The Baryon Diffusion Constant of a Hot Hadron gas - a Boltzmann Approach

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Diffusion

- Stochastic process clearing out inhomogenities in particle-number densities
- Maximizes entropy

Fick's first law for dilute fluids

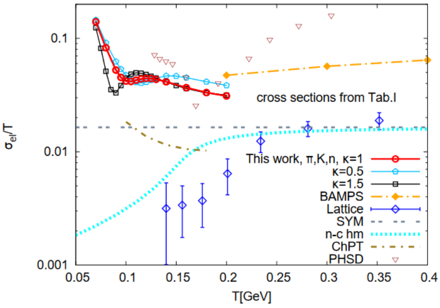
$$\vec{j}(\vec{x}, t) = -D\vec{\nabla} n(\vec{x}, t)$$

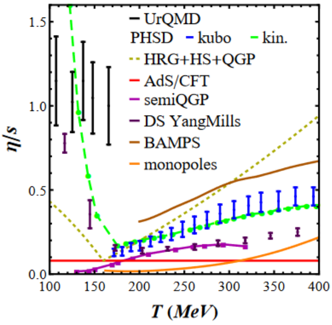
Adolf Fick 1897

Fick, Annalen der Physik, vol. 170, no. 1 (1855), pp. 59-86
Einstein, Annalen der Physik, vol. 322, no. 8 (1905), pp. 549-560
figure: *https : //de.wikipedia.org/wiki/Adolf_Fick*

Transport Coefficients

Diffusion currents are **macroscopic observables**
Diffusion constant is a **transport coefficient**
Transport coefficients take their extremum at phase transitions:



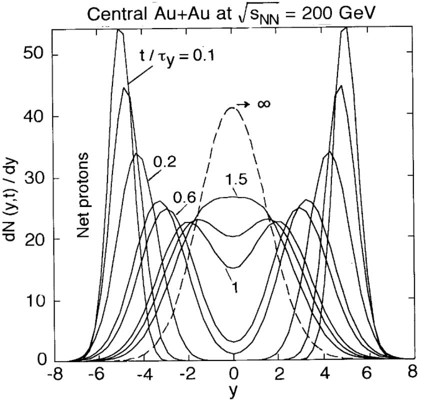


Greif, Greiner, Denicol Phys. Rev. D **93** (2013) for electric conductivity

Noronha-Hostler, axiv: 1512.06315 (2015) for shear viscosity

Rapidity Diffusion

In heavy-ion collisions: baryon stopping / diffusion of baryons to mid-rapidity



Wolschin, Phys. Rev., C **69** (2004), 024906

Method and goal

Calculating the **baryon** diffusion constant of a hot hadron gas nearby local equilibrium (**no** heavy-ion collisions)

Using **linear response** method proposed in (among others):

Denicol, Noronha, Niemi, Rischke Phys. Rev. D **83** (2011)
Denicol, Gale, Jeon, Noronha Phys. Rev. C **88** (2013)
main reference: Greif, Greiner, Denicol Phys. Rev. D **93** (2016)

Investigating the response of a small volume of gas due to a small gradient in baryo-chemical potential
$\Rightarrow$ **neglecting non-linearities**

Assumptions

- massive nucleon-pion-kaon gas with either constant or simplified **GiBUU**[1] **isotropic** cross sections
- **classical** particles $\Rightarrow$ Maxwell-Jüttner distribution
- dilute gas with **elastic** $2 \leftrightarrow 2$ particle collisions $\Rightarrow$ **Boltzmann approach**
- **small deviations** from (local) equilibrium and **small, constant gradients** in baryo-chemical potential $\Rightarrow$ **linear response**

[1] Greif, Greiner, Denicol Phys. Rev. D **93** (2016)
Buss, Gaitanos, Gallmeister, Hees et al. Phys. Rept. **512**, 1 (2012)

Relativistic kinetic theory

Relativistic Boltzmann equation for multi-component system

$$p_i^\mu \frac{\partial f_p^i}{\partial x^\mu} = \hat{C}(p_i)[f_p^i]$$

Small deviations in equilibrium distribution:

$$f_p^i = f(x_i, p_i) = f_{0p}^i + \delta f_p^i(x_i)$$

Maxwell-Jüttner distribution for classical particles:

$$f_{0p}^i = \exp\left[-\beta\left(u_\mu p_i^\mu - \mu_i\right)\right]$$

linearized Collision term (higher orders in deviation δf neglected):

$$\hat{C}(p_i)[f_p^i] = \sum_{j=1}^{N_s} \int \frac{\mathrm{d}^3 p_j'}{(2\pi)^3 E_{jp'}} \frac{\mathrm{d}^3 k_i}{(2\pi)^3 E_{ik}} \frac{\mathrm{d}^3 k_j'}{(2\pi)^3 E_{jk'}} \; \gamma_{ij} s \sigma_{ij}(s,\theta)(2\pi)^6$$

$$\times\, \delta^{(4)}\left(k_i + k_j' - p_i - p_j'\right) f_{0p}^i f_{0p'}^j \left[\frac{\delta f_k^i}{f_{0k}^i} + \frac{\delta f_{k'}^j}{f_{0k'}^j} - \frac{\delta f_p^i}{f_{0p}^i} - \frac{\delta f_{p'}^j}{f_{0p'}^j} \right]$$

$\sigma_{ij}(s,\theta)$: Cross section of interaction between i-th and j-th particle species

N_S: number of particle species

Method

General particle-number current:

$$N^\mu = \underbrace{n_0 u^\mu}_{\text{equilibrium}} + \underbrace{j^\mu}_{\text{non-equilibrium currents}}$$

Assumptions

- only **gradients in baryo-chemical potential** μ_0
- fluid velocity u^μ = velocity energy current (**Landau frame**) $\Rightarrow j^\mu$ = particle diffusion current
- fluid neither accelerating nor expanding (gradients in u^μ vanish)

Boltzmann equation with source term:

$$p^\mu \partial_\mu \delta f_p^i + \underbrace{\lambda_i \beta_0 f_{0p}^i p_i^\nu \nabla_\nu \mu_0}_{\text{source term: external "force field"}} = \hat{C}(p_i)[f_p^i]$$

Fourier-transformed Boltzmann equation gives

$$\delta \tilde{f}_p^i = B^\nu \widetilde{\nabla_\nu \mu_0}$$

B^ν fulfills Boltzmann equation divided by gradient in μ_0. It is space-like and only dependent on energy and momentum.

Ansatz: power series in energy

$$B^\nu(p_i) = f_{0p}^i \Delta_\mu^\nu p_i^\mu \sum_{n=0}^{\infty} a_n^{(i)} E_{ip}^n \quad , \quad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

Diffusion coefficient D:

$$D \propto \sum_{i=1}^{N_s} \lambda_i \int \mathrm{d}P_i \Delta_{\mu\nu} p_i^\mu B^\nu$$

Goal

To calculate the coefficients $a_n^{(i)}$

Calculations

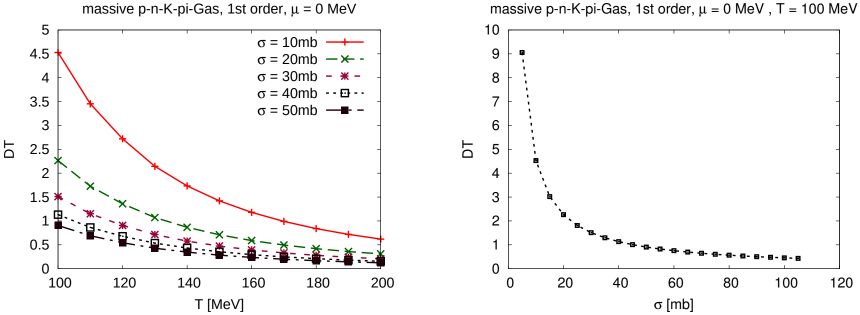
Calculating the baryon diffusion constant for
hot ($T = 80 - 250$ MeV) nucleon-pion-kaon gas

massive

assuming in general finite baryo-chemical potential and isotropic simplified GiBUU cross sections: Δ,ρ and K^* resonances with Breit-Wigner shapes, all other: mean value
(taken from **Greif et al. Phys. Rev. D 93 (2013)**)

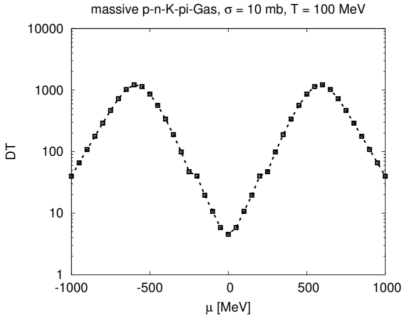
Calculations done with **MATHEMATICA**

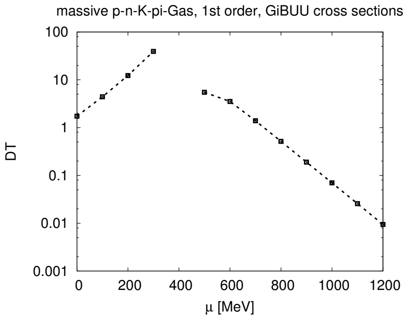
Cross sections and temperature



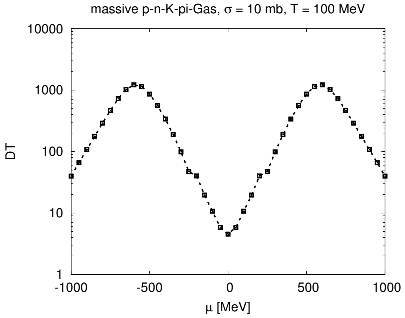
- stronger interactions lead to smaller diffusion constants
- increase in temperature ⇒ increase in total density of gas ⇒ decreasing diffusion constant

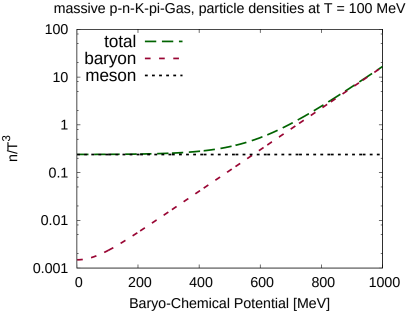
Chemical potential and numerical issues





- Ratio of baryon and meson density also determines behavior
- Numerical issues for finite baryo-chemical potential


massive p-n-K-pi-Gas, σ = 10 mb, T = 100 MeV
10000
1000
100
10
1
DT
-1000
-500
0
500
1000
μ [MeV]

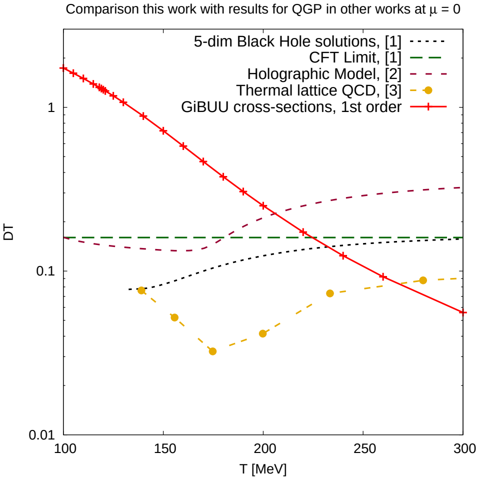

massive p-n-K-pi-Gas, particle densities at T = 100 MeV
100
10
1
0.1
0.01
0.001
n/T3
total
baryon
meson
0
200
400
600
800
1000
Baryo-Chemical Potential [MeV]

Comparison to diffusion in QGP

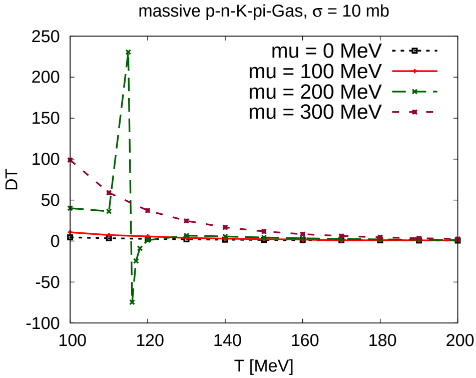
[1]: Rougemont, Noronha et al. Phys. Rev. Lett. **115**, no.20, 202301 (2015)

[2]: DeWolfe, Gubser et al. Phys. Rev. D **84**, 126014 (2011)

[3]: Aarts, Allton, Amato et al. , Gubser et al. JHEP **02**, 186 (2015)



Issues



numerical issues at $T = 110 - 120$ MeV (low precision?)

Conclusion and Outlook

Summary and conclusion

- Baryon diffusion: macroscopic observable of hadron gases related to rapidity diffusion and phase transition
- Linear response method in kinetic theory as proposed by Denicol, Rischke, Niemi, Greif et al. to calculate diffusion constant of nucleon-pion-kaon gas
- Diffusion constant is sensitive to total number-density, baryon- and meson-number-density ratios and baryon-number-density of hadron gas
- Approaches comparable values at critical temperature
- However: numerical issues at finite chemical potential

Conclusion and Outlook

Outlook

- Baryon Diffusion processes in heavy-ion collisions: e.g. how is rapidity diffusion connected to baryon diffusion?
- Baryon Diffusion for quantum gases
- Using more realistic baryo-chemical potential for calculations