Diffusion of multiple conserved charges in relativistic dissipative fluid dynamics

CRC–TR 211
Strong-interaction matter under extreme conditions

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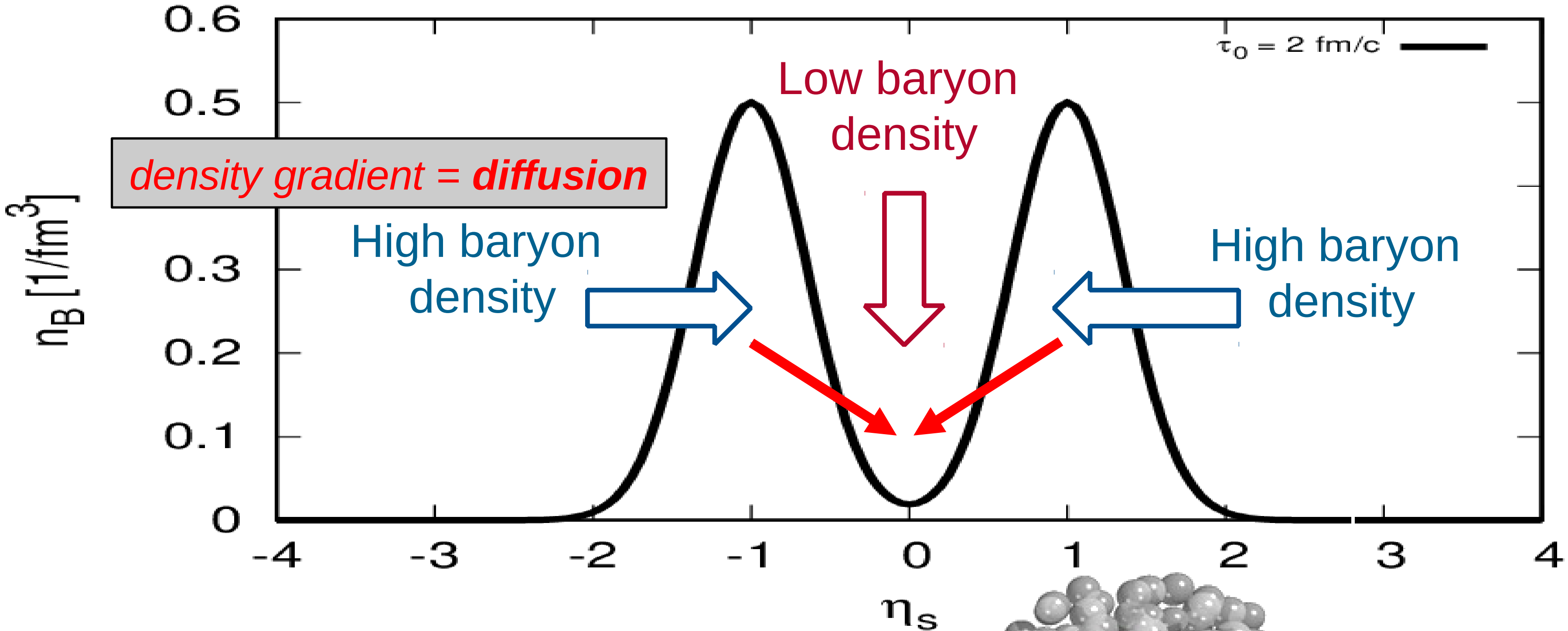
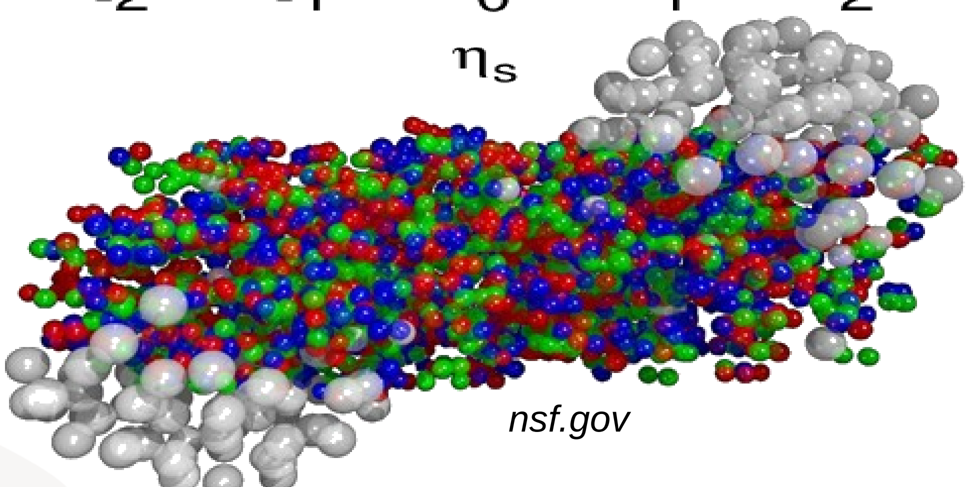
Transport Meeting 2019

Motivation



density gradient = ***diffusion***



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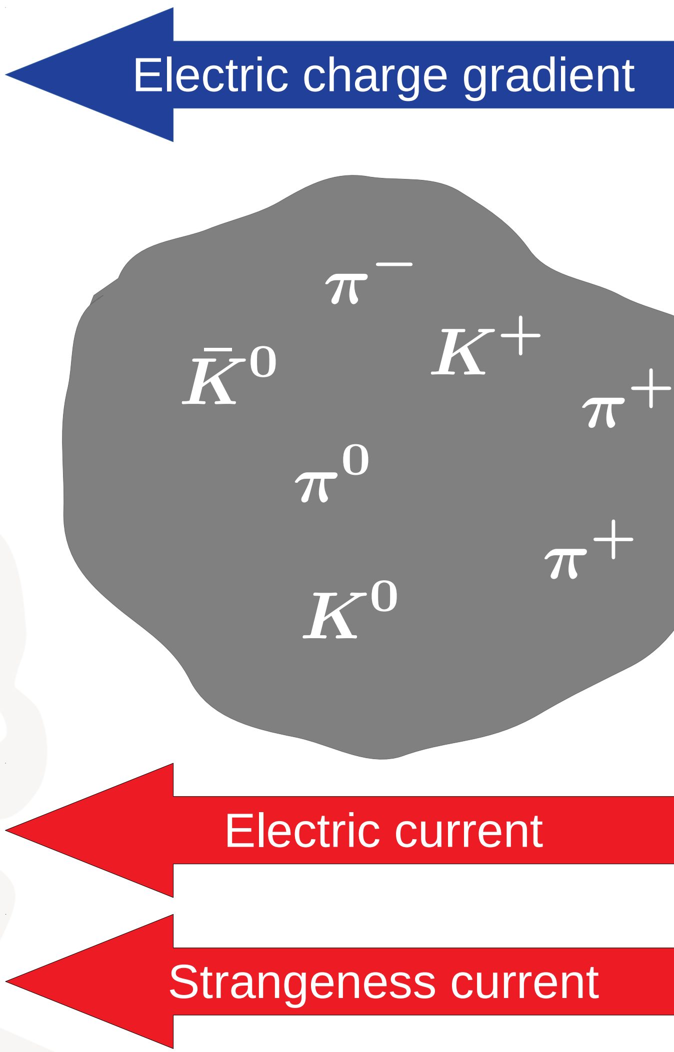
Motivation



Particles carry a multitude of quantum numbers (“*mixed chemistry*”)

= **currents are correlated/coupled!**

Fluid dynamics

Bulk matter ***close to local equilibrium*** is characterized by ***macroscopic*** quantities:

- Thermal densities (energy, quantum number)
- Equation of state (isotropic pressure, temperature, chemical potentials)
- Velocity field
- Dissipative currents (bulk viscousity, **diffusion**, shear viscosity)

Energy-momentum current

$$\mathrm{T}^{\mu\nu} = \epsilon\, u^{\mu} u^{\nu} - (P(\epsilon, \{n_q\}) + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

Current of conserved quantum number *q*

(net electric charge, net baryon number, net strangeness, ...)

$$\mathrm{N}_q^{\mu} = n_q u^{\mu} + V_q^{\mu}$$

Fluid dynamics: equations of motion

Dynamics determined by ...

… energy-momentum conservation: $\partial_\nu T^{\mu\nu} = 0$

… conservation of quantum numbers *q*: $\partial_\mu N_q^\mu = 0$

- 4 + N equations, but ...
- … 10 ($T^{\mu\nu}$) + 4N ($\{N_q^\mu\}$) degrees of freedom (d.o.f.)
- 1 d.o.f. is determined by the equation of state
- Additional 5 + 3N equations needed (**dissipation**)

Denicol-Niemi-Molnar-Rischke theory (DNMR)

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- Here: neglecting bulk viscosity ($\Pi \equiv 0$)
- Shear-stress:

Second order terms

Relaxation time

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} - \frac{4}{3}\pi^{\mu\nu}\theta - 2\pi_\lambda^{\langle\mu}\omega^{\nu\rangle\lambda} - \frac{10}{7}\pi^{\lambda\langle\mu}\sigma^{\nu\rangle}_\lambda$$

Navier-Stokes term

Notation:

Expansion parameter: $\theta \equiv \partial_\mu u^\mu$

Shear tensor: $\sigma^{\mu\nu} = \frac{1}{2}(\nabla^\mu u^\nu + \nabla^\nu u^\mu) - \frac{1}{3}\theta\Delta^{\mu\nu}$

Vorticity: $\omega^{\mu\nu} = \frac{1}{2}(\nabla^\mu u^\nu - \nabla^\nu u^\mu)$

*G. Denicol, H. Niemi, E. Molnar, D. Rischke, Phys. Rev. D **85**, 114047 (2012)*

*K. Gallmeister, H. Niemi, C. Greiner, D. Rischke, Phys. Rev. C **98**, 024912 (2018)*

Coupled charges in DNMR

- System with **one conserved quantum number** *q* only

$$\tau_q \, \dot{V}_q^{\langle\mu\rangle} + V_q^\mu = \boldsymbol{\kappa_q \nabla^\mu \alpha_q} + \mathcal{O}(2)(\theta, \pi^{\mu\nu}, \ldots)$$

- For system with **multiple conserved quantum numbers**: mixed chemistry introduces ***coupling of charges through diffusion coefficient matrix***!

$$V_B^\mu \sim \kappa_B \nabla^\mu \alpha_B \;\; \rightarrow \;\; \begin{pmatrix} V_B^\mu \\ V_Q^\mu \\ V_S^\mu \end{pmatrix} \sim \begin{pmatrix} \kappa_{BB} & \kappa_{BQ} & \kappa_{BS} \\ \kappa_{QB} & \kappa_{QQ} & \kappa_{QS} \\ \kappa_{SB} & \kappa_{SQ} & \kappa_{SS} \end{pmatrix} \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_Q \\ \nabla^\mu \alpha_S \end{pmatrix}$$

$$\tau_q \, \dot{V}_q^{\langle\mu\rangle} + V_q^\mu = \boldsymbol{\sum_{q'} \kappa_{qq'} \nabla^\mu \alpha_{q'}} + \mathcal{O}(2)$$

M. Greif, J. Fotakis, G. Denicol, C. Greiner, Phys. Rev. Lett. ***120****, 242301 (2018)*

Equation of state

- Hadronic system including lightest 19 species

$$\pi^{\pm}, \pi^0, K^{\pm}, K^0, \bar{K}^0, p, \bar{p}, n, \bar{n}, \Lambda^0, \bar{\Lambda}^0, \Sigma^0, \bar{\Sigma}^0, \Sigma^{\pm}, \bar{\Sigma}^{\pm}$$

- Assume classical statistics and non-interacting limit

$$P_0(T, \{\mu_q\}) \equiv \frac{1}{3} \sum_{i=1}^{N_{\text{species}}} \int \frac{\mathrm{d}p^3}{(2\pi)^3 E_{i,p}} \left(E_{i,p}^2 - m_i^2\right) \times g_i \exp(-E_{i,p}/T + \sum_q q_i \alpha_q)$$

- Only assume baryon number and strangeness, neglect electric charge

- Tabulate state variables over energy density ϵ and net charge densities n_q

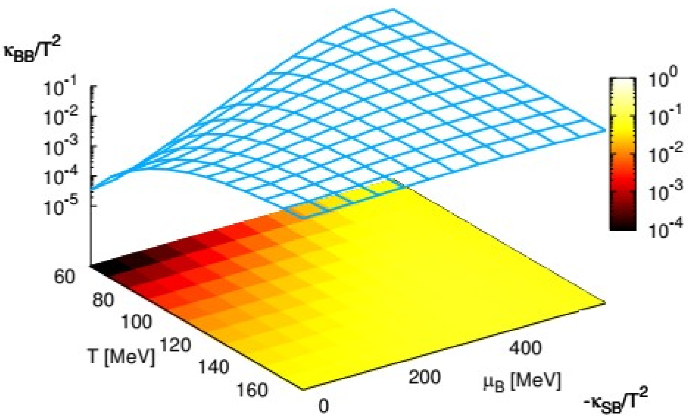
$$T \equiv T(\epsilon, \{n_q\}), \quad \mu_q \equiv \mu_q(\epsilon, \{n_q\}), \quad P_0 \equiv P_0(\epsilon, \{n_q\})$$

Diffusion coefficient matrix

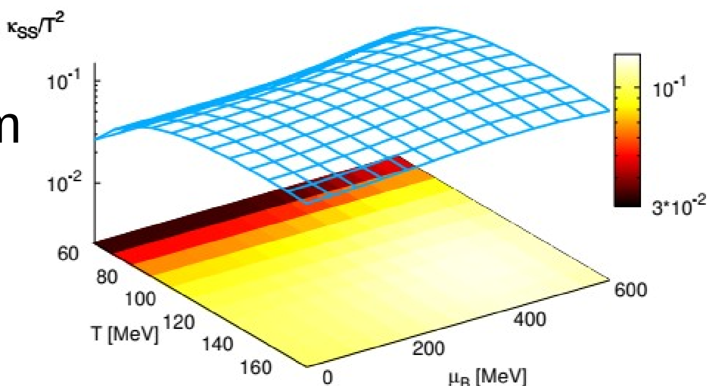
$$\begin{pmatrix} V_B^\mu \\ V_S^\mu \end{pmatrix} \sim \begin{pmatrix} \kappa_{BB} & \kappa_{BS} \\ \kappa_{SB} & \kappa_{SS} \end{pmatrix} \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_S \end{pmatrix}$$

- Matrix is symmetric

L. Onsager, Phys. Rev. ***37****, 405 (1931) & Phys. Rev.* ***38****, 2265 (1931)*

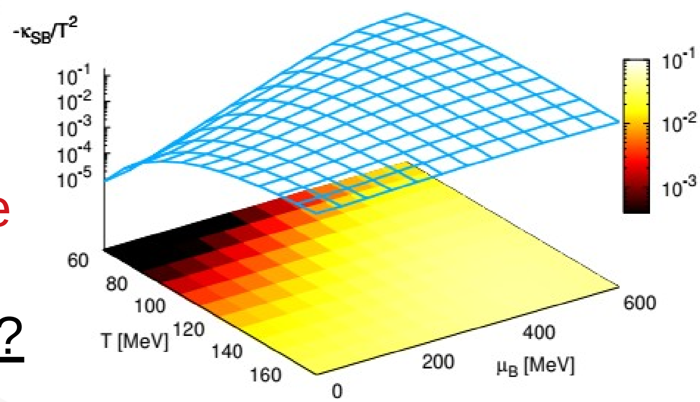


- Elastic isotropic cross sections from PDG, SMASH, GiBUU, UrQMD



κ_{SB} is negative and has similar magnitude as κ_{BB}

$\Rightarrow$ significant coupling?



- Tabulate coefficient matrix over T, μ_B, μ_S
- $\mu_Q = 0$

M. Greif, J. Fotakis, G. Denicol, C. Greiner, Phys. Rev. Lett. ***120****, 242301 (2018)*

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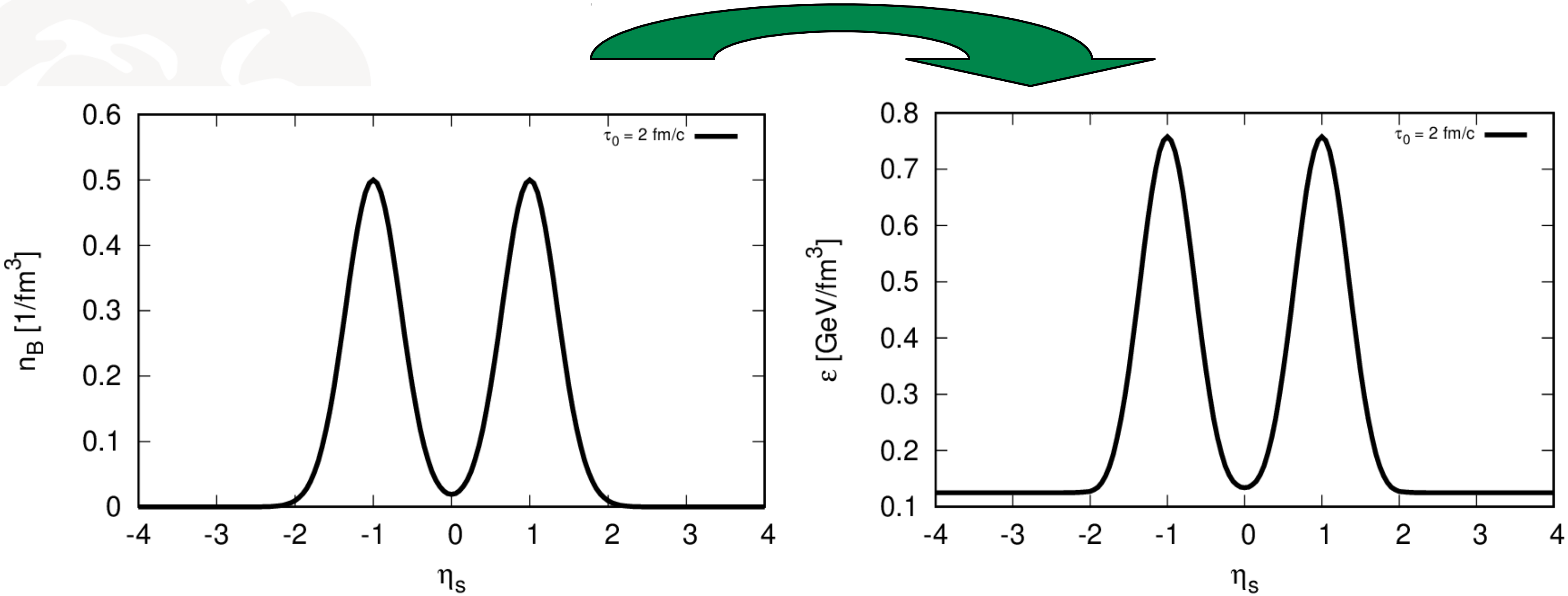
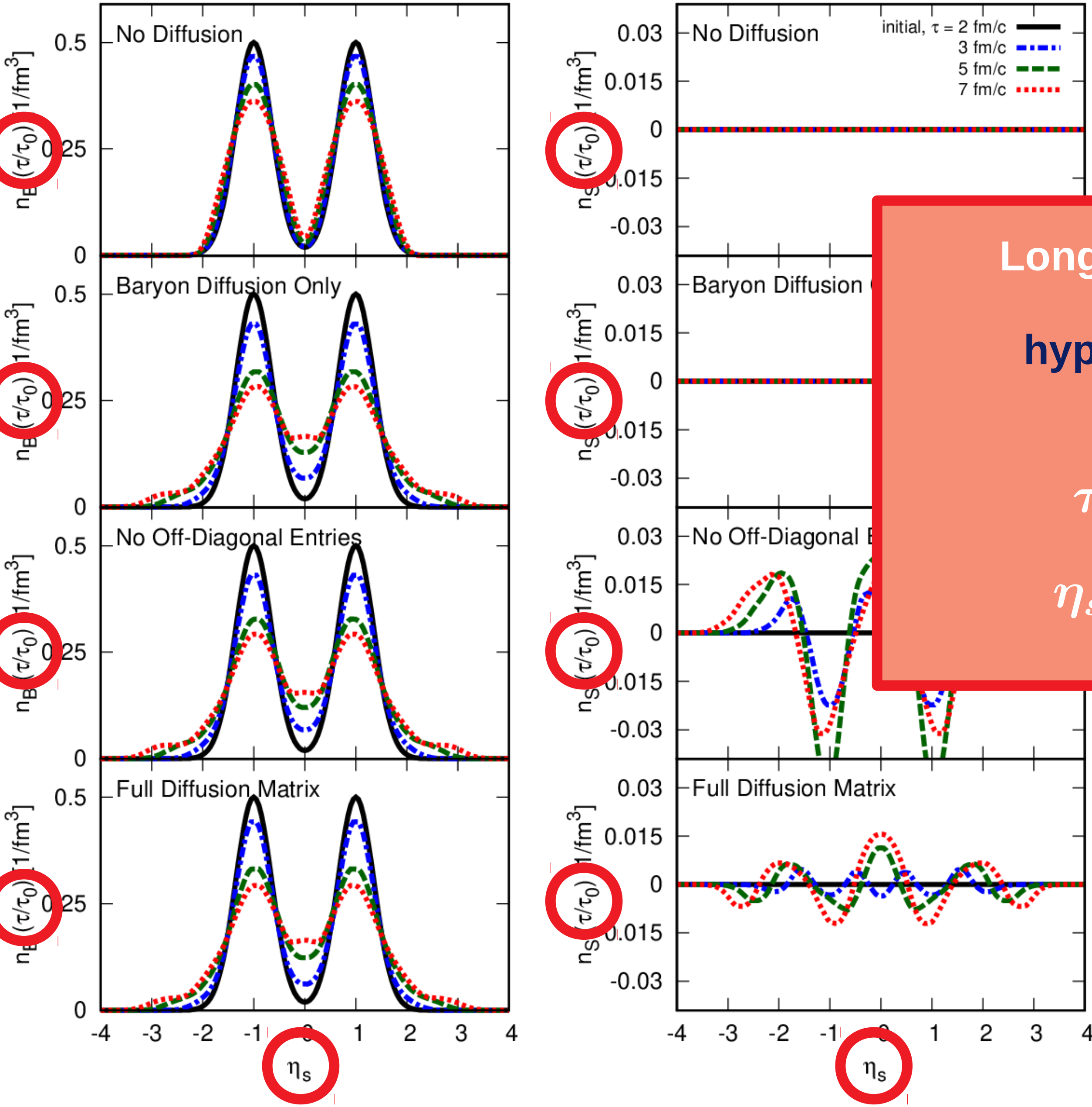
Initial conditions

- $\tau_0 = 2 \text{ fm/c}$
- Initially: no dissipation and only **Bjorken scaling flow**
- Temperature = 160 MeV
- Double-gaussian profile in net baryon number
- No initial net strangeness
- From **EoS**: get energy density



Results (Framework)



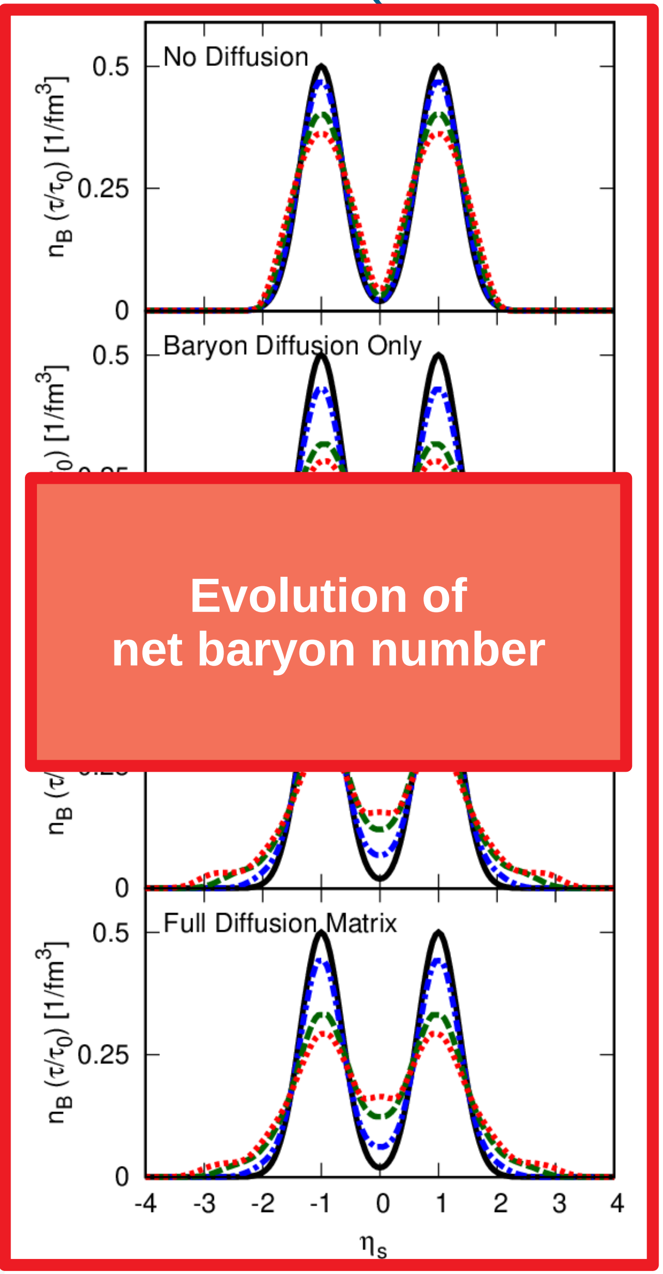
Longitudinal evolution is evaluated in hyperbolic coordinates with SHASTA

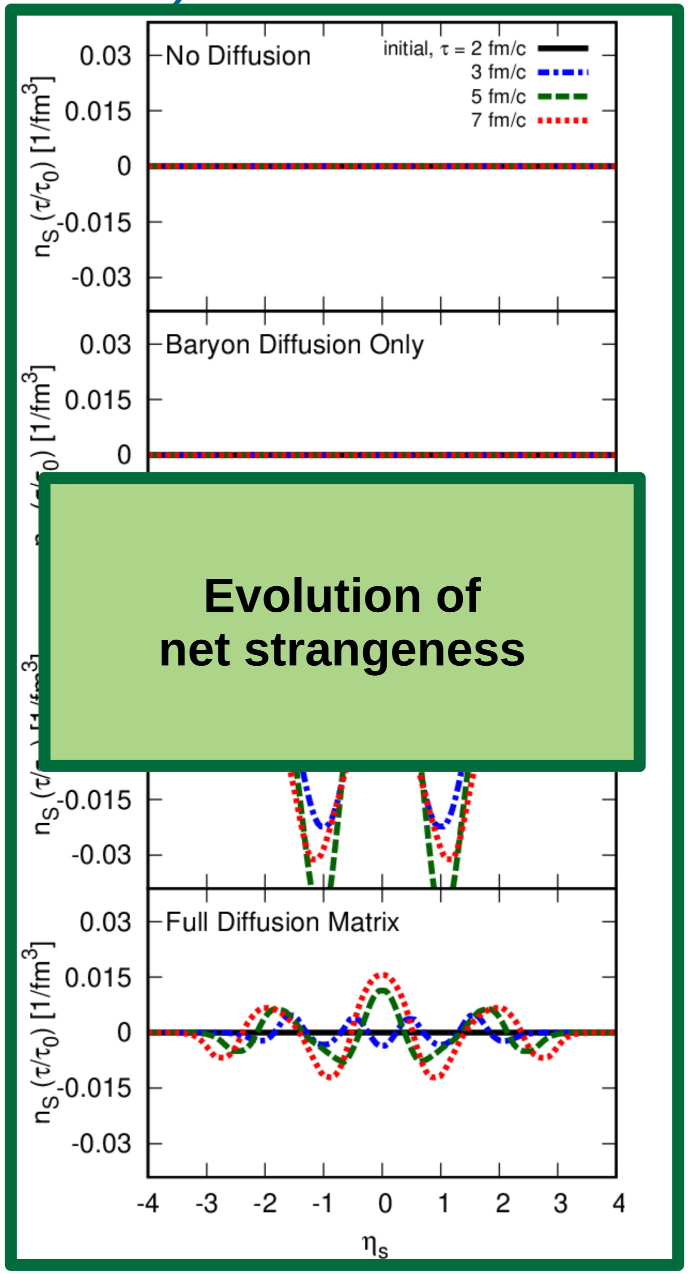
$$\tau \equiv \sqrt{t^2 - z^2}$$

$$\eta_s \equiv \operatorname{arctanh}\left(\frac{z}{t}\right)$$

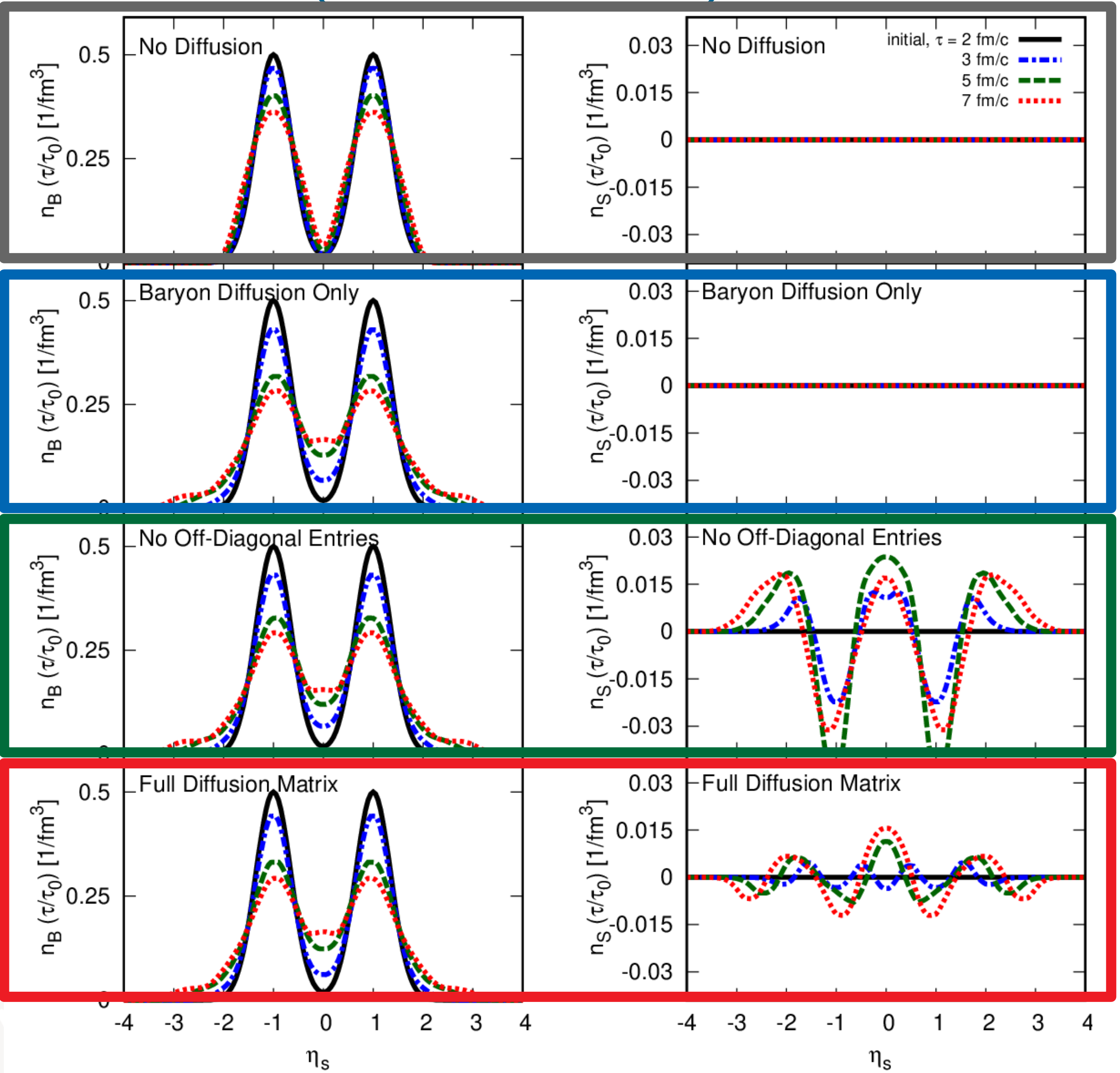
E. Molnar, H. Niemi, D. Rischke, Eur. Phys. J. C ***65****, 615-635 (2010)*

Results (Orientation)





Results (Orientation)



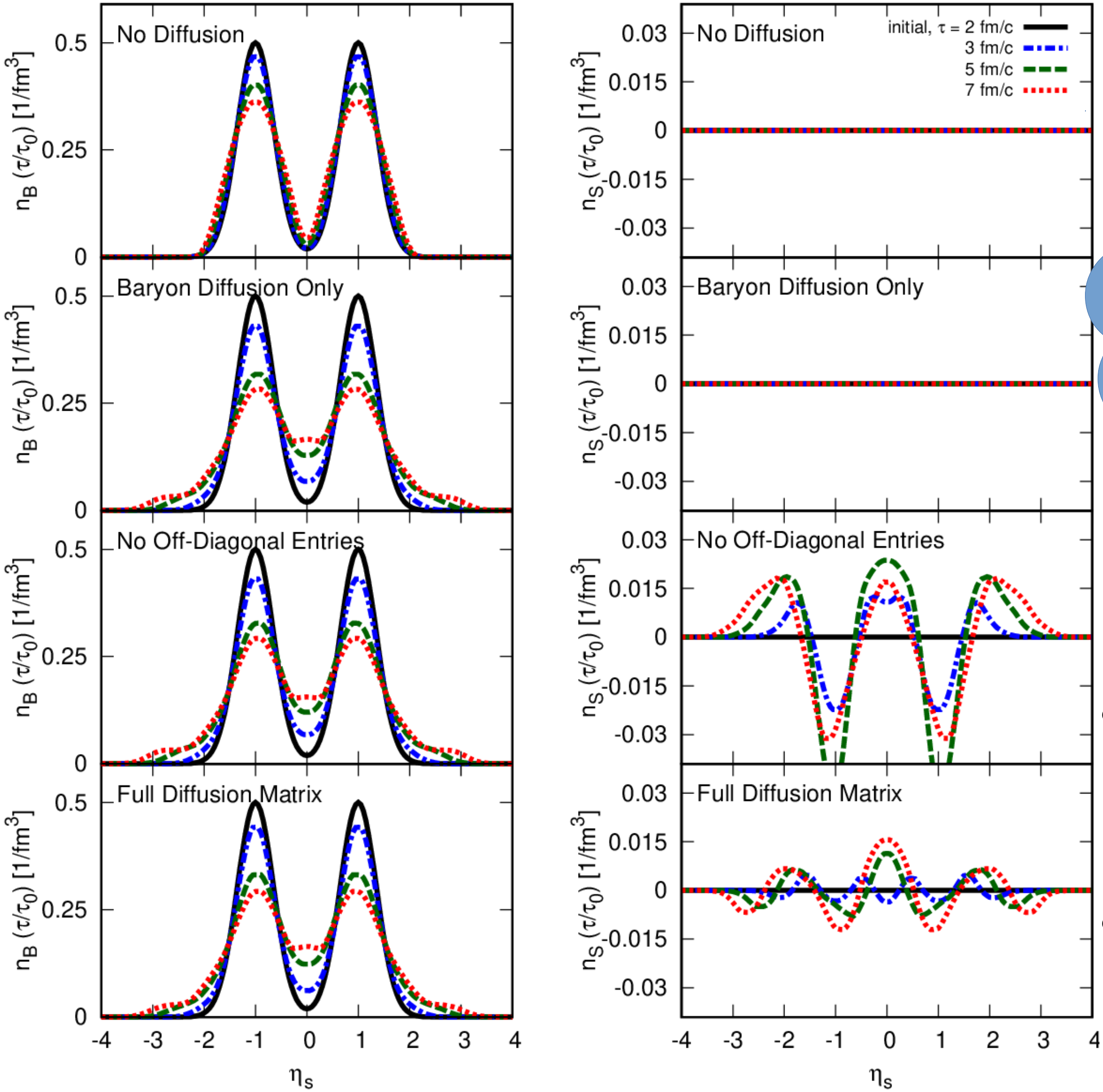
$$\kappa_{BB} = 0$$
$$\kappa_{SS} = 0 = \kappa_{SB}$$

$$\kappa_{BB} \neq 0$$
$$\kappa_{SS} = 0 = \kappa_{SB}$$

$$\kappa_{BB} \neq 0, \kappa_{SS} \neq 0$$
$$\kappa_{SB} = 0$$

$$\kappa_{BB} \neq 0$$
$$\kappa_{SS} \neq 0, \kappa_{SB} \neq 0$$

Results (Navier-Stokes only)



$$\tau_q \dot{V}_q^{\langle\mu\rangle} + V_q^{\mu} = \sum_{q'} \kappa_{qq'} \nabla^{\mu} \alpha_{q'}$$

- Appearence of non-trivial net strangeness profile
- Baryon diffusion similar in all diffusive cases

Results (Navier-Stokes only)

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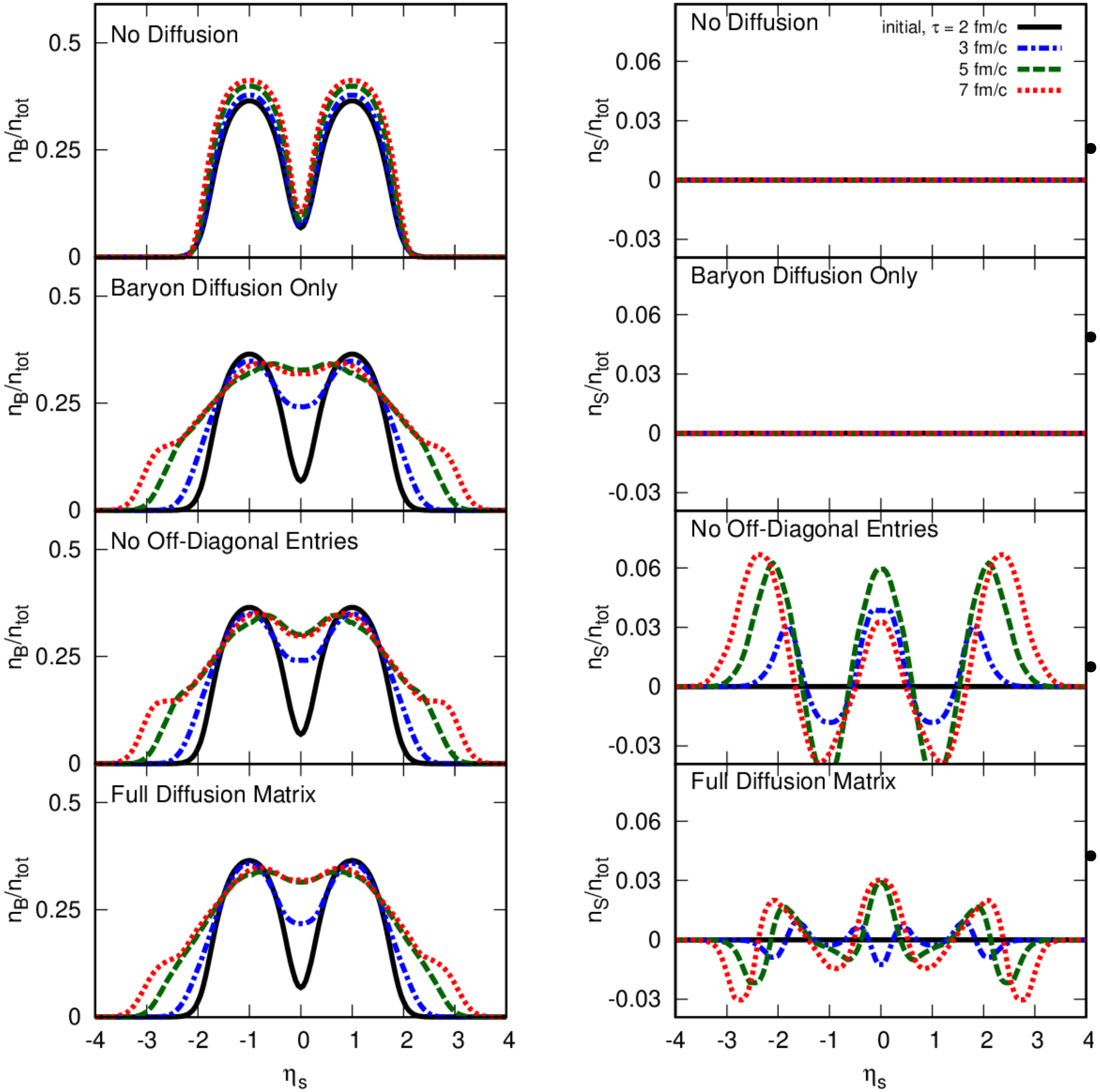


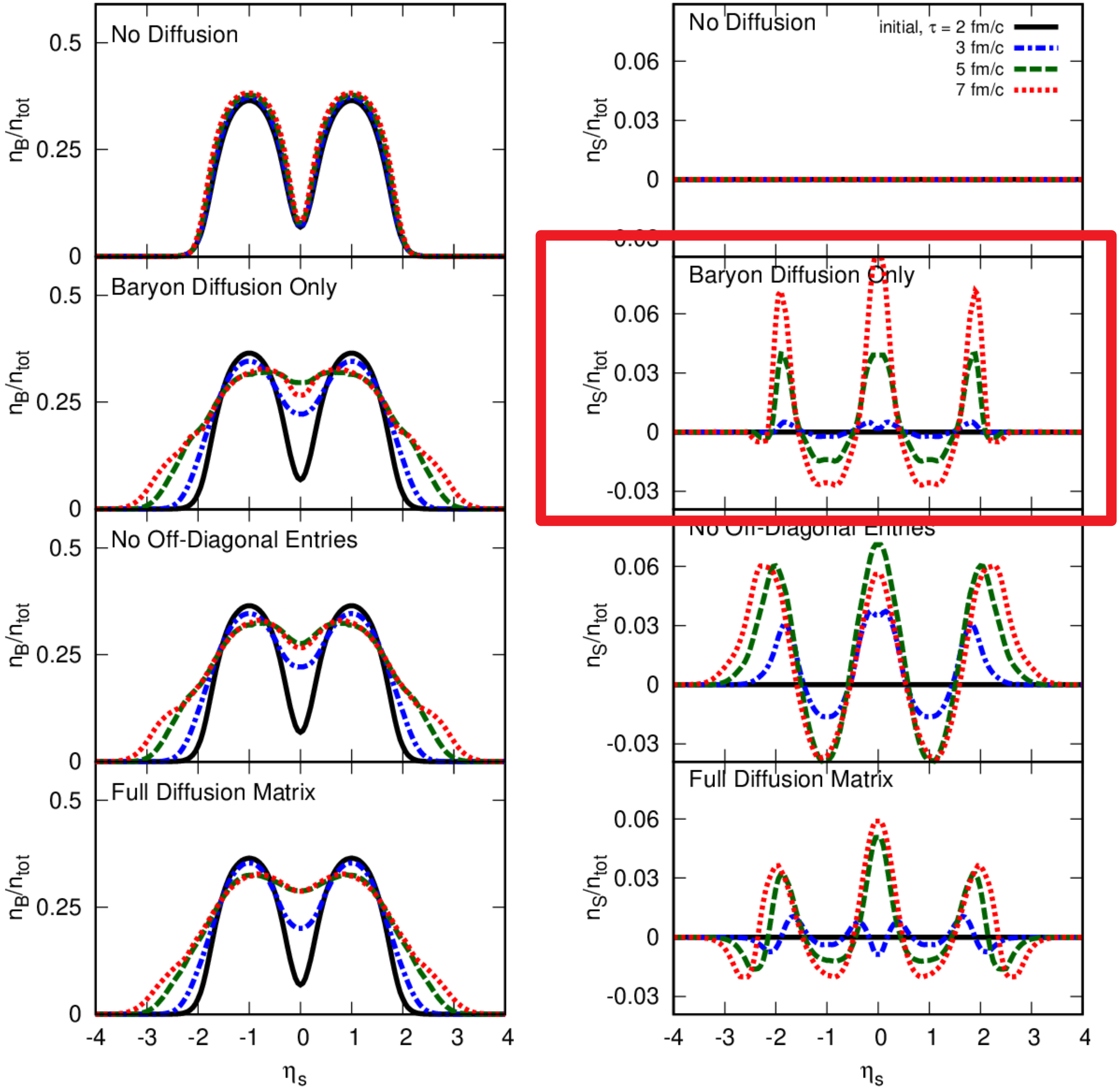
- Chemistry causes baryon-strangeness correlation through ...
- … the EoS which affects the gradients

$$\mu_S \equiv \mu_S(\epsilon, n_B, n_S)$$
$$\nabla^\mu \alpha_S \sim \nabla^\mu n_B$$

- … the Navier-Stokes terms give diffusive correlation
- Magnitude of effect in ‘full’ case:

$$\max\left(\frac{n_S}{n_{\text{tot}}}\right) \approx 3\%$$

Results (all terms + shear)



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$$\frac{\eta}{s} = \frac{1}{4\pi}$$

- Shear seems to enhance diffusive effects
- Magnitude of effect in 'full' case:

$$\max\left(\frac{n_S}{n_{\text{tot}}}\right) \approx 6\%$$

- At least for 'baryon diffusion only'-case this is problematic!
- Strangeness diffusion should not occur here

Results (all terms + shear)

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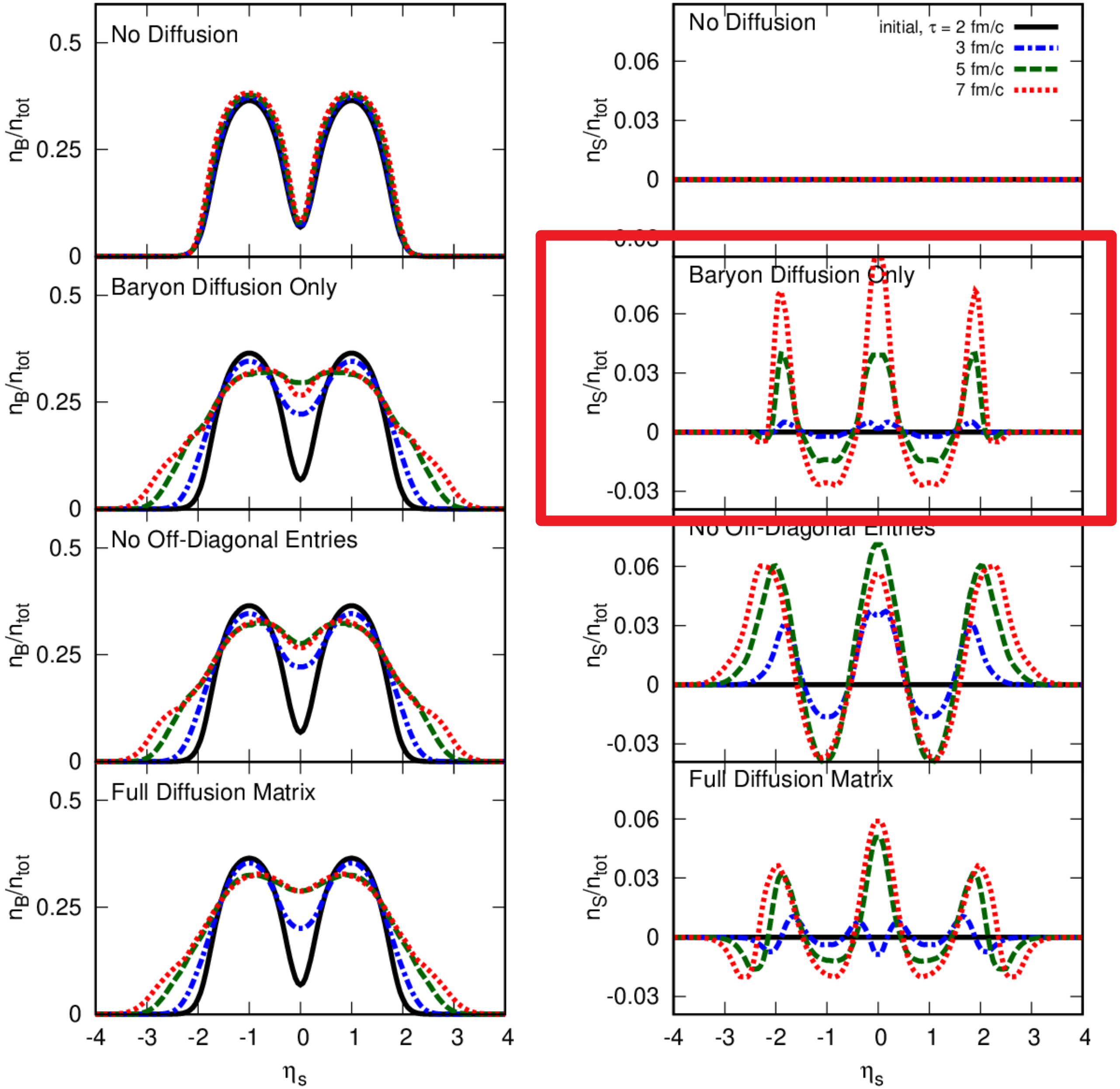

$$\frac{\eta}{s} = \frac{1}{4\pi}$$

- Reason:

  for second-order terms we assumed transport coefficients from **ultrarelativistic**, single-component case!

Conclusion

- Introduced multiple conserved charges to (3+1)D-hydro
- Computed diffusion coefficient matrix from linear response theory in relativistic kinetic theory for a hadronic system with realistic elastic, isotropic cross sections
- Investigated the *longitudinal* evolution of net baryon number and net strangeness for simple initial conditions
- Found **baryon-strangeness correlation** introduced by EoS and coupled diffusion currents; up-building of non-trivial strangeness profile
- Investigated second-order terms: shear-stress could have a significant impact on diffusive evolution
- Transport coefficients of second-order terms remain problematic

Outlook

- Investigate transverse and full 3D-evolution
- Use more realistic (fluctuating) initial conditions and equation of state
- Include **freeze-out and compare to experimental data**: are there sensitive observables for charge-correlation?
- Use temperature parametrizations for shear viscosity and use hadronic second-order transport coefficients
- Derive ***multi-component fluid dynamics***: coupling second-order terms?
- Compare to BAMPS and SMASH
  and other fluid dynamic approaches