Numerical magneto-hydrodynamics for relativistic nuclear collisions

Transport Meeting 16/11/16

Gabriele Inghirami,
in collaboration with:
L. Del Zanna, A. Beraudo,
M. Haddadi Moghaddam, F. Becattini, M. Bleicher

Frankfurt am Main, November, 16^{th}, 2016

FIAS Frankfurt Institute for Advanced Studies

H-QM Helmholtz Research School Quark Matter Studies

HGS-HIRe for FAIR Helmholtz Graduate School for Hadron and Ion Research

Outline

Outline of the talk:

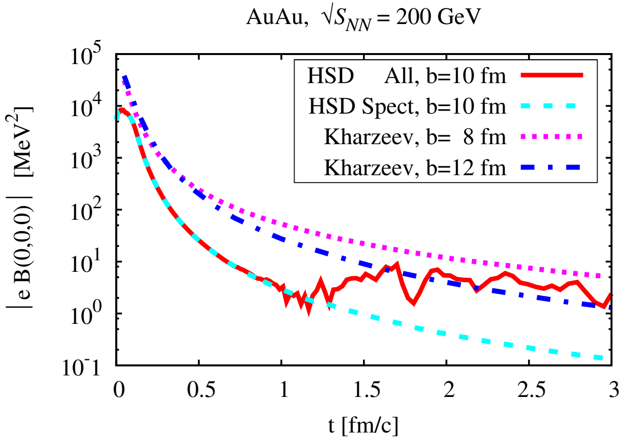
- Motivations
- Estimates of the B field in H.I.C.
- Implementation of ideal magnetohydrodynamics in ECHO-QGP
- Tests to validate the code
- Preliminary application to H.I.C.
- Discussions and conclusions

Why to study magnetic fields in HIC?

Strong magnetic fields may produce many interesting effects:

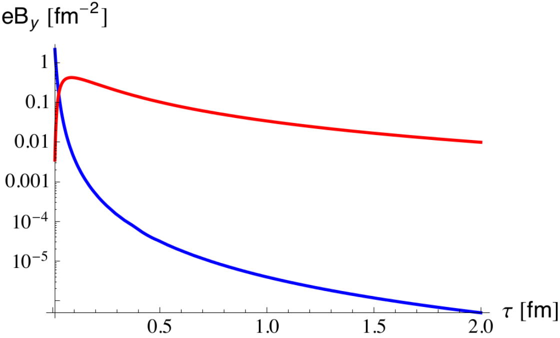
- The Chiral Magnetic Effect
  Kharzeev, McLerran, Warringa - Nuclear Physics A 803 (2008)
- Pressure anisotropy in QGP
  Bali, Bruckmann, Endrődi et al. - Journal of High Energy Physics 08 177 (2014)
- A shift in meson masses
  Andersen - Phys. Rev. D 86, 025020 (2012), Luschevskaya and Larina - JETP Letters 98 (2014)
- Mass shifts in quarkonia states
  Suzuki and Yoshida - arXiv: 1601.02178
- Shift of the Critical Temperature
  Bali, Bruckmann, Endrődi et al. - Journal of High Energy Physics 02 044 (2012)
- Influence on the elliptic flow
  Bali, Bruckmann, Endrődi and Schäfer - Phys. Rev. Lett. 112 (2014)
  Pang, Endrődi and Petersen - arXiv: 1602.06176v1
- Influence on directed flow
  Gürsoy, Kharzeev and Rajagopal - Phys. Rev. C 89 (2014)

How large are magnetic fields in HIC?



Voronyuk et al. - Phys. Rev. C 83 (2011)

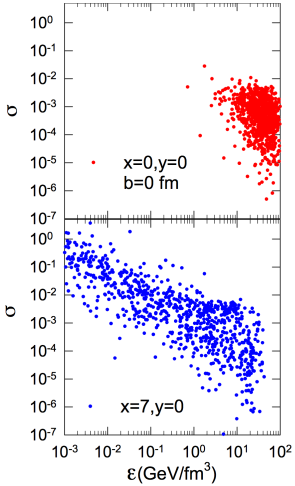
But how long do they stay so strong?

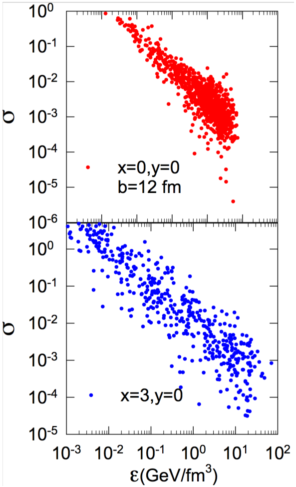


The medium plays a crucial role:
Blue line: $\sigma = 0.\,\mathrm{fm}^{-1}$
Red line: $\sigma = 0.023\,\mathrm{fm}^{-1}$

Gürsoy, Kharzeev and Rajagopal - Phys. Rev. C 89, 054905 (2014)

Estimates by Roy and Pu

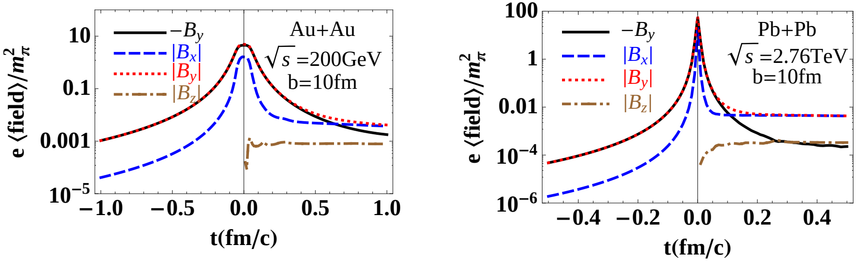




$\sigma(x,y,\vec{b}) = \frac{B^2(x,y,\vec{b})}{2\varepsilon(x,y,\vec{b})}$, Au-Au collision at $\sqrt{s}_{\mathrm{NN}}$=200 GeV, Glauber-M.C.

Plots taken from: Roy, Pu - Phys. Rev. C 92 (2015)

Estimates by Deng and Huang



Based on HIJING, a Monte-Carlo event generator, to model the A+A collisions and on Li´enard-Wiechert potentials to compute the electromagnetic field.

Plots taken from: Deng, Huang - Phys. Rev. C 85, 044907 (2012)

Our initial conditions: basic formula for point charge

Reference article: Tuchin, Phys. Rev. C 88 (2013)
For an observer at $\mathbf{r} = z\hat{\boldsymbol{z}} + \mathbf{b}$, $(\mathbf{b} \cdot \hat{\boldsymbol{z}} = 0)$, if $\gamma = 1/\sqrt{1-v^2} \gg 1$:

$$\mathbf{H}(t, \boldsymbol{b}r) = H(t, \mathbf{r})\hat{\boldsymbol{\phi}} =$$

$$\frac{e}{2\pi\sigma}\hat{\boldsymbol{\phi}} \int_0^\infty \frac{J_1(k_\perp b)k_\perp^2}{\sqrt{1+\frac{4k_\perp^2}{\gamma^2\sigma^2}}} \exp\left\{\frac{1}{2}\sigma\gamma^2 x_- \left(1 - \sqrt{1+\frac{4k_\perp^2}{\gamma^2\sigma^2}}\right)\right\} dk_\perp \tag{1}$$

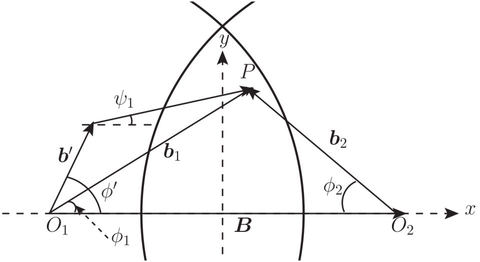
where $x_- = t - z/v$ and $\hat{\boldsymbol{\phi}}$ is the unit vector of the angular polar coordinates in the transverse plane x, y.
Electrical conductivity σ is constant. Ohm law simply: $\vec{J} = \sigma\vec{E}$.
We model the nuclei as uniformly charged spheres which freely propagate into a medium, before and after the collision.

Our initial conditions: formula for two colliding nuclei

Assuming that the nucleus is a sphere uniformly charged:

$$\boldsymbol{b}H_Z(x_-,\boldsymbol{b}b_1) = \int 2\sqrt{R_A^2 - b'^2}\rho\, H(x_-,|\boldsymbol{b}b_1-\boldsymbol{b}b'|)(-\sin\psi_1\hat{\boldsymbol{x}}+\cos\psi_1\hat{\boldsymbol{y}}) \quad (2)$$

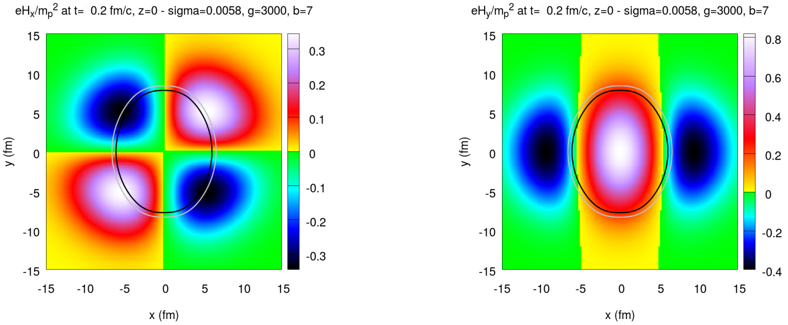
A similar expression holds for the other nucleus and the total magnetic field is the sum of the two.



Tuchin - Phys. Rev. C 88 (2013)

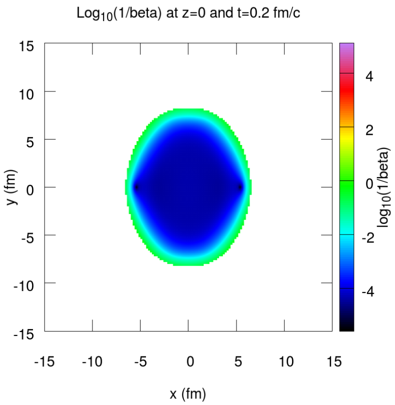
Estimates with Geometrical Glauber initial conditions

Collision at $\sqrt{s_{\mathrm{NN}}}$=5.5 TeV, b=7 fm:



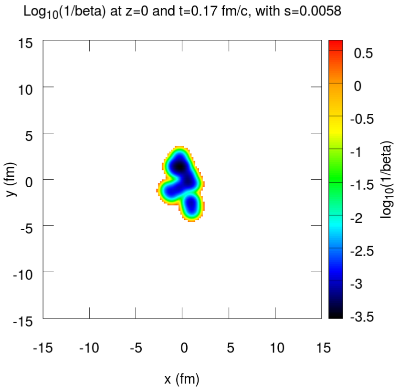
Magnetic permeability $\mu \sim 1$
Black line: $e = 1\mathrm{GeV/fm}^3$, gray line: $e = 150\mathrm{MeV/fm}^3 \sim 140\mathrm{MeV}$.

How do magnetic fields compare with thermal pressure?



$\log_{10}\beta^{-1}$, where $\beta = 2\,p/B^2$, at at $\sqrt{s}_{\rm NN}$=5.5 TeV, b=7 fm

Our estimates using Glauber-Monte Carlo initial conditions



$\log_{10}\beta^{-1}$, where $\beta = 2\,p/B^2$, at at $\sqrt{s}_{\rm NN}$=200 GeV

See: Holopainen, Niemi, and Eskola, Phys. Rev. C 83 (2011)

Alternative equations - 1

Li, Sheng and Wang, Phys. Rev. C 94, 044903 (2016)

$$
\begin{aligned}
B_\phi(t,\mathbf{x}) &= \frac{Q}{4\pi}\cdot\frac{v\gamma x_T}{\Delta^{3/2}}\left(1+\frac{\sigma v\gamma}{2}\sqrt{\Delta}\right)e^{A},\\
B_r(t,\mathbf{x}) &= -\sigma_\chi\frac{Q}{8\pi}\cdot\frac{v\gamma^2 x_T}{\Delta^{3/2}}\cdot\left[\gamma(vt-z)+A\sqrt{\Delta}\right]e^{A},\\
B_z(t,\mathbf{x}) &= \sigma_\chi\frac{Q}{8\pi}\cdot\frac{v\gamma}{\Delta^{3/2}}\cdot\\
&\quad\left[\gamma^2(vt-z)^2\left(1+\frac{\sigma v\gamma}{2}\sqrt{\Delta}\right)+\Delta\left(1-\frac{\sigma v\gamma}{2}\sqrt{\Delta}\right)\right]e^{A}
\end{aligned}
$$

where: σ is the electric conductivity, σ_χ the chiral magnetic conductivity, $\Delta\equiv\gamma^2(vt-z)^2+x_T^2$, $A\equiv(\sigma v\gamma/2)[\gamma(vt-z)-\sqrt{\Delta}]$

What is ECHO-QGP

ECHO-QGP derives from the Eulerian Conservative High-Order astrophysical code for general relativistic magnetohydrodynamics, developed by L. Del Zanna.

(Del Zanna, Zanotti, Bucciantini, and Londrillo, A&A 473 (2007))

A collaboration lead by F. Becattini adapted ECHO-QGP to run second order dissipative hydrodynamical simulations of heavy ion collisions, including the computation of particle spectra following the Cooper-Frye prescription.

Del Zanna, Chandra, Inghirami, Rolando, Beraudo, De Pace, Pagliara, Drago, and Becattini, Eur.Phys.J. C73 (2013)

Floerchinger, Wiedemann, Beraudo, Del Zanna, Inghirami, Rolando, PLB 735 (2014)

Becattini, Inghirami, Rolando, Beraudo, Del Zanna, De Pace, Nardi, Pagliara, Chandra, Eur. Phys. J. C 75 (2015)

Inghirami, Del Zanna, Beraudo, Haddadi, Becattini, Bleicher, arXiv:1609.03042 - Accepted by Eur. J. Phys. C on November, 14th - 2016

Website: http://theory.fi.infn.it/echoqgp/

The basis

The fundamental equations

Energy and momentum conservation: $d_\mu T^{\mu\nu} = 0$
Baryonic number conservation: $d_\mu N^\mu = 0$
Second law of thermodynamics: $d_\mu s^\mu \geq 0$
Maxwell equations: $d_\mu F^{\mu\nu} = -J^\nu \quad (d_\mu J^\mu = 0) \qquad d_\mu F^{\star\mu\nu} = 0$

The fundamental assumptions

- We neglect all dissipative effects
- We neglect polarization and magnetization effects
- We assume infinite electrical conductivity
- We assume local thermal equilibrium

The ideal RHMD energy-momentum tensor

Polarization and magnetization neglected

$T_f^{\mu\nu} = F^{\mu}{}_{\lambda} F^{\nu\lambda} - \frac{1}{4}(F^{\lambda\kappa} F_{\lambda\kappa}) g^{\mu\nu}$

from Maxwell equations: $d_\mu T_{\mathrm{f}}^{\mu\nu} = J_\mu F^{\mu\nu}$

Dissipative effects neglected:

Eckart frame = Landau frame ⇒ single fluid u^μ $(u_\mu u^\mu = -1)$

Infinite electrical conductivity

Ohm's law: $J^\mu = \rho_{\mathrm{e}} u^\mu + j^\mu; \quad j^\mu = \sigma^{\mu\nu} e_\nu \Rightarrow e^\mu = 0$

Energy-momentum tensor $T^{\mu\nu}$

$T^{\mu\nu} = T_m^{\mu\nu} + T_f^{\mu\nu}$

Matter: $T_m^{\mu\nu} = (e+p) u^\mu u^\nu + p g^{\mu\nu}$

Electromagnetic field: $T_f^{\mu\nu} = b^2 u^\mu u^\nu + \frac{1}{2} b^2 g^{\mu\nu} - b^\mu b^\nu$

The energy momentum tensor components

Lorentz transformations from the laboratory to the comoving frame:
$e^\mu = (\gamma v_k E^k, \gamma E^i + \gamma \varepsilon^{ijk} v_j B_k)$
$b^\mu = (\gamma v_k B^k, \gamma B^i - \gamma \varepsilon^{ijk} v_j E_k)$ where:
ε_{ijk} is the Levi-Civita pseudo-tensor of the spatial three-metric
γ = Lorentz factor, $g_{ij} = \mathrm{diag}(1,1,1)$ or $g_{ij} = \mathrm{diag}(1,1,\tau^2))$
e and p are measured in the *comoving fluid frame*,
$\vec{E}$ and $\vec{B}$ are measured in the *laboratory frame*

Components of the energy-momentum tensor

Energy density $\mathcal{E} \equiv -T^0_0 = (e+p)\gamma^2 - p + \frac{1}{2}(E_k E^k + B_k B^k)$
Momentum density $S_i \equiv T^0_i = (e+p)\gamma^2 v_i + \varepsilon_{ijk} E^j B^k$
Stresses $T^i_j = (e+p)\gamma^2 v^i v_j + (p + \frac{1}{2}(E_k E^k + B_k B^k))\delta^i_j - E^i E_j - B^i B_j$

The evolution equations

Ideal Ohm'law in the laboratory frame

$$e^\mu = 0 \Rightarrow E_i = -\varepsilon_{ijk} v^j B^k$$

The evolution equations in conservative form

$$\partial_0 \mathbf{U} + \partial_i \mathbf{F}^i = \mathbf{S}$$

where

$$\mathbf{U} = |g|^{\frac{1}{2}} \begin{pmatrix} \gamma n \\ S_j \equiv T^0_j \\ \mathcal{E} \equiv -T^0_0 \\ B^j \end{pmatrix}, \quad \mathbf{F}^i = |g|^{\frac{1}{2}} \begin{pmatrix} \gamma n v^i \\ T^i_j \\ S^i \equiv -T^i_0 \\ v^i B^j - B^i v^j \end{pmatrix}, \quad \mathbf{S} = |g|^{\frac{1}{2}} \begin{pmatrix} 0 \\ \frac{1}{2} T^{ik} \partial_j g_{ik} \\ -\frac{1}{2} T^{ik} \partial_0 g_{ik} \\ 0 \end{pmatrix}$$

Numerical schemes

Primitive variables: $n, v^x, v^y, v^z, p, B^x, B^y, B^z$, evaluated at the center of the cells.

For each time step we perform:

- conversion from primitive to conservative variables
- reconstruction of primitive variables values at left and right sides of the cells (TVD2, CENO3, WENO3, WENO5, MPE3, MPE5)
- computation of fluxes
- approximate Riemann solver (HLL)
- time integration (RK2)
- conversion from conservative to primitive variables

Enforcement of the solenoidal condition $\partial_i(|g|^{\frac{1}{2}} B^i) = 0$:
Hyperbolic Divergence Cleaning
(See: A. Dedner et al., Journal of Comp. Physics **175** (2002) 645)

The EoS and the retrieving of "primitive" variables.

Relativistic gas Equation of State $p = e/3$.
At the end of each timestep, we know: γn, S_i, $\mathcal{E}$, B_i
and we want to find: n, v_i and p.
We introduce the new variables: $x = v^2 = v_i v^i, \quad y = 4p\gamma^2$
and we compute them by solving:

$$(y + B^2)^2 x - y^{-2}(S_i B^i)^2(2y + B^2) - S^2 = 0,$$

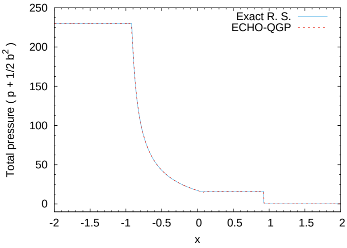
$$\frac{3+x}{4} y + \frac{1}{2}(1+x)B^2 - \frac{1}{2} y^{-2}(S_i B^i)^2 - \mathcal{E} = 0.$$

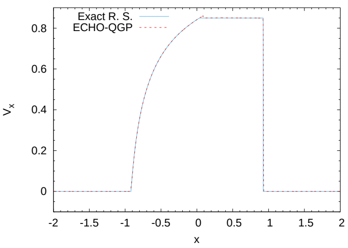
Then we can compute:

$$v^i = \frac{S^i + (S_k B^k) B^i / y}{y + B^2}, \quad n = \frac{\gamma n}{\gamma}, \quad p = \frac{e}{3} = \frac{1}{4}(1-x)y$$

Magnetized shock tube

Comparison with the exact Riemann Solver by Giacomazzo and Rezzolla, EoS $p = (\Gamma - 1)(e - \rho)$
(See: B. Giacomazzo and L. Rezzolla, J. Fluid Mech. **562** (2006) 223)





Left side ($x < 0$)		**Right side** ($x > 0$)	
ρ	1	ρ	0.1
p	30	p	1
B_y	20	B_y	0

The large amplitude CP Alfvén wave test

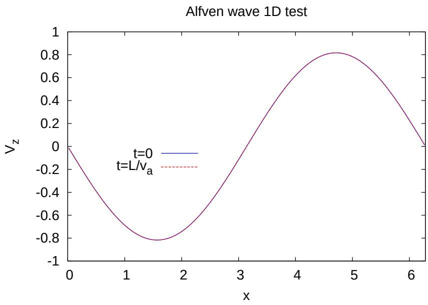


Figure : v_z after one period $t = L/v_a$

ρ and p unaffected by the wave

$B_y = \eta B_0 \cos[k(x - v_A t)]$

$B_z = \eta B_0 \sin[k(x - v_A t)]$

$v_y = -v_A B_y / B_0$

$v_z = -v_A B_z / B_0$

From the momentum equation we get:

$[e+p+(1+\eta^2-\eta^2 v_A^2)B_0^2]v_A^2 = B_0^2$

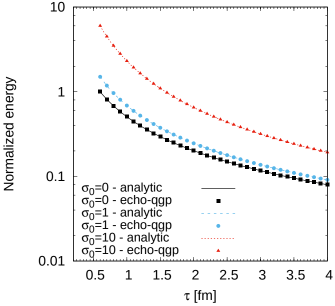
$\Rightarrow v_a^2 = \frac{B_0^2}{e+p+B_0^2(1+\eta^2)} \cdot \left[\frac{1}{2}\left(1+\sqrt{1-\left(\frac{2\eta B_0^2}{e+p+B_0^2(1+\eta^2)}\right)^2}\right)\right]^{-1}$

For the test we took:

$\rho = p = B_0 = \eta = 1, \; \mathrm{L} = 2\pi.$

See: Del Zanna et al. - A& A 473 (2007)

The 1D Bjorken flow test



Evolution of the normalized total energy density $e/e_0 + \frac{1}{2}\sigma_0(B/B_0)^2$ $(\sigma_0 = B_0^2/e_0)$

One-dimensional Bjorken flow:
$u^\mu = \gamma\,(1, 0, 0, v^z)$ $(v^z = z/t)$
Transverse MHD:
$B^\mu = (0, B^x, B^y, 0)$
Milne coordinates: $(\tau, x, y, \eta) \equiv$

$$\left(\sqrt{t^2 - z^2}, x, y, \frac{1}{2}\ln\left(\frac{t+z}{t-z}\right)\right)$$

$\Rightarrow u^\mu = \gamma(1, 0, 0, 0)$
EOS: $p = e/3$
Energy conservation equation:
$\partial_\tau\left(e + \frac{B^2}{2}\right) + \frac{e+p+B^2}{\tau} = 0$
Ideal-MHD limit:
$B(\tau) = B_0\frac{\rho(\tau)}{\rho_0} \Leftrightarrow B_0\frac{s(\tau)}{s_0}$
$\Rightarrow \frac{e(\tau)}{e_0} = \left(\frac{\tau_0}{\tau}\right)^{4/3}$ and $\frac{B(\tau)}{B_0} = \frac{\tau_0}{\tau}$

(See: Roy, Pu, Rezzolla, Rischke, Physics Letters B, 750 (2015))

Self-similar expansion in vacuum

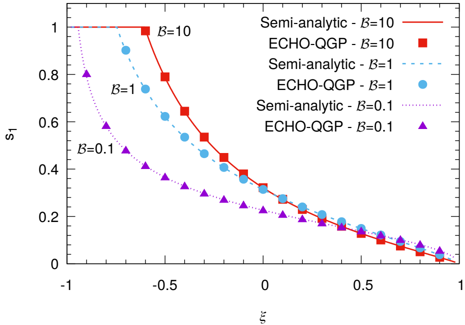


Figure at the left:
$s_1 = s/s_0$ vs $\xi = z/t$ at $t = 20$
Transverse MHD ($\vec{v} \perp \vec{B}$)
Initial pressure:
left side ($z \leq 0$) $p_0 = 1000$
right side ($z > 0$) $p_0 = 5 \cdot 10^{-5}$

$$\mathcal{B} = \frac{2p_0}{B_0^2}, \quad s_1 = \frac{s}{s_0}, \quad \xi = \frac{z}{t}$$

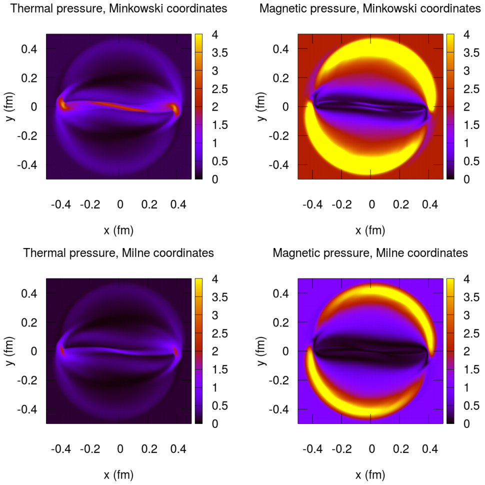
$$\delta_v \equiv \sqrt{\frac{1+v}{1-v}}, \quad \delta_\xi \equiv \sqrt{\frac{1+\xi}{1-\xi}}, \quad \frac{\delta_v^2}{\delta_\xi^2} \equiv f^2(s_1) \equiv \frac{(4\mathcal{B}+3s_1^{2/3}) \pm \sqrt{(4\mathcal{B}+3s_1^{2/3})^2 - 4\mathcal{B}^2}}{2\mathcal{B}}$$

$$\ln \frac{\delta_\xi(s_1)}{\delta_{\xi 0}} = \int_1^{s_1} d\alpha \frac{f(\alpha)(1-f^2(\alpha)) - \alpha f'(\alpha)(1+f^2(\alpha))}{\alpha\, f(\alpha)(1+f^2(\alpha)}$$

$$\delta_{\xi 0} = \sqrt{\frac{1-c_{f,0}}{1+c_{f,0}}}, \quad \text{where} \quad c_{f,0}^2 = \frac{2\mathcal{B}+3}{3(2\mathcal{B}+1)}$$

(See: Lyutikov and Hadden, Physical Review E 85, 026401 (2012))

Rotor test

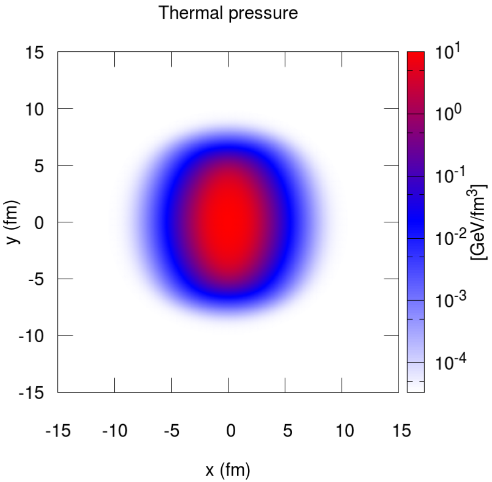


The initial velocity of the fluid is null outside of the disk. Inside the disk its components are: $v^x = \frac{\omega\, y}{r_0}$, $v^y = -\frac{\omega\, x}{r_0}$, $v^z = 0$

Initial conditions for the other variables:

r_0	disk radius	0.1
ω	Rot. param.	0.995
B^x	(everywhere)	2
B^y	(everywhere)	0
B^z	(everywhere)	0
p	th. press. $(r \leq r_0)$	5
p	th. press. $(r > r_0)$	1
t_i	start time	1
t_f	end time	1.4

Initial conditions for pressure/energy density



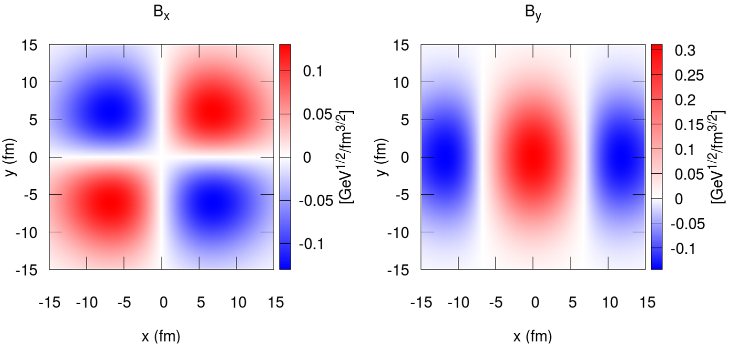
2D+1 simulation in Milne coordinates

Au+Au collision at 200 GeV $\sqrt{s_{NN}}$

Geometrical Glauber initial conditions.

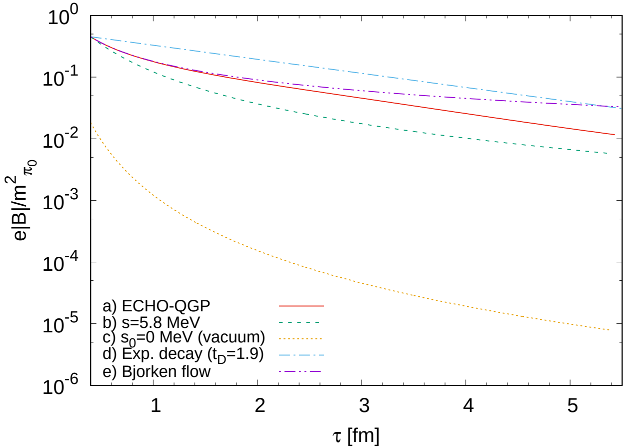
Parameter	Value
b	10 fm
τ_0	0.4 fm/c
$e_{f.o.}$	$150\,\mathrm{MeV/fm^3}$
ϵ_0	$55.\,\mathrm{GeV/fm^3}$
ϵ_{min}	$0.1.\,\mathrm{MeV/fm^3}$
σ_{in}	40 mb
α_H	0.05
EoS	$p = e/3$

Initial conditions for the magnetic field



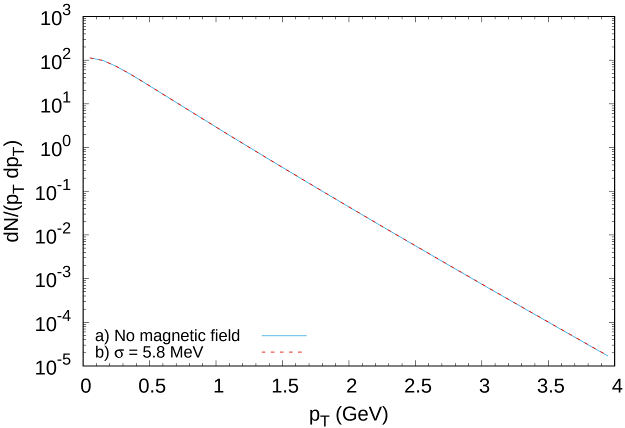
Electromagnetic field computed with Tuchin's model.
Electrical conductivity of the medium ($\tau \leq \tau_0$): $\sigma = 5.8$ MeV, constant. Electrical conductivity of the QGP ($\tau > \tau_0$): $\sigma = \infty$.

Time evolution of the magnetic field at the center of the grid



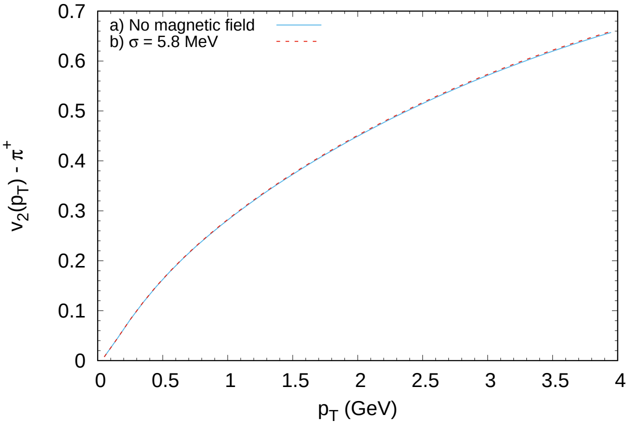
Magnetic field (lab frame) at the center of the grid.

The pion spectra



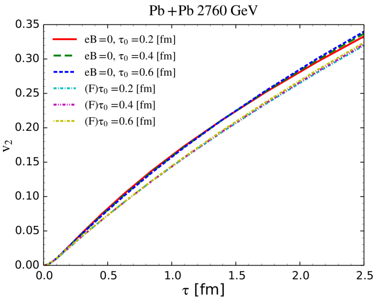
Transverse momentum distribution of π^+, computed with the Cooper-Frye formula.

The pion elliptic flow

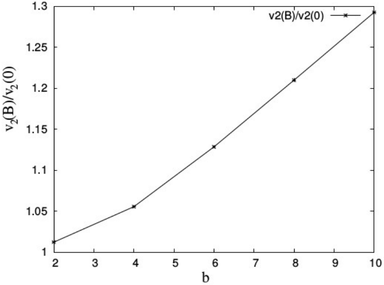


v_2 of π^+, computed with the Cooper-Frye formula.

Results from other groups



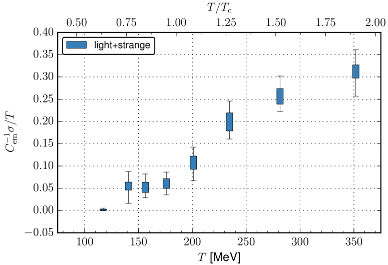
(Plot from: Pang et al.,
Phys. Rev. C 93, 044919 (2016))



(Plot from: Mohapatra et al.,
Mod. Phys. Lett. A26, 2477 (2011))

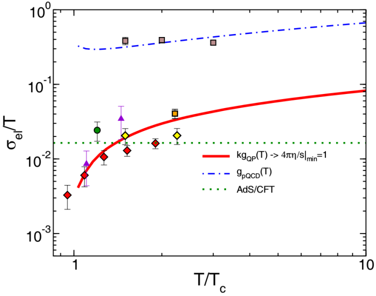
Assuming that magnetic fields may be larger than in our estimates, what is their effect on v_2?

The electrical conductivity of the Quark Gluon Plasma



$(C_{em} = e^2 \sum_f q_f^2)$

(Plot from: Aarts et al, JHEP **1502** (2015) 186)

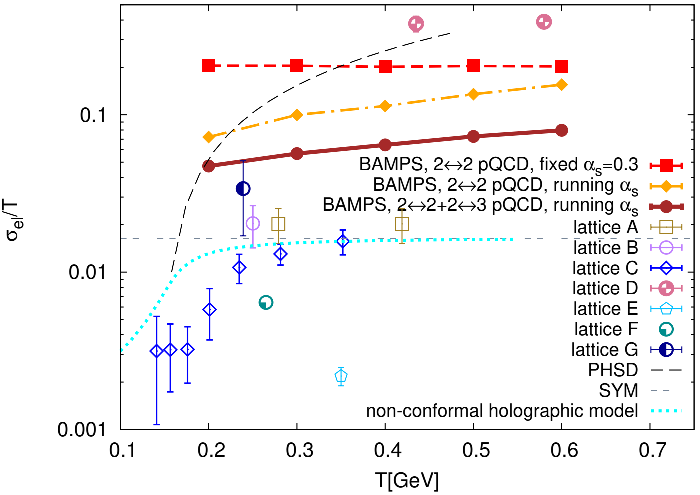


(Plot from: Puglisi et al, EPJ Web of Conferences 117 (2016))

Electrical conductivity is finite and temperature dependent!

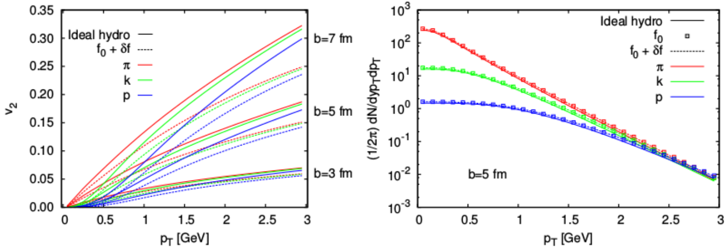
And it is not isotropic! (See: Hattori and Satow, https://arxiv.org/abs/1610.06818)

The electrical conductivity of QGP - BAMPS



(Plots from: Greif, Bouras, Xu and Greiner, Phys. Rev. D 90, 094014 (2014))

And the dissipative effects?...



(Plots from: presentation of V. Rolando at QM 2014)

Resistive RMHD is not implemented in ECHO-QGP, but dissipative hydro is.
If the magnetic fields really influence $v2$,
should we reconsider the effects attributed to the shear viscosity?

Conclusions and future perspectives

- Magnetic fields may produce some relevant effects on several observable quantities
- It is uncertain whether they are strong and persistent enough to produce measurable effects in HIC produced ad RHIC and LHC
- The new version of ECHO-QGP may help to better investigate the influence and the evolution of magnetic fields in the QGP phase
- Future work will involve full 3D+1 simulations with different sets of initial conditions

Thank you!

Conversions

Constants

$\alpha = 0.0072973525664$
$m_{\pi^0} = 139.57018 \mathrm{MeV}$
$m^2_{\pi^0} = 0.01932 \mathrm{MeV}^2$
$4\pi\alpha = e^2 \;\Rightarrow\; e = \sqrt{4\pi\alpha} = 0.30282212$
$\hbar c = 0.197326\,\mathrm{GeV fm}$
$(\hbar c)^{\frac{3}{2}} = 0.087655\,(\mathrm{GeV fm})^{\frac{3}{2}}$