $T_{eff}(R, \tau) = \frac{30}{g_Q \pi^2}(\epsilon(R, \tau) - B)^{1/4}$ with bag constant $B = (239\ \text{MeV})^4$.
- The simulation ends when the boundary of mixed phase and hadron gas phase is reached (i.e. when $T < T_C$).

Central collisions

The nuclear modification factor R_{AA} for π^0:

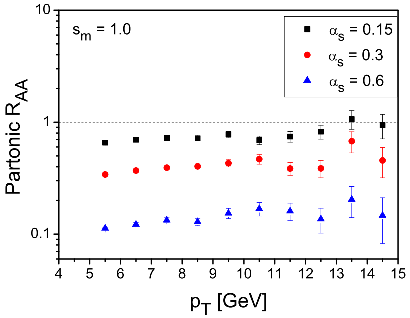


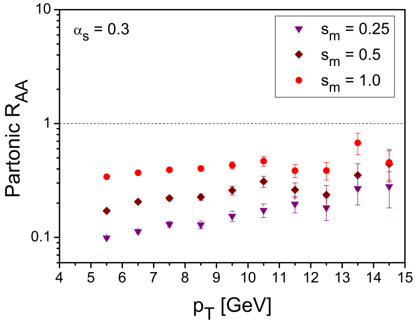
PHENIX data from A. Adare *et al.* [PHENIX Collaboration], Phys. Rev. Lett. 101, 232301 (2008).

J. Auvinen, K. J. Eskola and T. Renk, Phys. Rev. C 82, 024906 (2010).

Central collisions

Sensitivity to the parameter values:

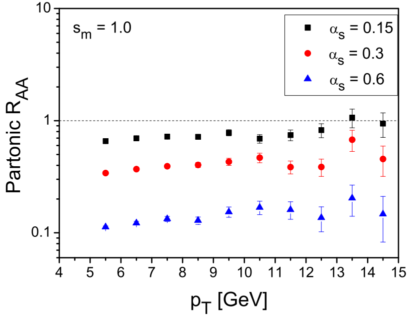


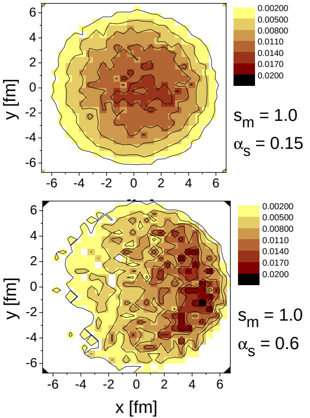


J. Auvinen, K. J. Eskola and T. Renk, Phys. Rev. C 82, 024906 (2010).

Central collisions

Stronger suppression leads to surface bias:

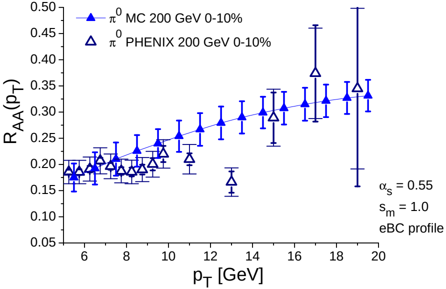


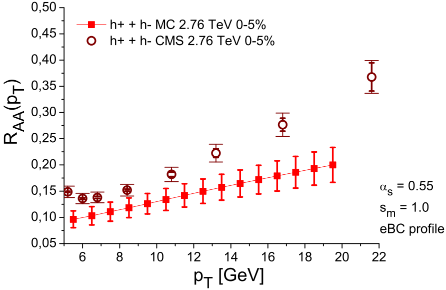


J. Auvinen, K. J. Eskola and T. Renk, Phys. Rev. C 82, 024906 (2010).

From RHIC to LHC

Running coupling needed?





J. Auvinen, K. J. Eskola, H. Holopainen and T. Renk, J. Phys. G **38**, 124160 (2011).

(2+1)-d hydro with eBC profile by H. Holopainen, see Phys. Rev. C **84**, 014906 (2011).

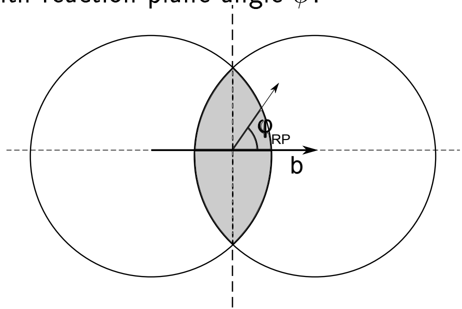
RHIC data from A. Adare *et al.* [PHENIX Collaboration], Phys. Rev. Lett. **101**, 232301 (2008).

LHC data from S. Chatrchyan *et al.* [CMS Collaboration], Eur. Phys. J. C **72**, 1945 (2012).

Non-central collisions: The pathlength dependence of energy loss

The π^0 nuclear modification as a function of the reaction plane angle $\Delta\phi_{RP}$:

- More matter in out-of-plane direction than in-plane
- R_{AA} varies with reaction plane angle ϕ?



Non-central collisions

Hydrodynamical background [7]:

- The smooth sWN profile[8] is used as an initial state.
- Assuming longitudinal boost-invariance reduces the hydrodynamical evolution equations into (2+1) dimensions.
- Equation of state by Laine and Schröder[9] (no separate QGP and mixed phases).
- Centrality classes defined using the optical Glauber model.

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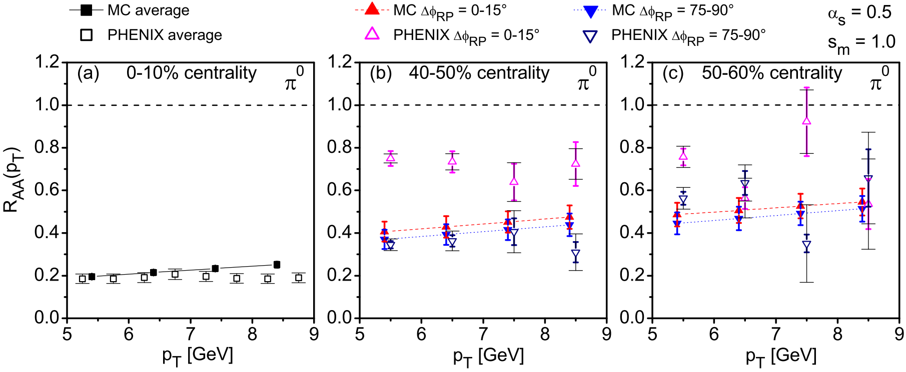
[7] H. Holopainen, H. Niemi and K. J. Eskola, Phys. Rev. C **83**, 034901 (2011).

[8] P. F. Kolb, U. W. Heinz, P. Huovinen, K. J. Eskola and K. Tuominen, Nucl. Phys. A **696**, 197 (2001).

[9] Phys. Rev. D **73**, 085009 (2006).

Non-central collisions

The π^0 nuclear modification as a function of the reaction plane angle $\Delta\phi_{RP}$:



J. Auvinen, K. J. Eskola, H. Holopainen and T. Renk, Phys. Rev. C 82, 051901 (2010).

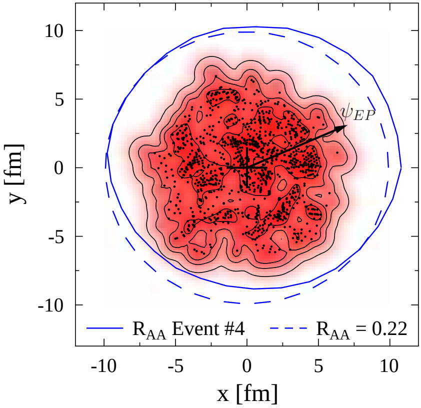
$R_{AA}(\phi_{RP})$ data from S. Afanasiev *et al.* [PHENIX Collaboration], Phys. Rev. C 80, 054907 (2009).

The effect of initial state density fluctuations

Event-by-event hydro calculations with fluctuating initial state:
H. Holopainen, H. Niemi and K. J. Eskola,

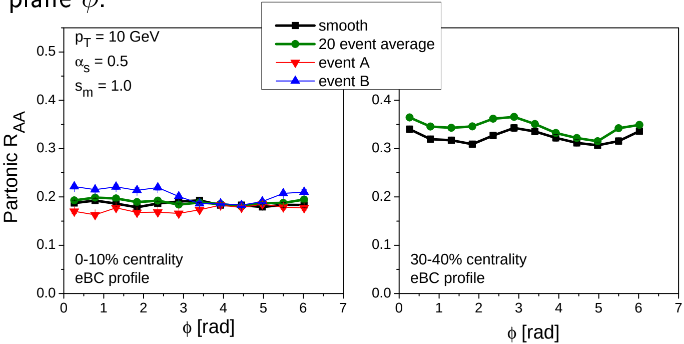
Phys. Rev. C 83, 034901 (2011).

- The eBC profile is used as an initial state.
- Centrality classes defined using the Monte Carlo Glauber.



Initial state fluctuations

R_{AA} at $p_T = 10$ GeV as a function of the angle of outgoing partons with the event plane ϕ:



- The initial state fluctuations average out in central collisions, no re-tuning of α_s required.
- In non-central collisions, the average over fluctuations gives slightly less suppression compared to smooth hydro.

T. Renk, H. Holopainen, J. Auvinen and K. J. Eskola, Phys. Rev. C **85**, 044915 (2012).

Conclusions & Outlook

Conclusions:

- The Monte Carlo model presented here agrees qualitatively with other similar models, differences due to regularisation ⇒ Supports $\hat{t}$-channel dominance assumption.
- The measured dependencies of R_{AA} on the reaction plane angle, centrality or collision energy are not matched with the same parameter values ⇒ Large elastic component of parton energy loss ruled out?
- The initial state fluctuations average out in central collisions; small effect on the nuclear modification in non-central case.

Currently under investigation: Running coupling, heavy quarks.

To be implemented: In-medium jets, dihadron correlations, coherence effects & radiative energy loss.