Transport coefficients in the hadron gas

Jean-Bernard Rose

with A. Schäfer, D. Oliinychenko, J.M. Torres-Rincon, J. Hammelmann, H. Elfner, M. Greif, G. Denicol, J. Fotakis, C. Greiner

CRC Transport Meeting, Frankfurt, Germany, December 12th, 2019

Three talks for the price of one!

1. Shear Viscosity
2. Cross-Conductivity
3. Bulk Viscosity

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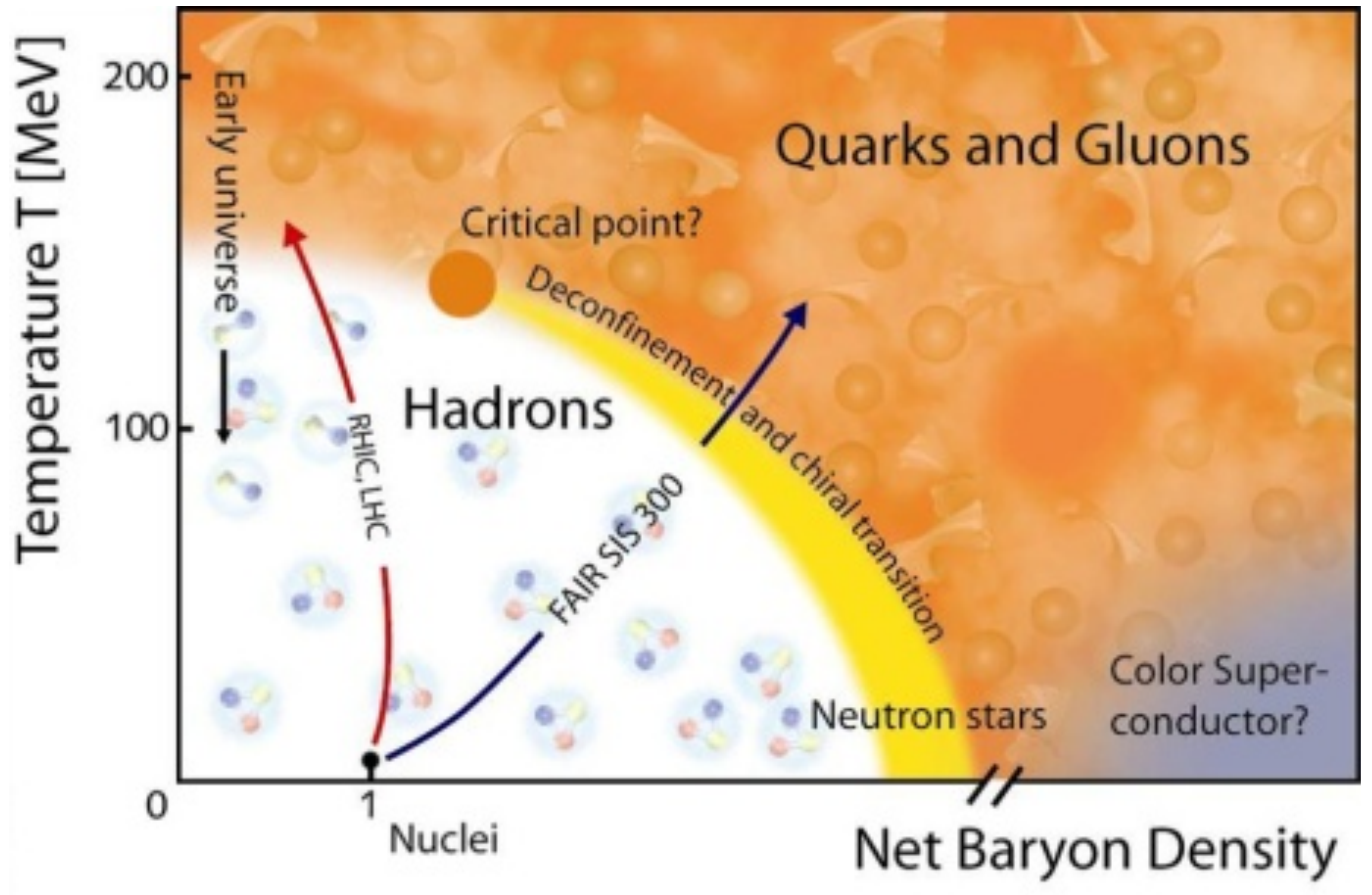
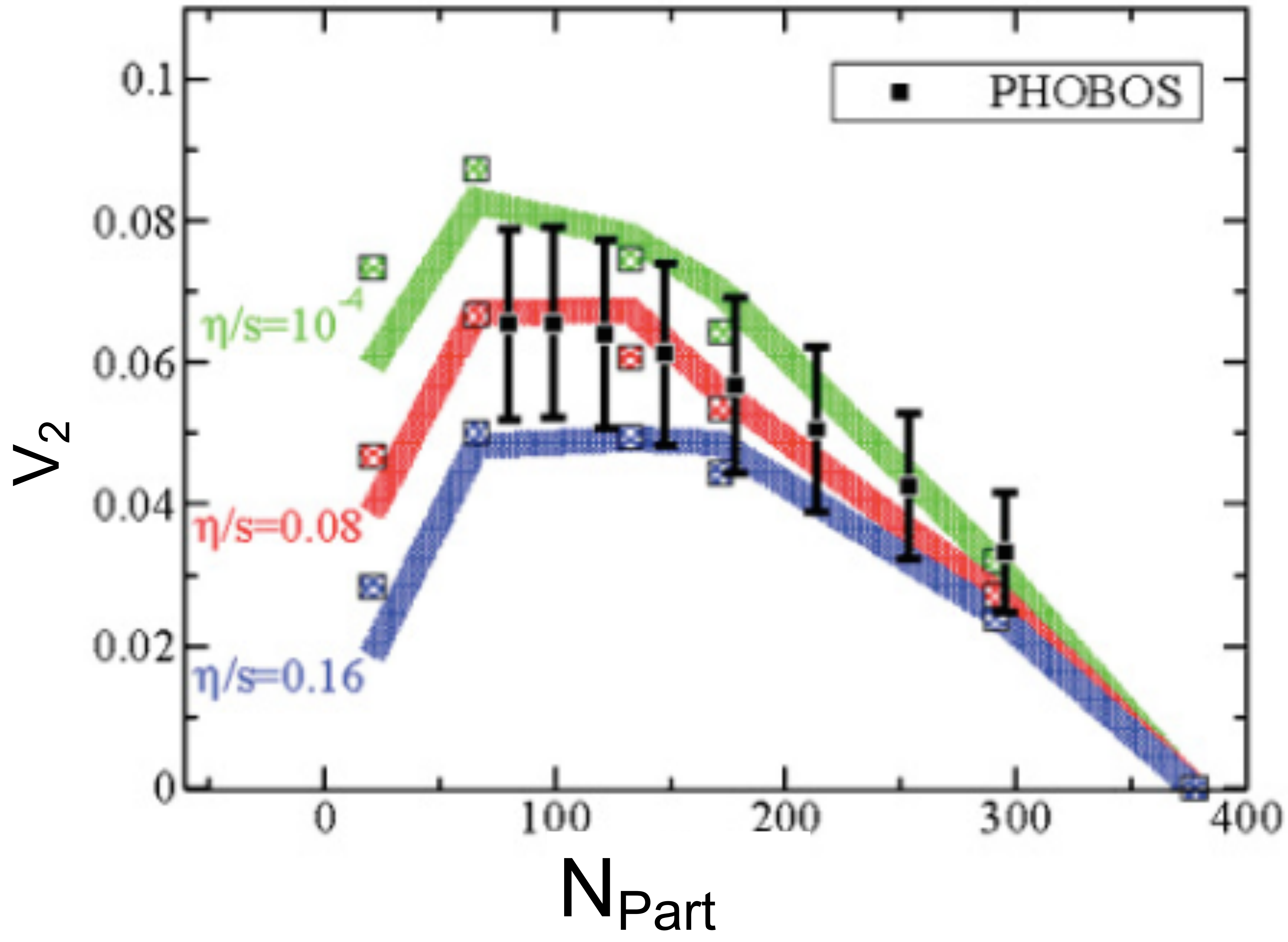
Viscosity in heavy ion collisions

- Investigating deconfinement requires a good knowledge of transport coefficients



Viscosity in heavy ion collisions

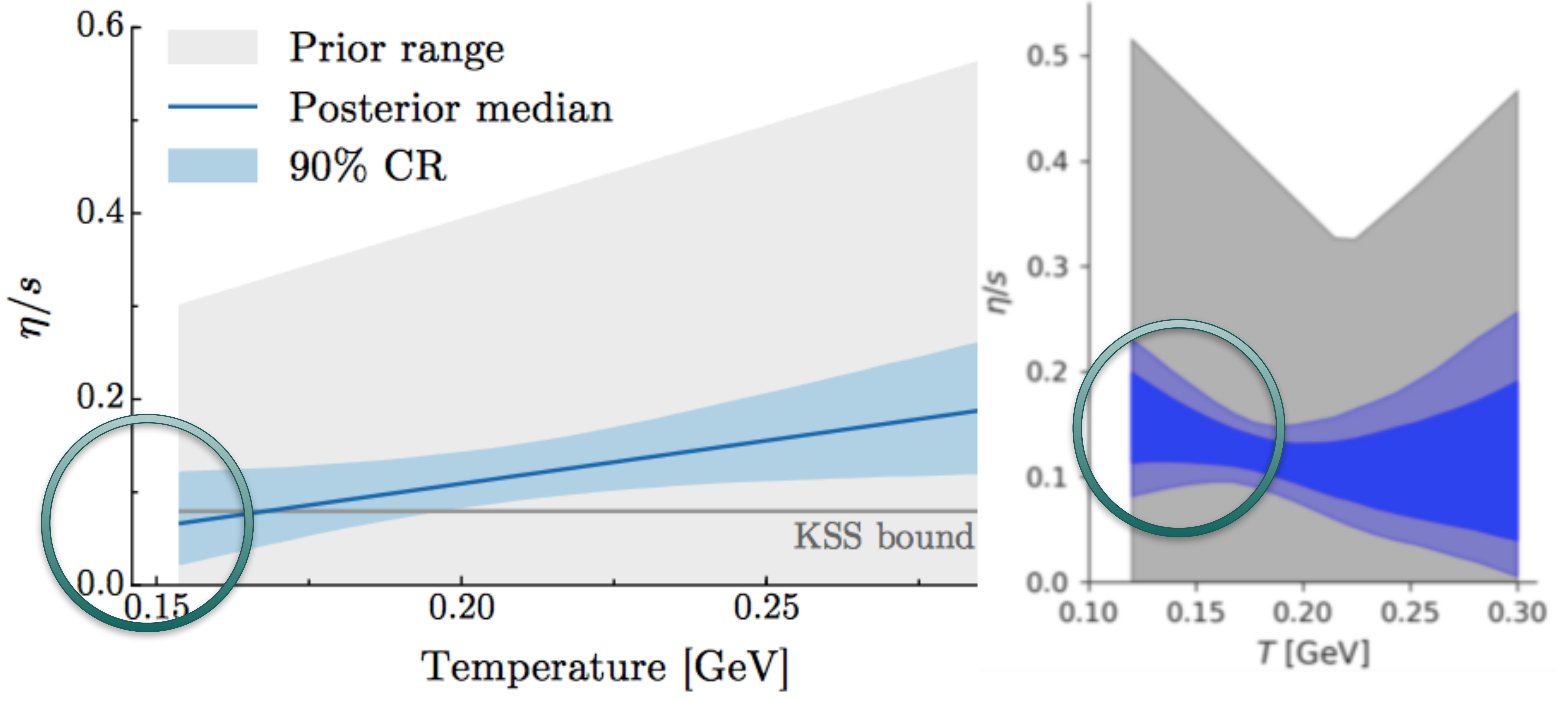
- Investigating deconfinement requires a good knowledge of transport coefficients
- Hydrodynamics relatively successful at explaining this with small *η/s* above the transition



Luzum & Romatschke 10.1103/Phys. Rev. C 78.034915

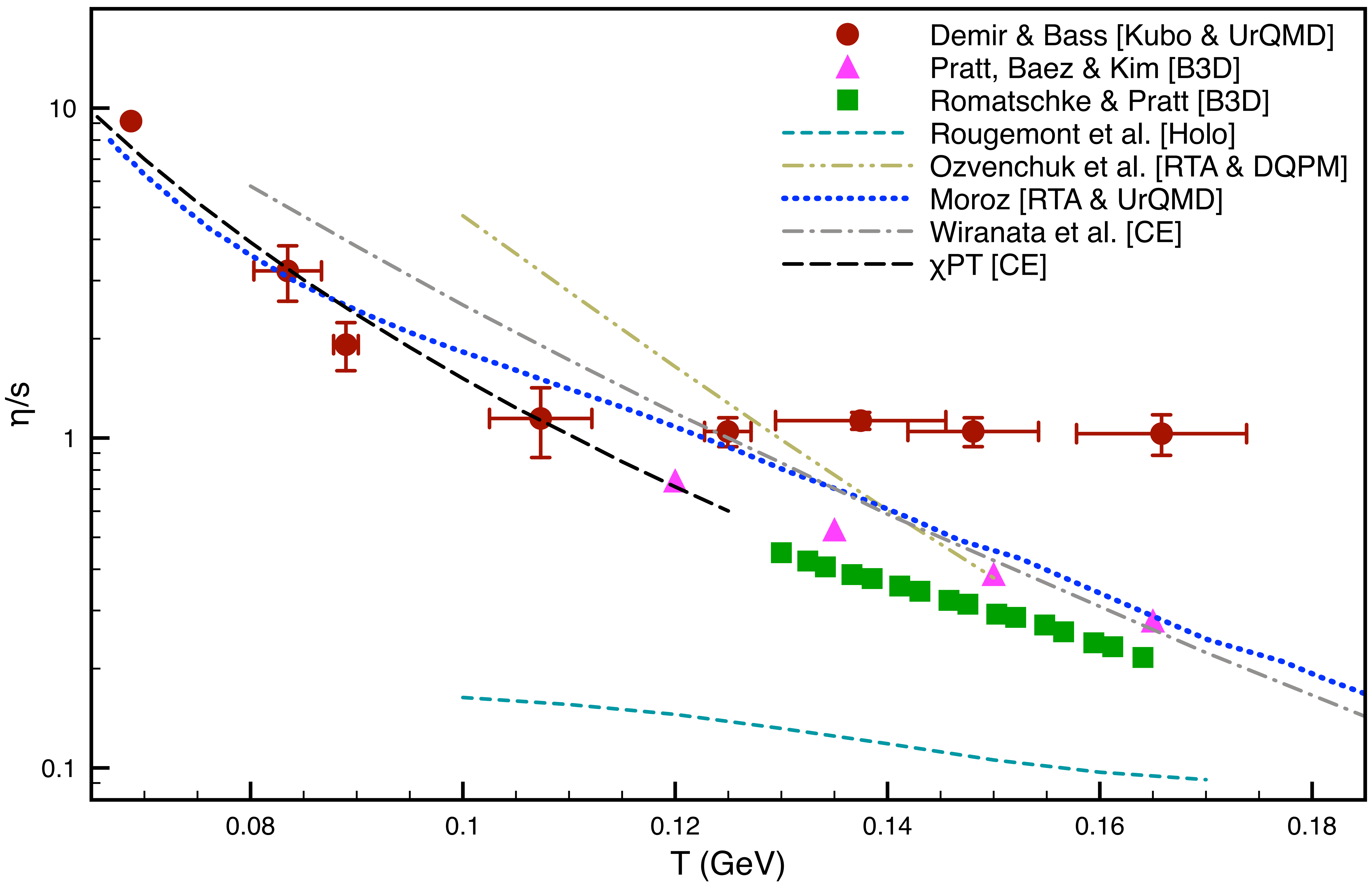
Viscosity in heavy ion collisions

- Investigating deconfinement requires a good knowledge of transport coefficients
- Hydrodynamics relatively successful at explaining this with small *η/s* above the transition
- Still not clear what the behavior of η/s is at low energies (FAIR, late stage RHIC/LHC)



-Bernhard, Moreland, Bass, Liu & Heinz Phys. Rev. C94 no. 2 (2016) 024907
-Paquet, QM2019 talk

Previous HG viscosity calculations



-Demir & Bass, Phys.Rev.Lett. 102 (2009) 172302
-Pratt, Baez & Kim, Phys. Rev. C95 (2017) 024901
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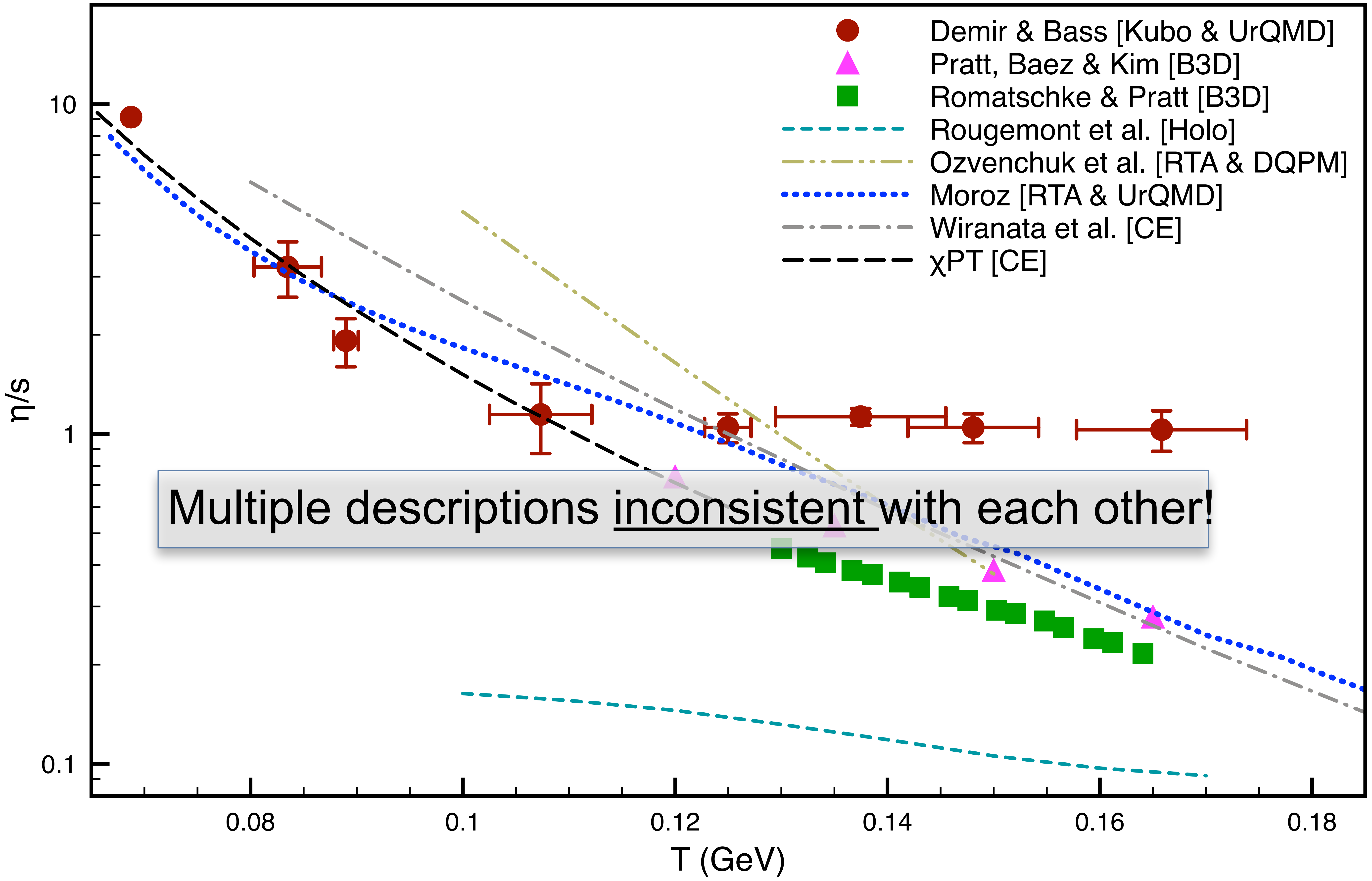

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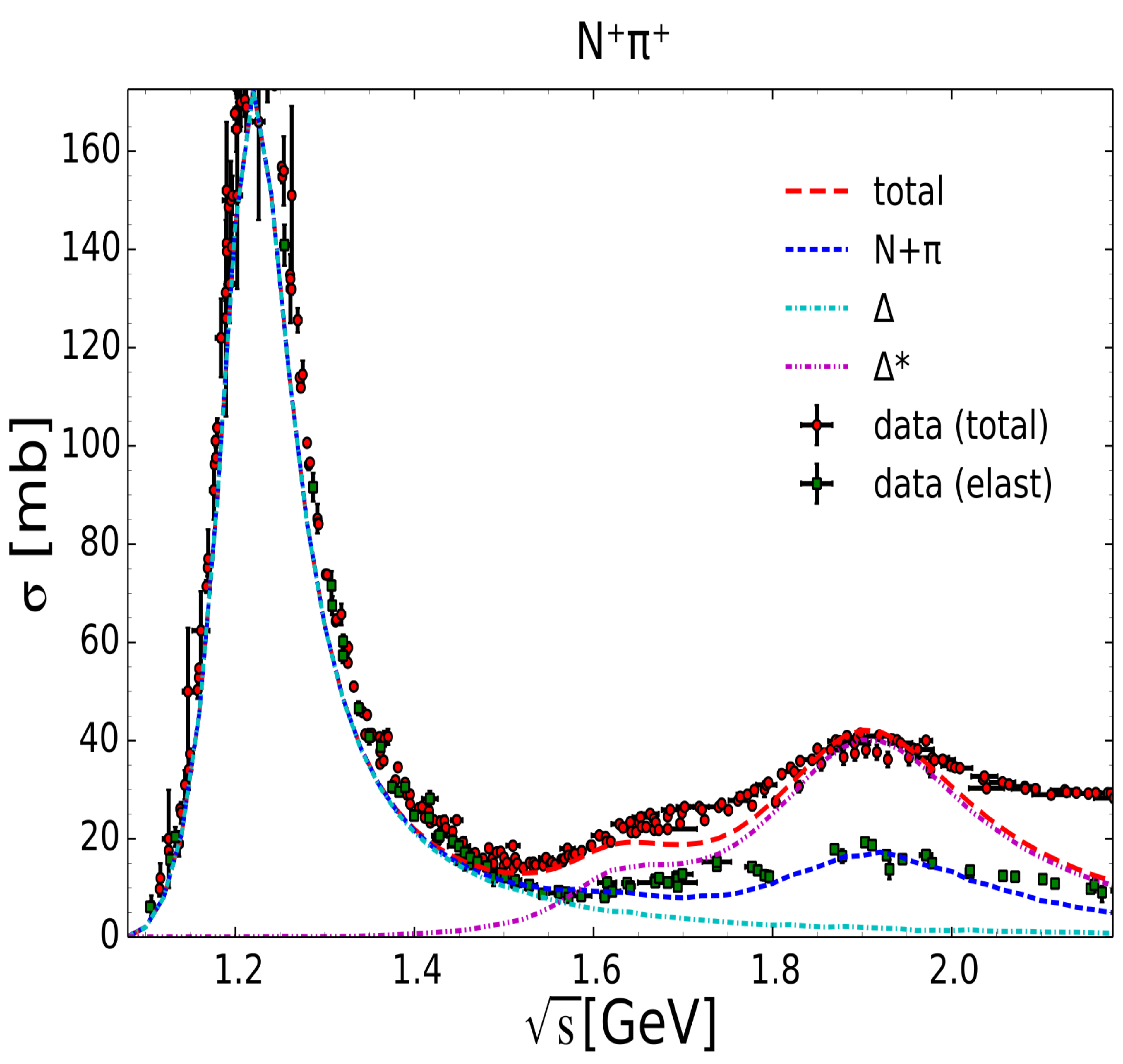
Modelling the hadron gas: smash



- **SMASH is a semi-classical transport approach for the hadron gas**
- Geometric collision criterion:

$$d_{trans} < d_{int} = \sqrt{\frac{\sigma_{tot}}{\pi}}$$

- Spectral functions of resonances are described by relativistic Breit-Wigner functions, with resonance lifetime

$$\tau_{res} = \frac{1}{\Gamma(m)}$$

- Elastic scatterings parameterized for NN; many other elastic scatterings assumed to go through resonances
- All other elastic scatterings go through Additive Quark Model
- Inelastic scatterings, currently include
  - NN↔NR, NN↔ ΔR
  - KN↔KN, KN↔πH
  - +antiparticles
- Strings (turned off for detailed balance)

Weil et al., Phys. Rev. C94 (2016) no.5, 054905

Green-Kubo formalism: Shear

The shear viscosity is calculated from

$$\eta = \frac{V}{T}\int_0^\infty C^{xy}(t)dt$$

where

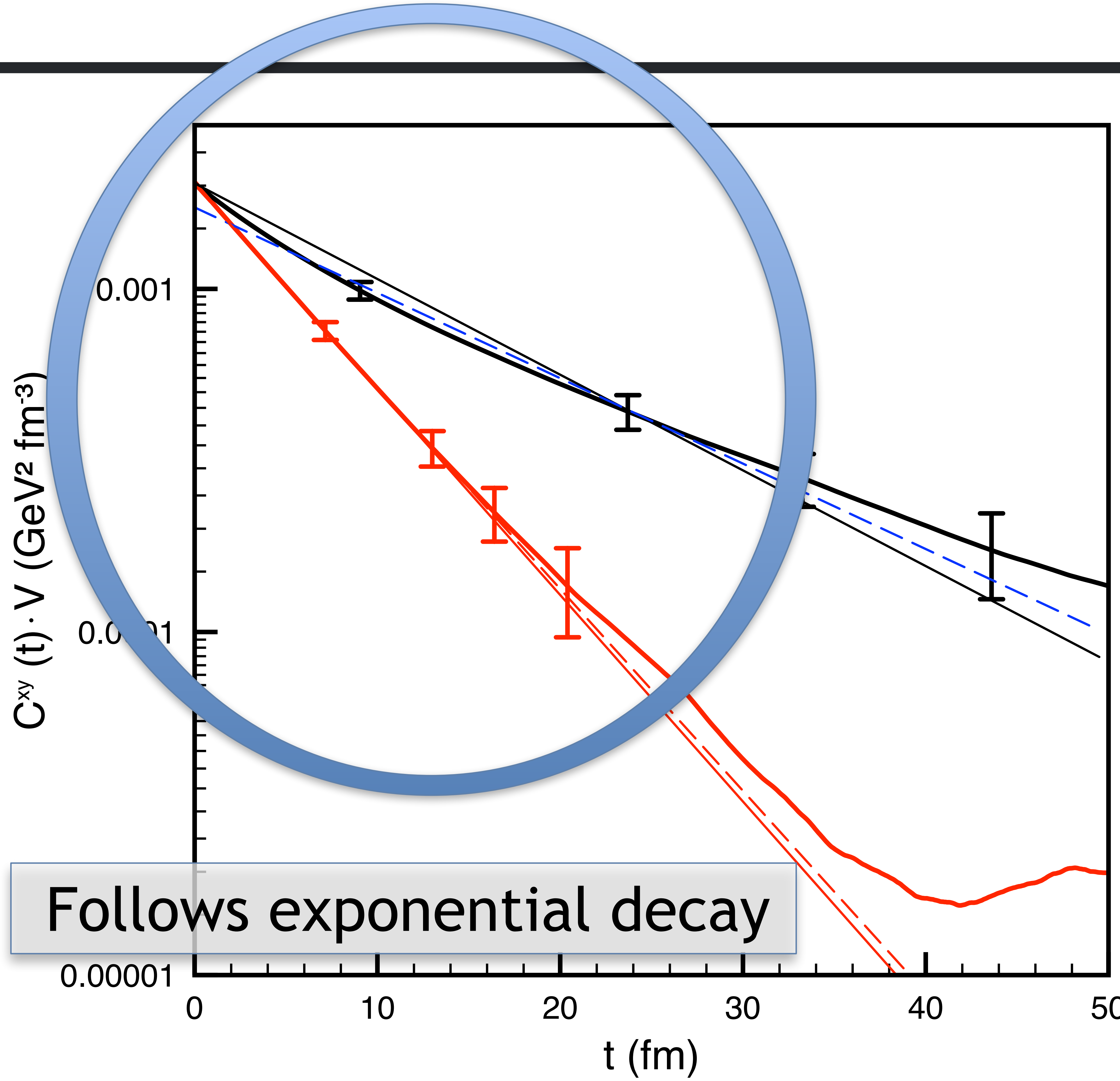
$$C^{xy}(t) \equiv \langle (T^{xy}(0) - \langle T^{xy}\rangle_{eq}) \cdot (T^{xy}(t) - \langle T^{xy}\rangle_{eq})\rangle_{eq}$$

In the dilute case, exponential *ansatz*

$$C^{xy}(t) = C^xy(0)\ e^{-\frac{t}{\tau_\eta}}$$

$$\eta = \frac{C^{xy}(0)\ V\ \tau_\eta}{T}$$

where τ_η is the shear relaxation time



Follows exponential decay

Green-Kubo formalism: Cross-Conductivity

We can extend the previous formalism:

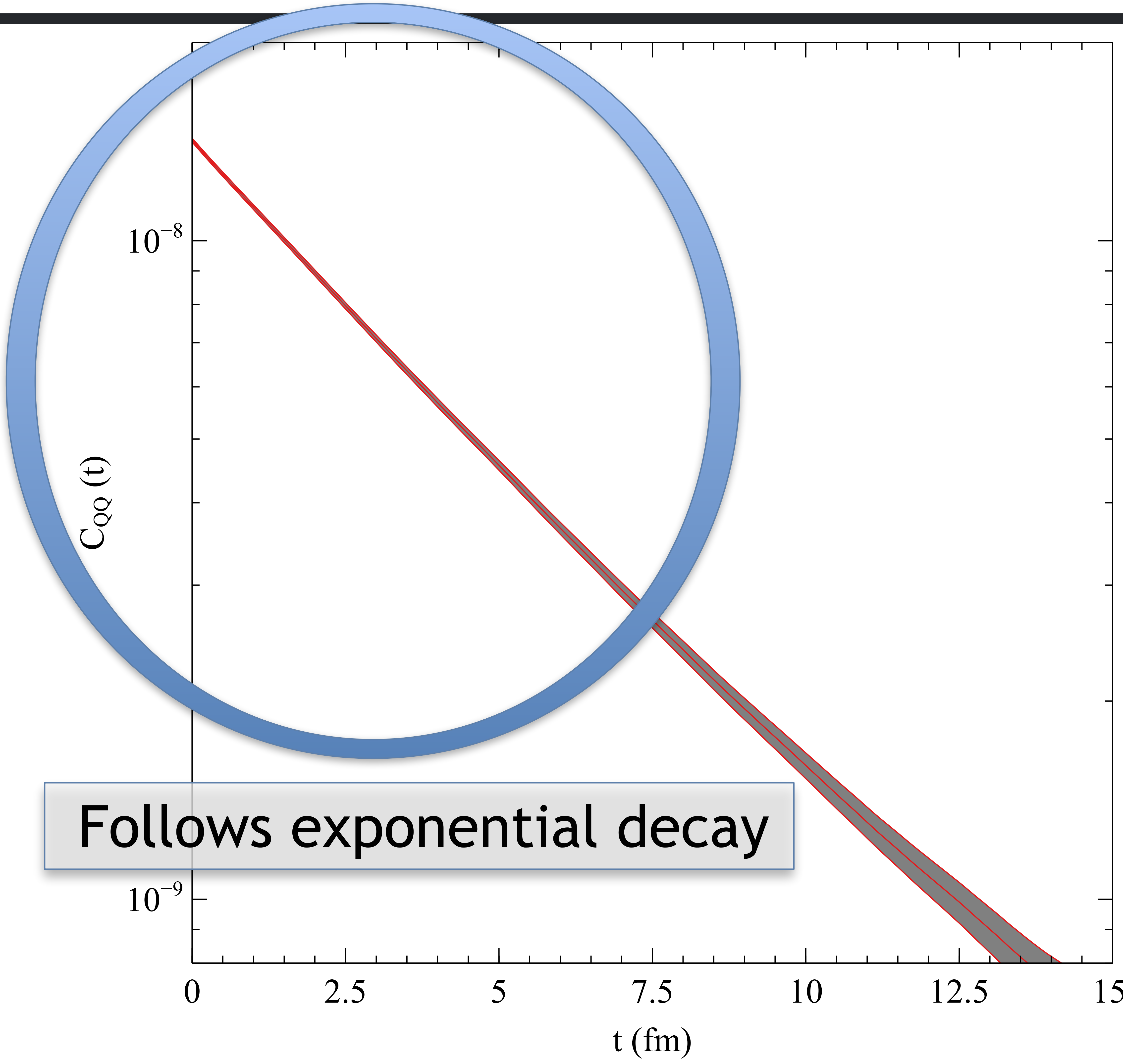
$$\eta = \frac{V}{T}\int_0^\infty \langle \pi^{xy}(0), \pi^{xy}(t)\rangle_{eq} dt$$

$$\zeta = \frac{V}{T}\int_0^\infty \langle p(0), p(t)\rangle_{eq} dt$$

$$\sigma_{QQ,QB,QS} = \frac{V}{T}\int_0^\infty \langle j^x_{Q,B,S}(0), j^x_Q(t)\rangle_{eq} dt$$

where

$$\langle A(t), B(t')\rangle_{eq} \equiv \langle (A(t) - \langle A\rangle_{eq}) \cdot (B(t') - \langle B\rangle_{eq})\rangle_{eq}$$



Green-Kubo formalism: Bulk

We can extend the previous formalism:

$$\eta = \frac{V}{T}\int_0^\infty \langle \pi^{xy}(0), \pi^{xy}(t)\rangle_{eq} dt$$

$$\zeta = \frac{V}{T}\int_0^\infty \langle p(0), p(t)\rangle_{eq} dt$$

$$\sigma_{QQ,QB,QS} = \frac{V}{T}\int_0^\infty \langle j^x_{Q,B,S}(0), j^x_Q(t)\rangle_{eq} dt$$

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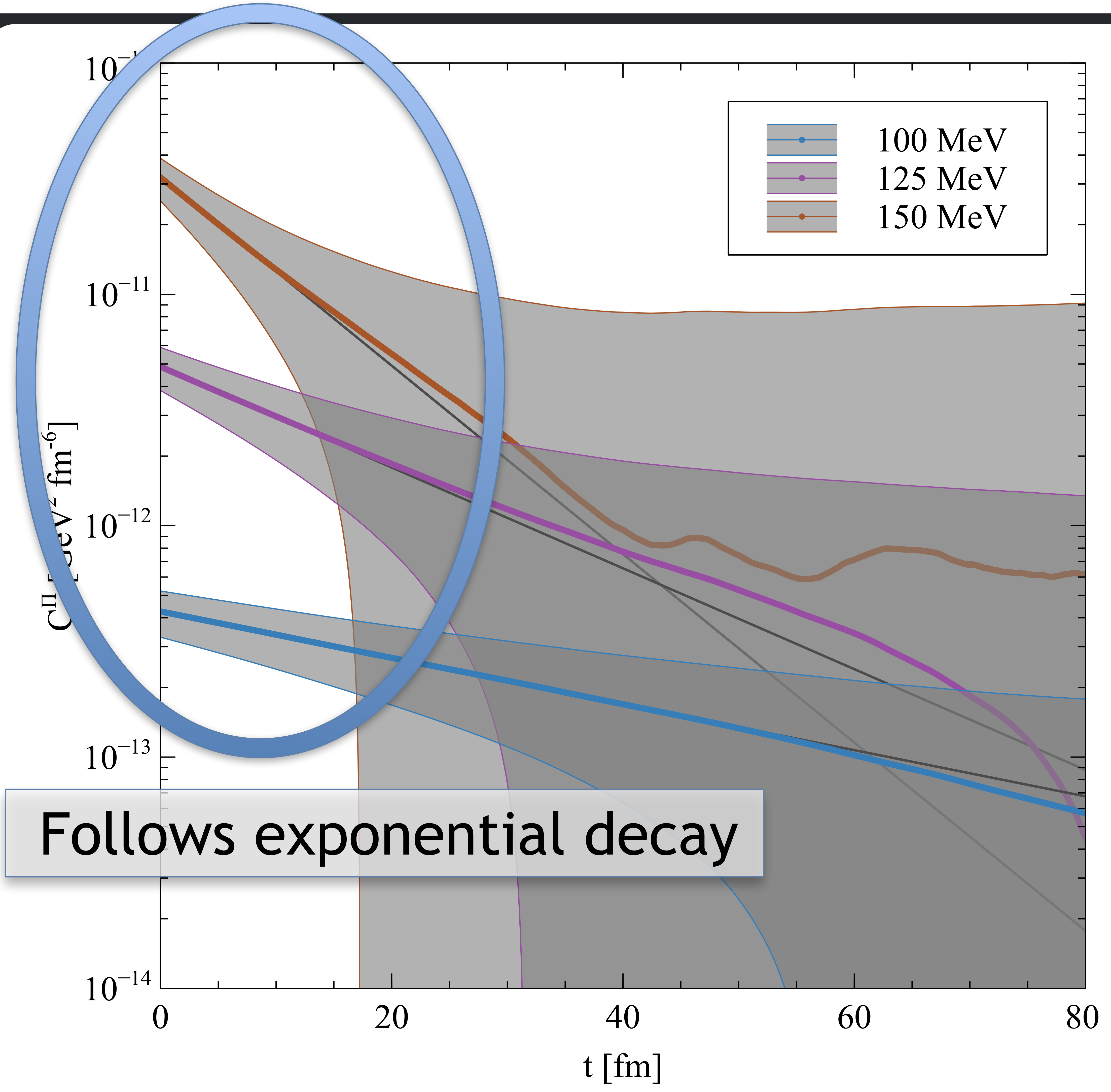
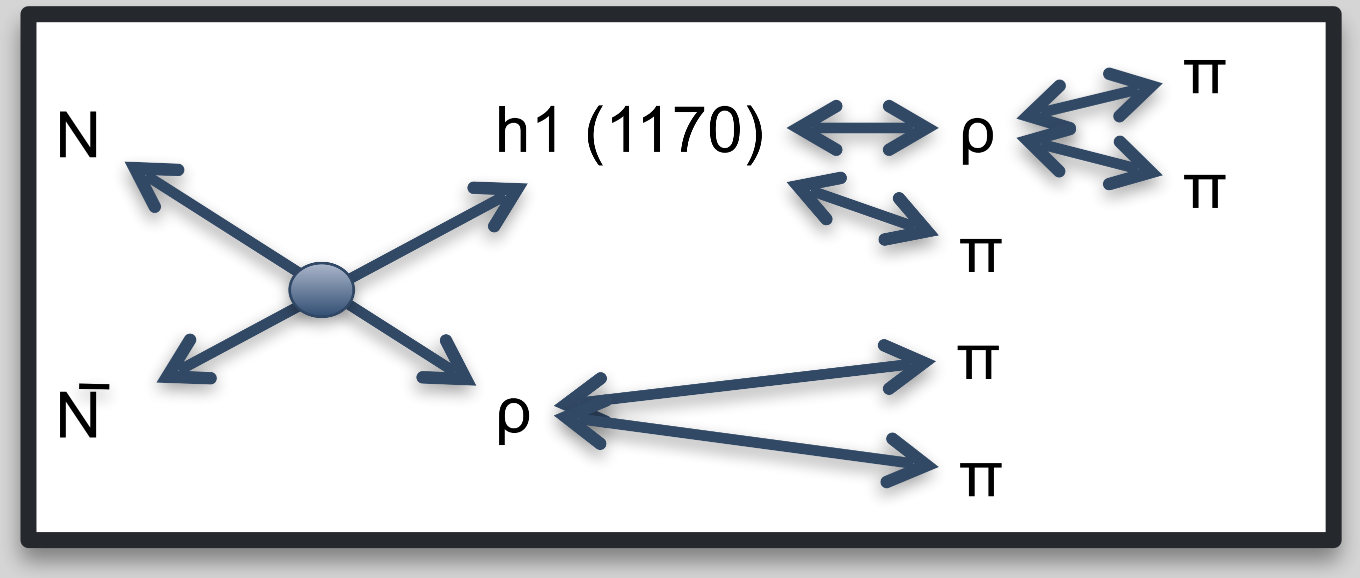
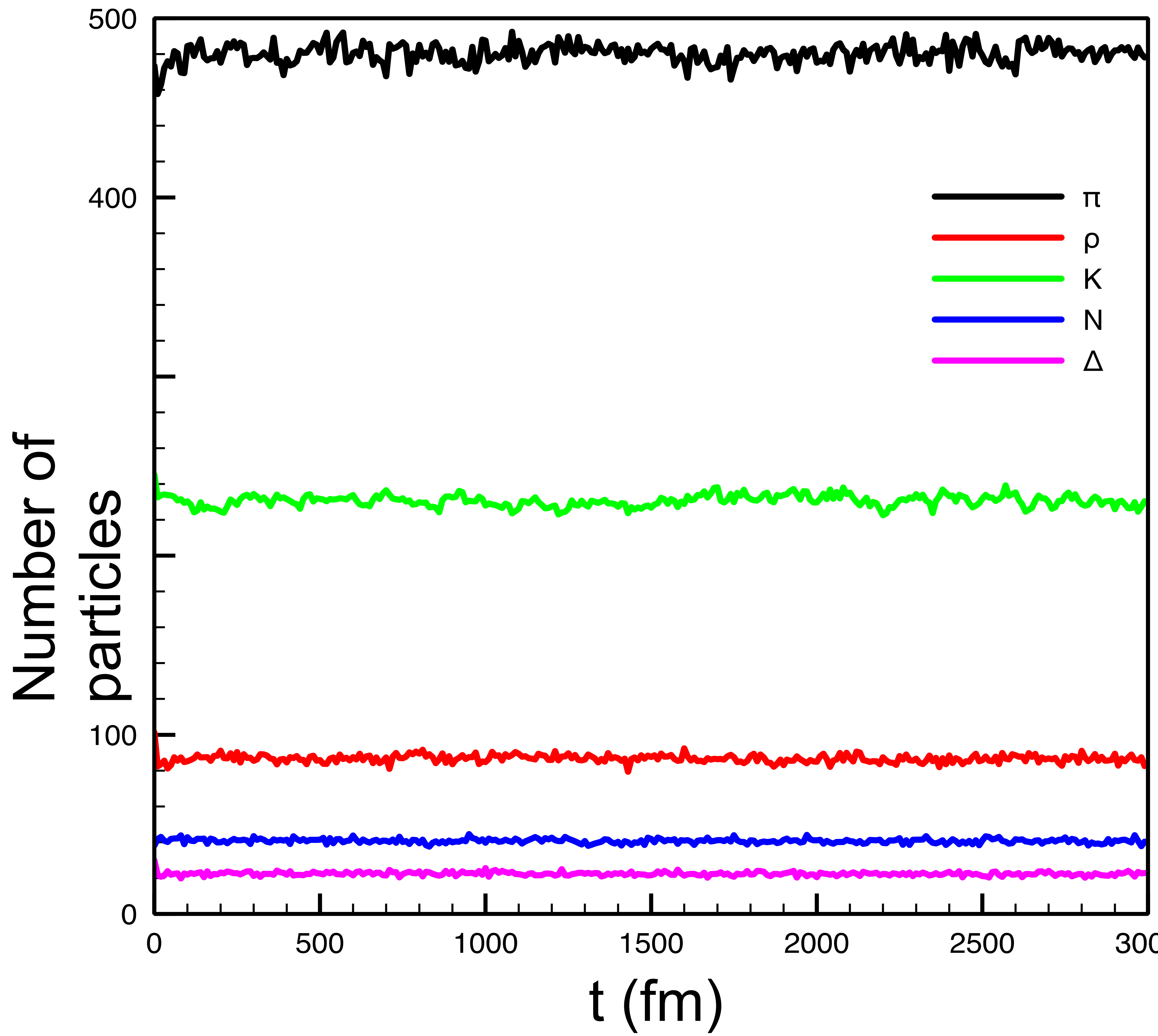


Method: Equilibrium in SMASH

- Box calculations simulating infinite matter to apply the Green-Kubo procedure
- MUST have thermal & **chemical** equilibrium = detailed balance
- Baryon/antibaryon annihilation implemented to conserve detailed balance via an average decay to 5π





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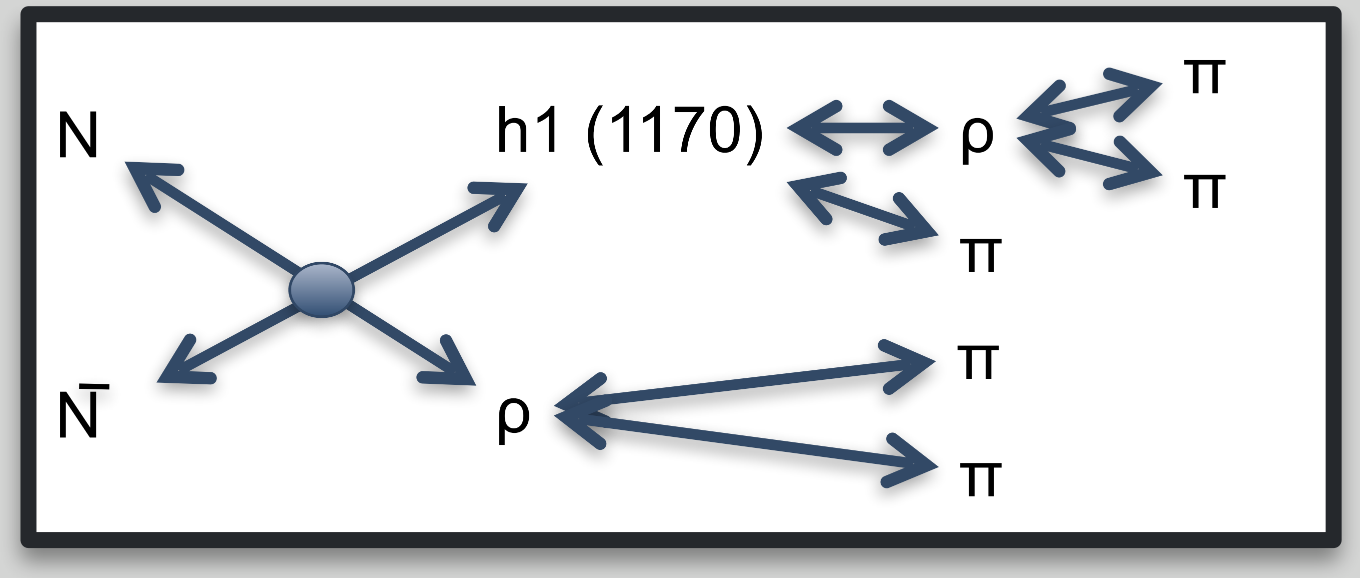
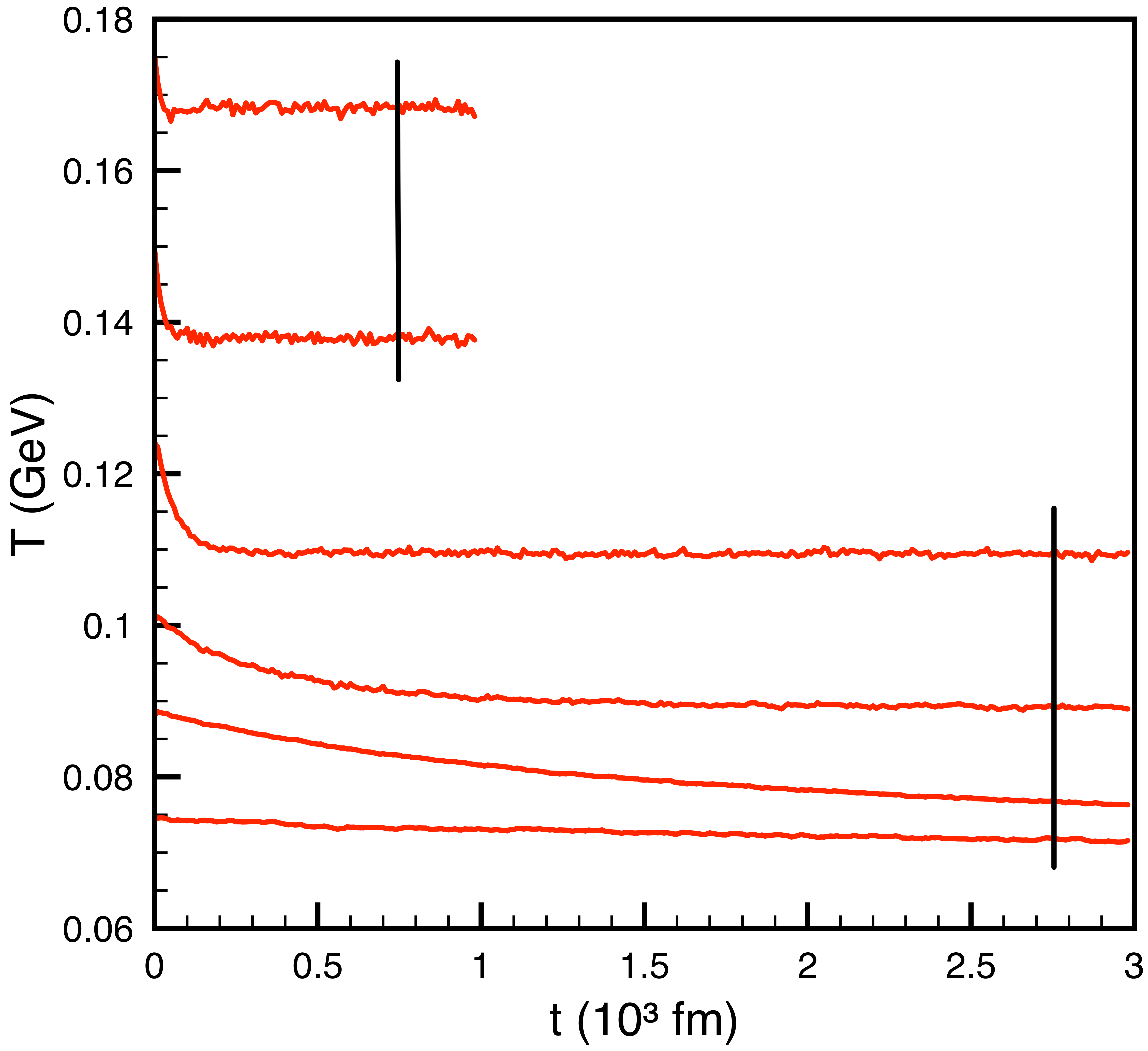
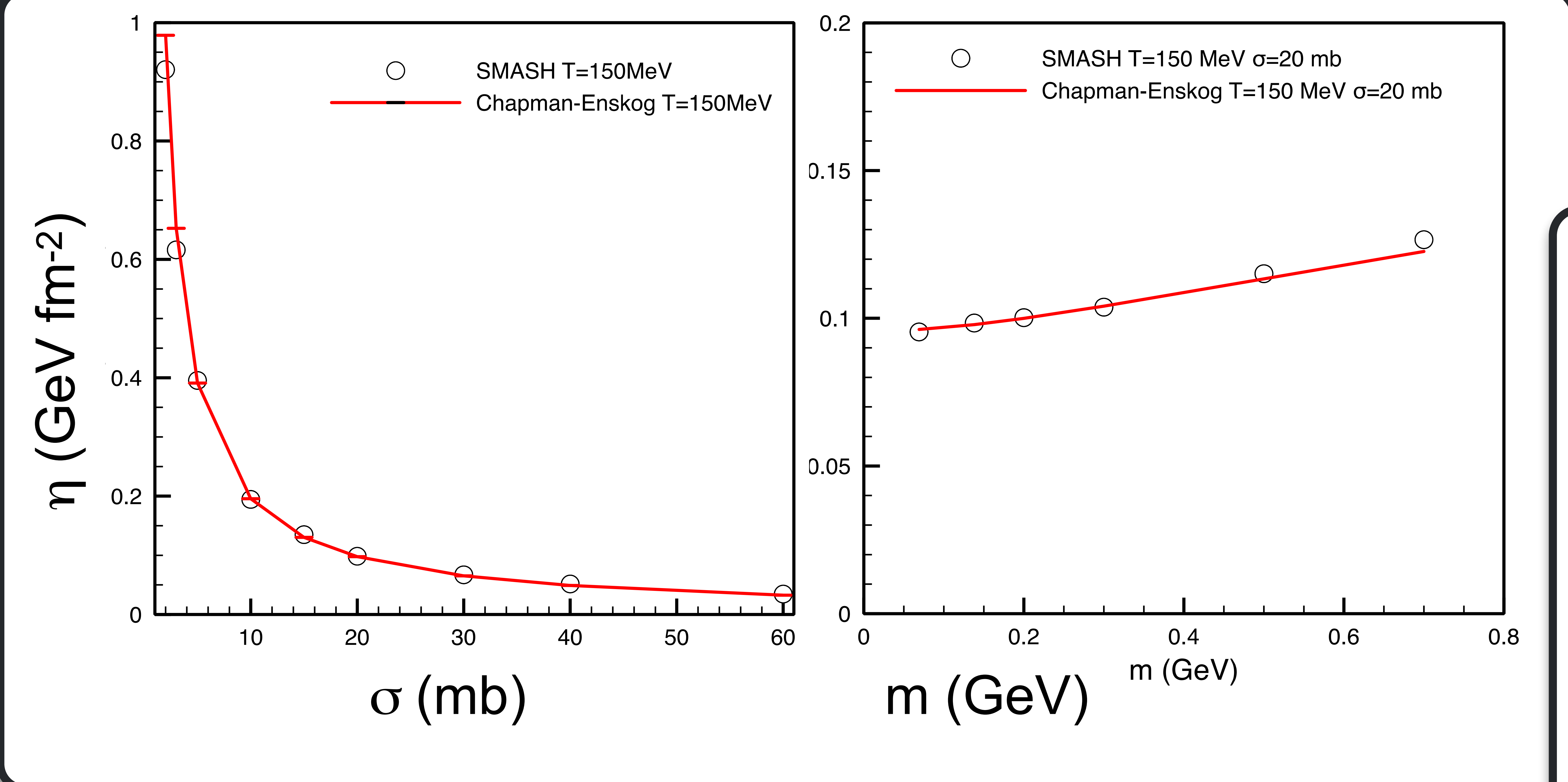
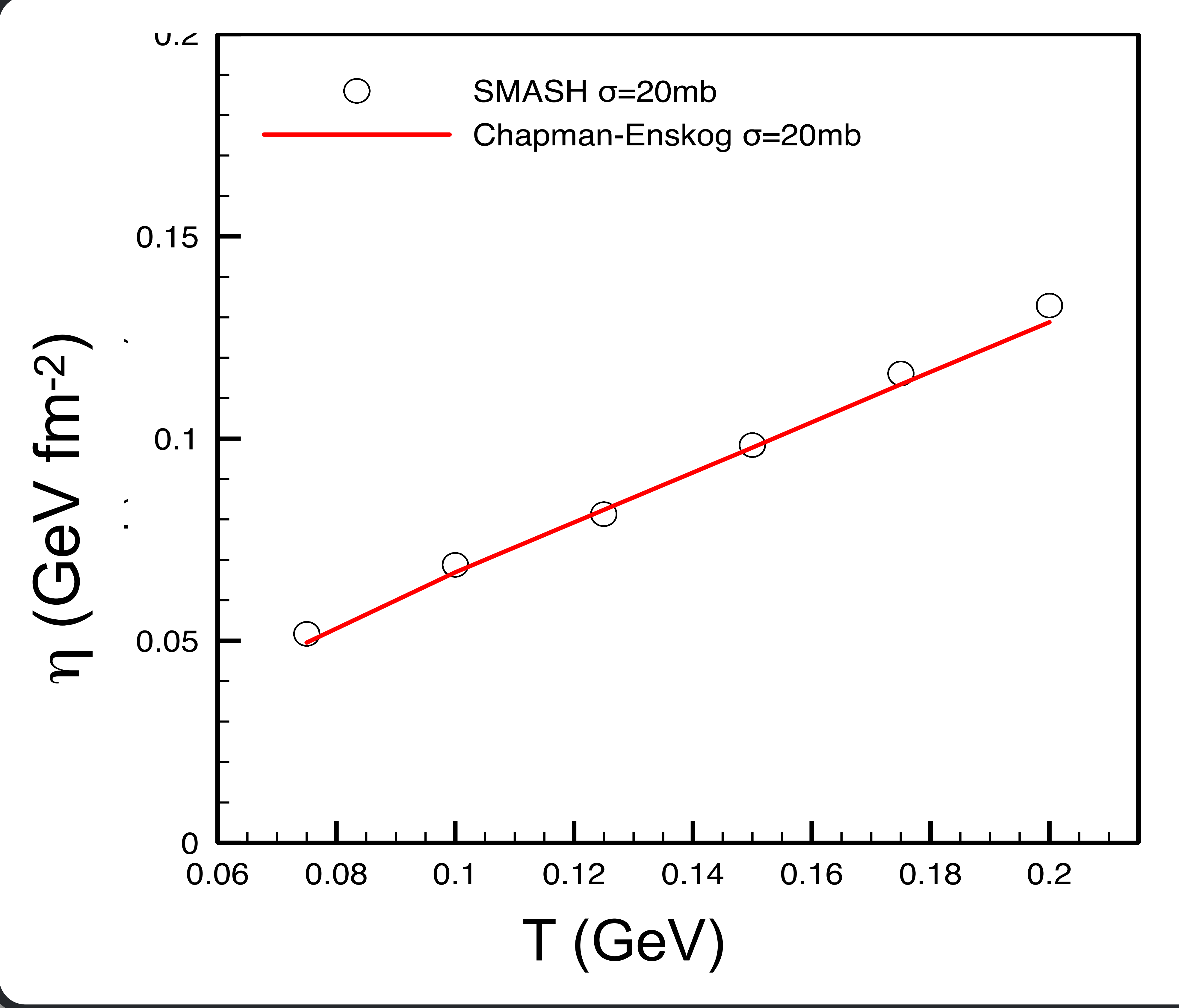




Test case #1: π with constant σ



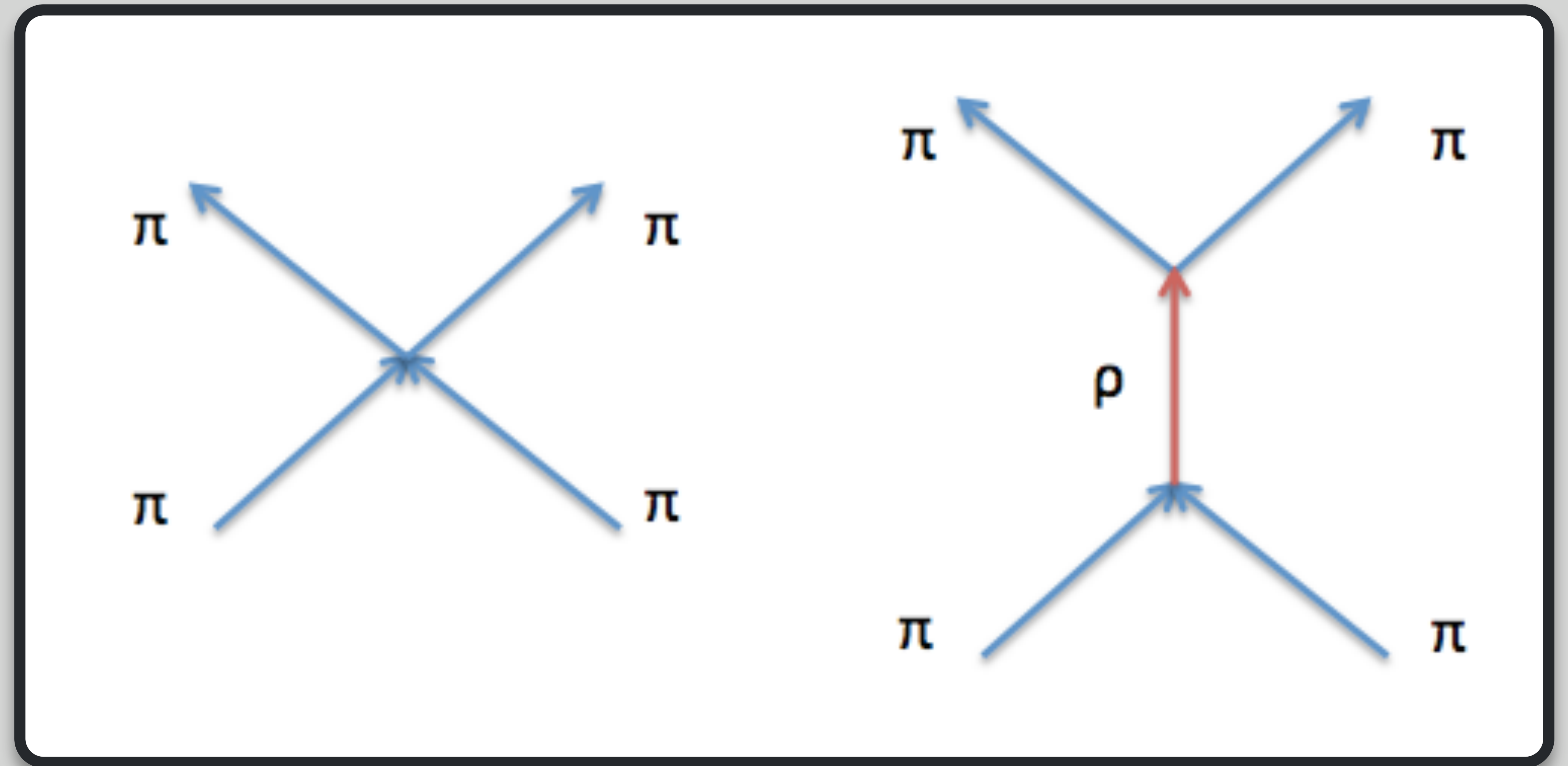


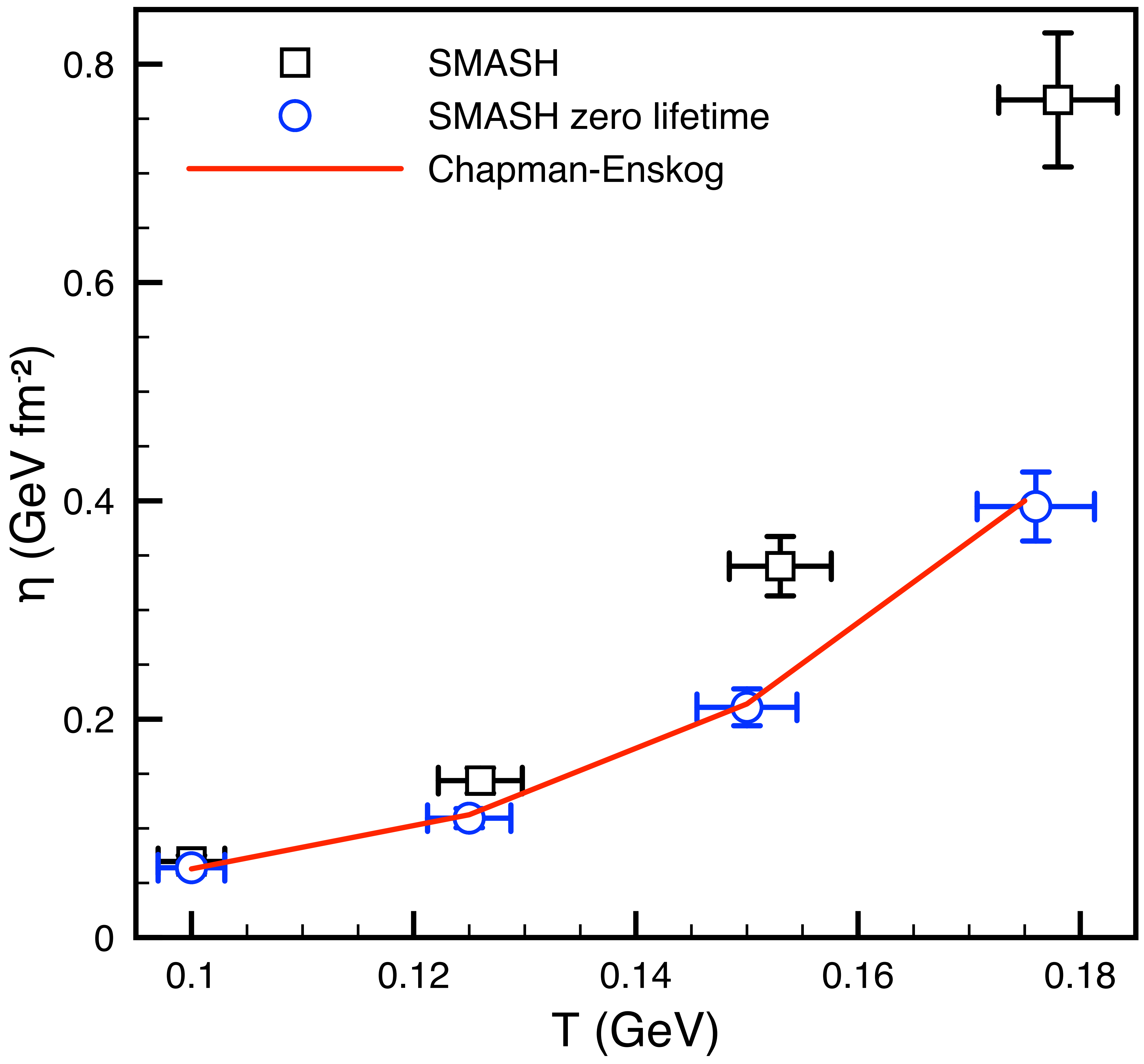
Main take-away:
The method is relatively insensitive to variations of parameters; maximum error is less than 10%

J. Torres-Rincon, PhD dissertation (2012), *Hadronic Transport Coefficients from Effective Field Theories*

Test case #2: π-ρ gas

- Normal SMASH run does not coincide directly with Chapman-Enskog
  - Resonance lifetimes





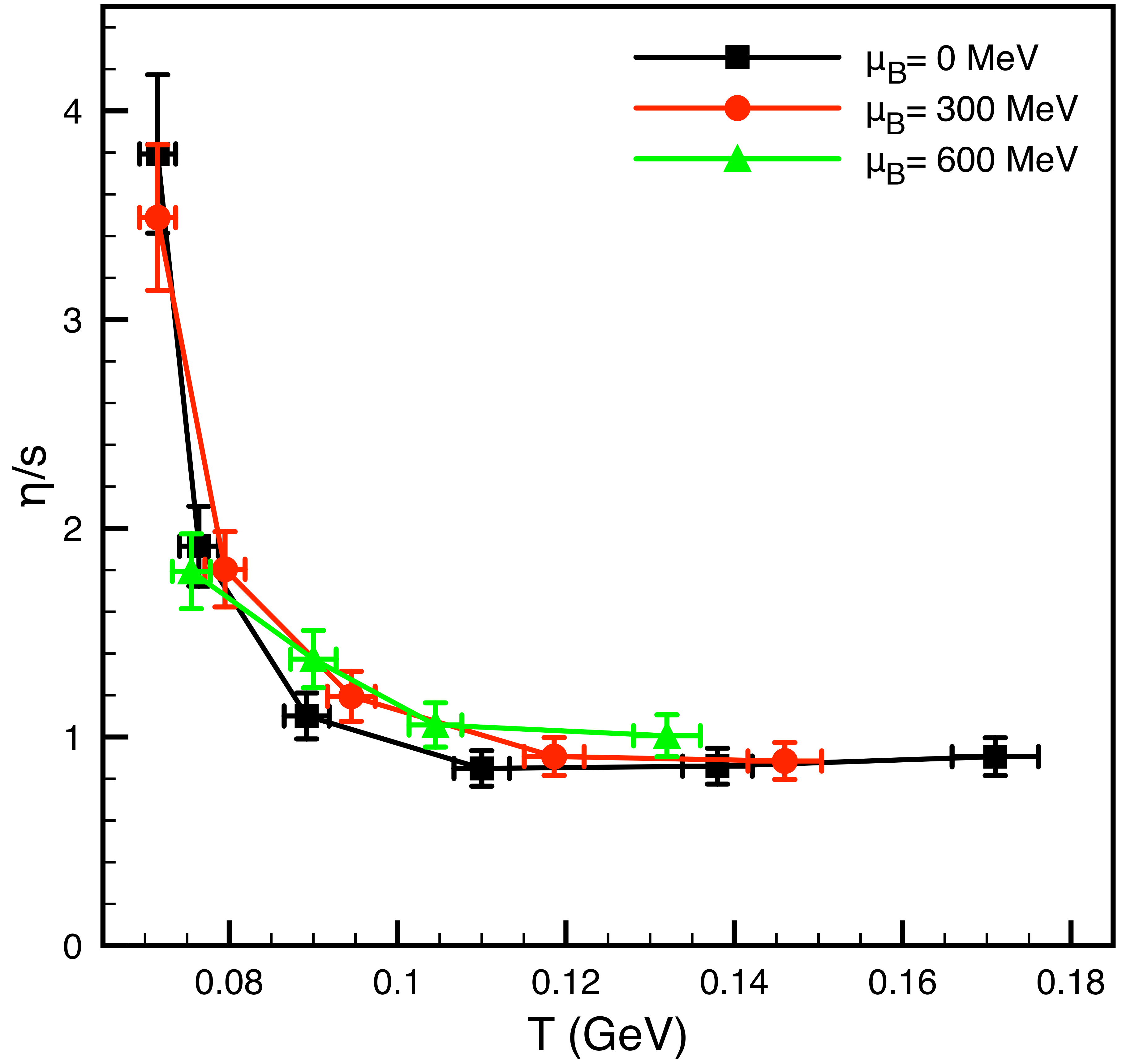
Full hadron gas: Degrees of freedom

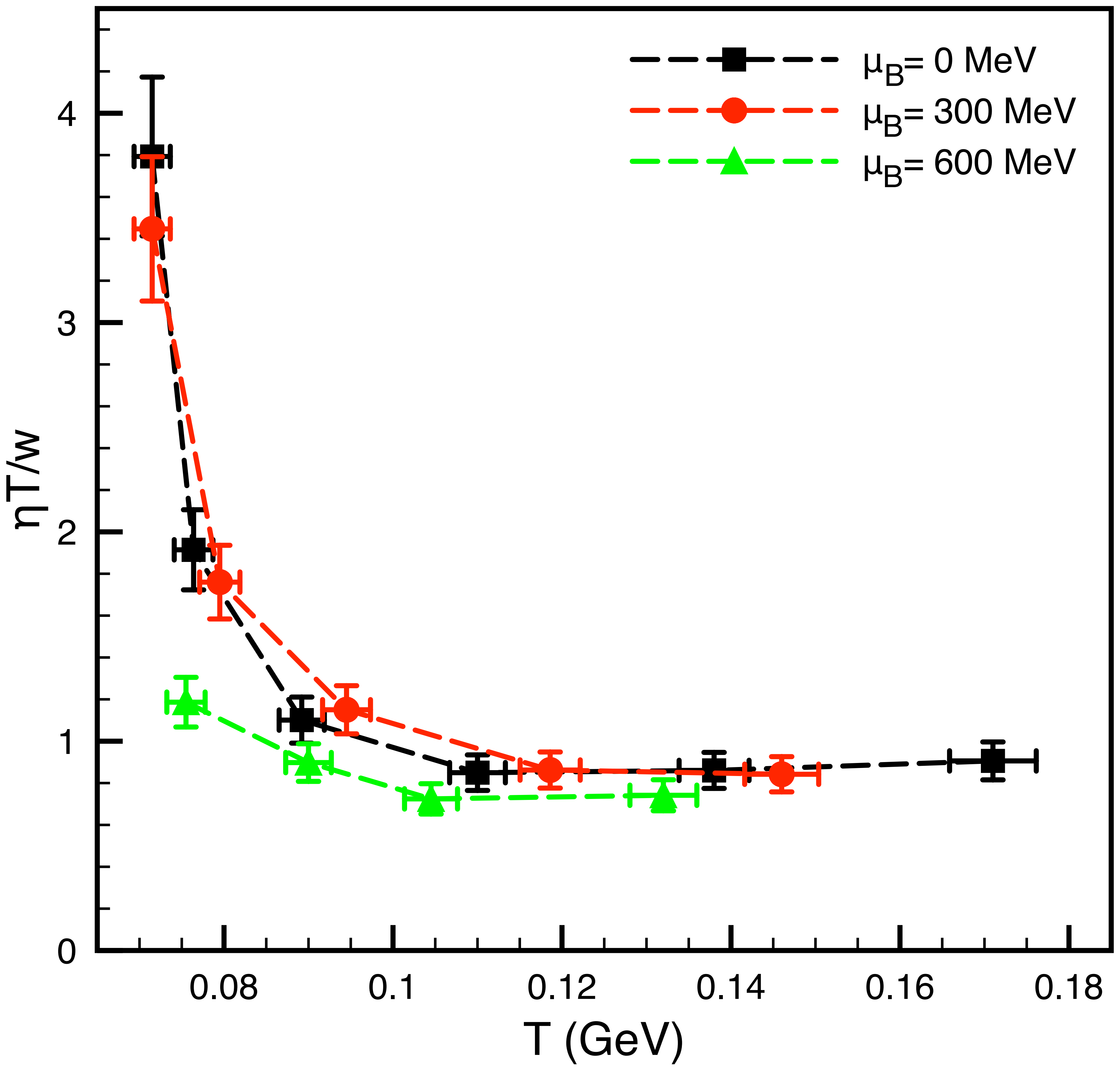
N	Δ	Λ	Σ	Ξ	Ω	Unflavored				Strange
N_{938}	Δ_{1232}	Λ_{1116}	Σ_{1189}	Ξ_{1321}	Ω^-_{1672}	π_{138}	$f_{0\ 980}$	$f_{2\ 1275}$	$\pi_{2\ 1670}$	K_{494}
N_{1440}	Δ_{1620}	Λ_{1405}	Σ_{1385}	Ξ_{1530}	Ω^-_{2250}	π_{1300}	$f_{0\ 1370}$	$f_2'_{1525}$		K^*_{892}
N_{1520}	Δ_{1700}	Λ_{1520}	Σ_{1660}	Ξ_{1690}		π_{1800}	$f_{0\ 1500}$	$f_{2\ 1950}$	$\rho_{3\ 1690}$	$K_{1\ 1270}$
N_{1535}	Δ_{1905}	Λ_{1600}	Σ_{1670}	Ξ_{1820}			$f_{0\ 1710}$	$f_{2\ 2010}$		$K_{1\ 1400}$
N_{1650}	Δ_{1910}	Λ_{1670}	Σ_{1750}	Ξ_{1950}		η_{548}		$f_{2\ 2300}$	$\varphi_{3\ 1850}$	K^*_{1410}
N_{1675}	Δ_{1920}	Λ_{1690}	Σ_{1775}	Ξ_{2030}		η'_{958}	$a_{0\ 980}$	$f_{2\ 2340}$		$K_0^*{}_{1430}$
N_{1680}	Δ_{1930}	Λ_{1800}	Σ_{1915}			η_{1295}	$a_{0\ 1450}$		$a_{4\ 2040}$	$K_2^*{}_{1430}$
N_{1700}	Δ_{1950}	Λ_{1810}	Σ_{1940}			η_{1405}		$f_{1\ 1285}$		K^*_{1680}
N_{1710}		Λ_{1820}	Σ_{2030}			η_{1475}	φ_{1019}	$f_{1\ 1420}$	$f_{4\ 2050}$	$K_{2\ 1770}$
N_{1720}		Λ_{1830}	Σ_{2250}				φ_{1680}			$K_3^*{}_{1780}$
N_{1875}		Λ_{1890}				σ_{800}		$a_{2\ 1320}$		$K_{2\ 1820}$
N_{1900}		Λ_{2100}					$h_{1\ 1170}$			$K_4^*{}_{2045}$
N_{1990}		Λ_{2110}				ρ_{776}		$\pi_{1\ 1400}$		
N_{2080}		Λ_{2350}				ρ_{1450}	$b_{1\ 1235}$	$\pi_{1\ 1600}$		
N_{2190}						ρ_{1700}				
N_{2220}							$a_{1\ 1260}$	$\eta_{2\ 1645}$		
N_{2250}						ω_{783}				
						ω_{1420}		$\omega_{3\ 1670}$		
						ω_{1650}				

- + anti-particles
- Isospin symmetry

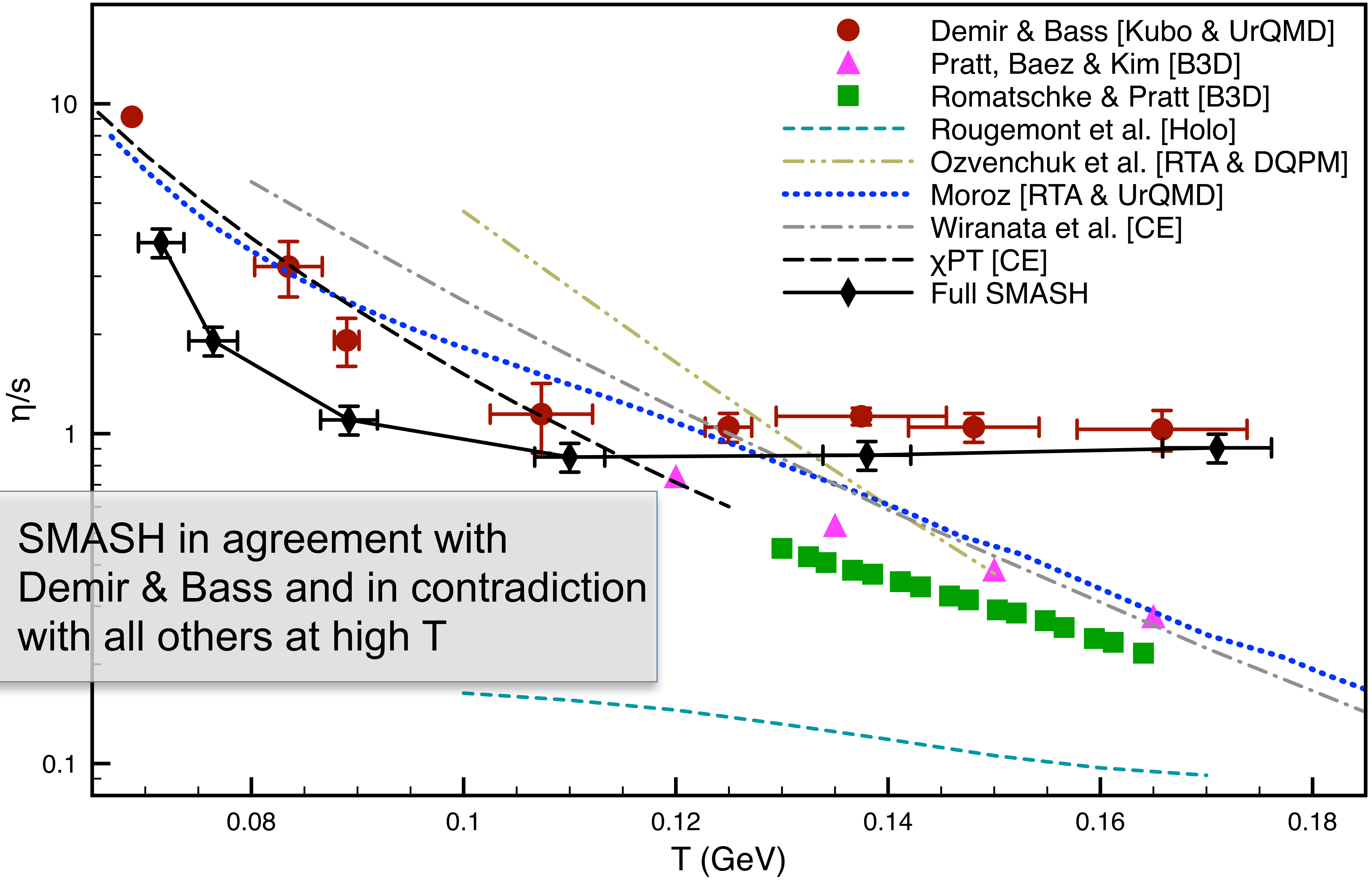
SMASH v1.6, smash-transport.github.io, DOI:10.5281/zenodo.3485108

Hadron Gas: T and μ_B dependence





HG: Viscosity Comparison



SMASH in agreement with Demir & Bass and in contradiction with all others at high T

-Demir & Bass, Phys.Rev.Lett. 102 (2009) 172302
-Pratt, Baez & Kim, Phys. Rev. C95 (2017) 024901
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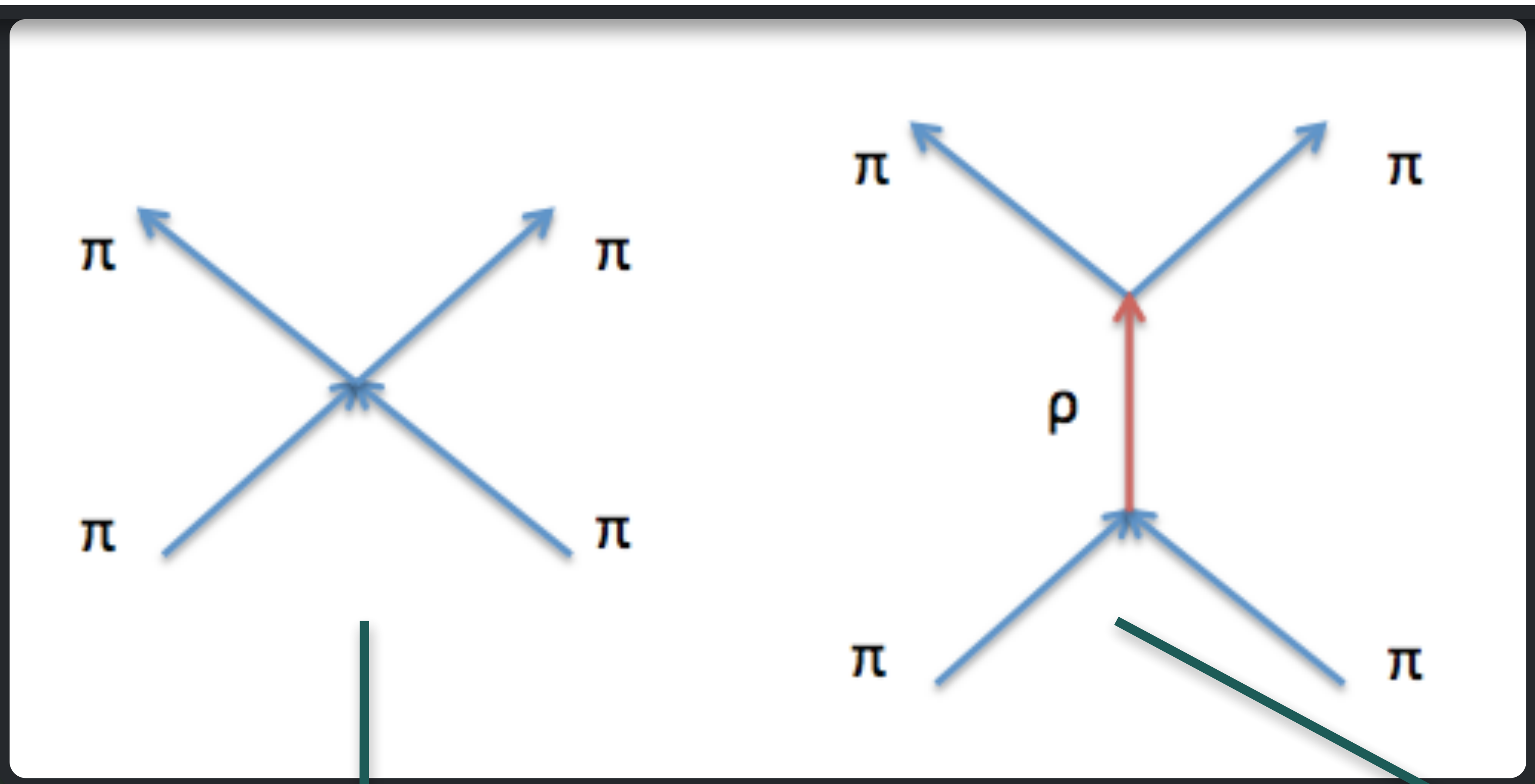
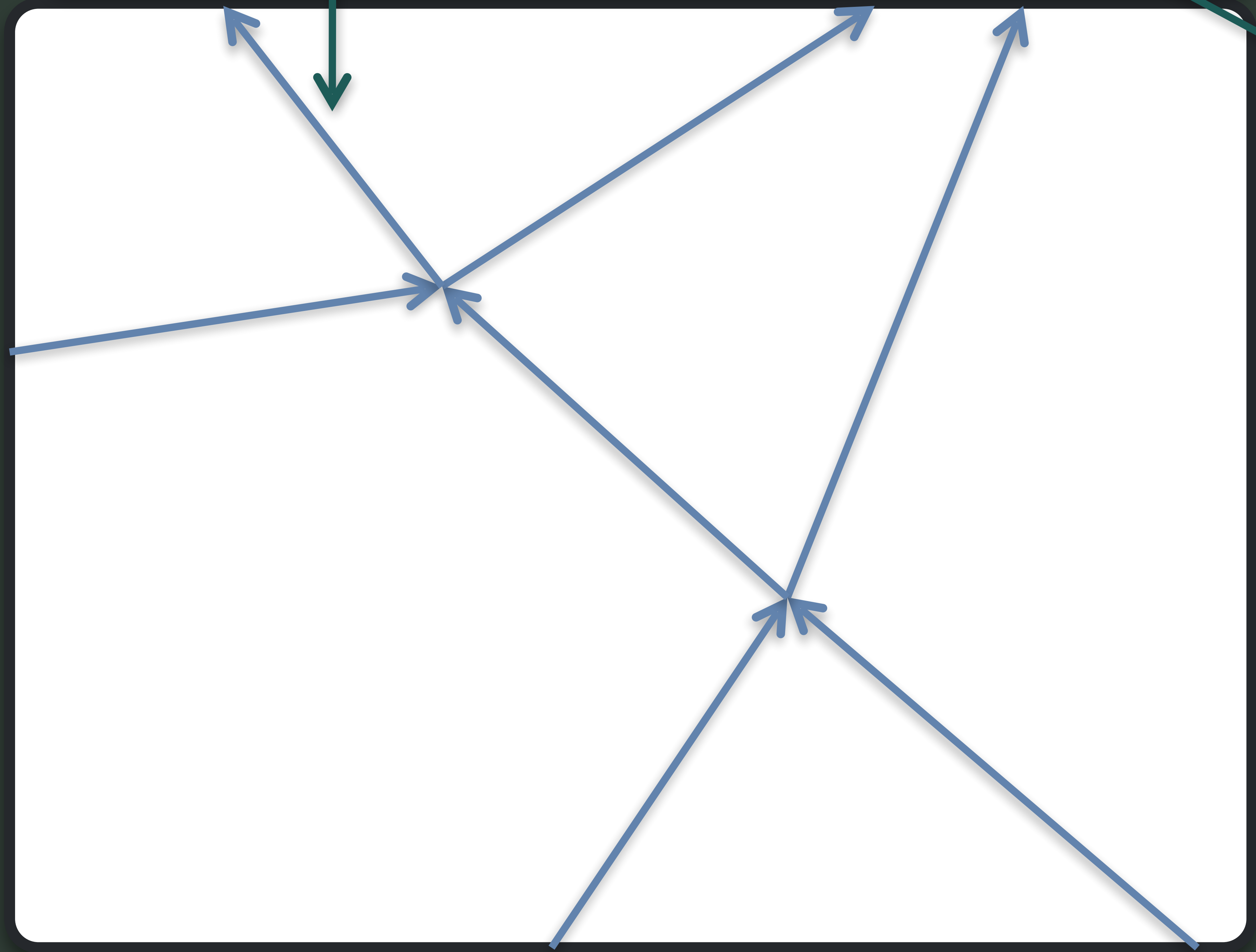
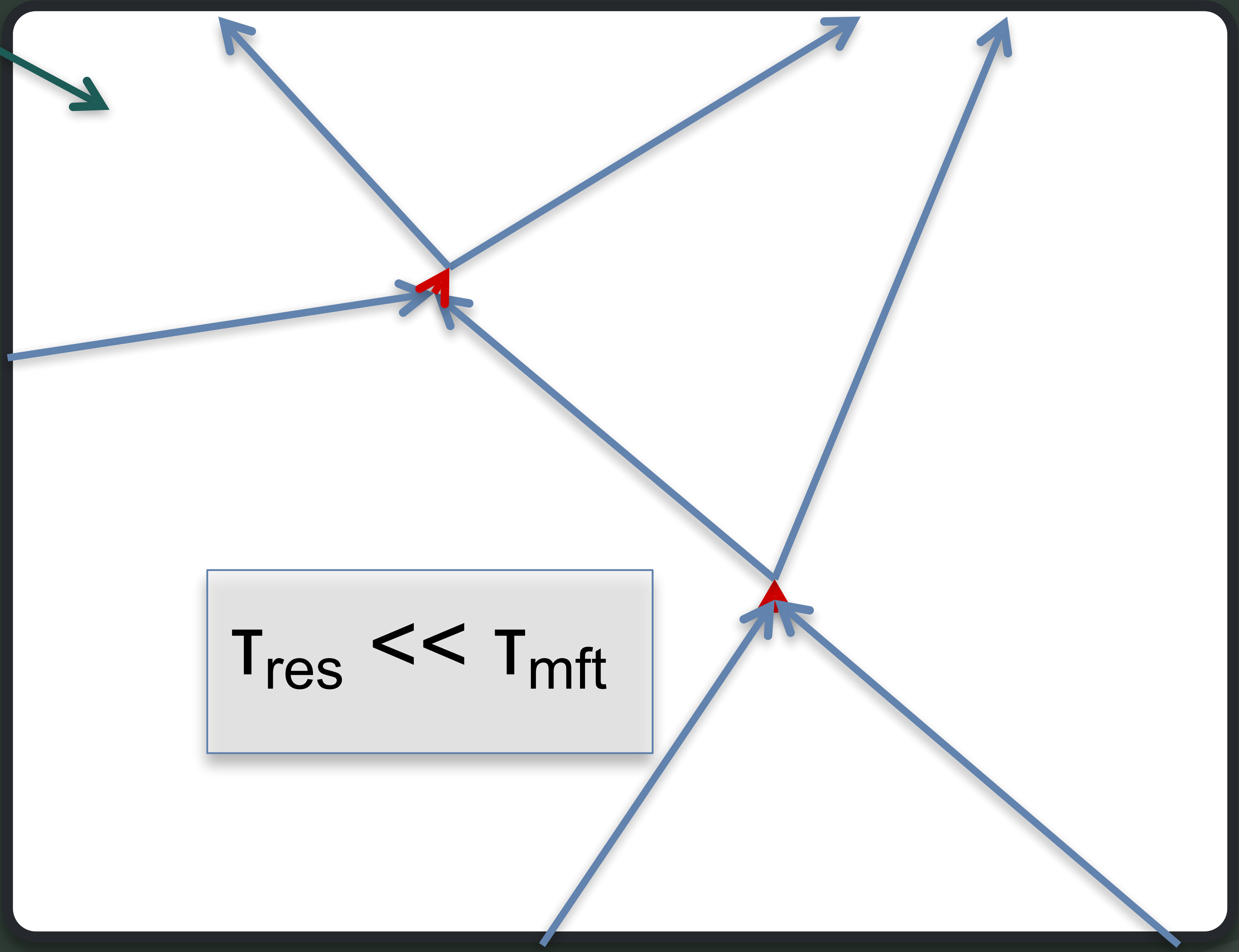
High temperature *η/s* : Resonance lifetimes



Must look at the microscopic picture from different descriptions
τ_{res} = resonance lifetime
τ_{mft} = mean free time

At high T and density:





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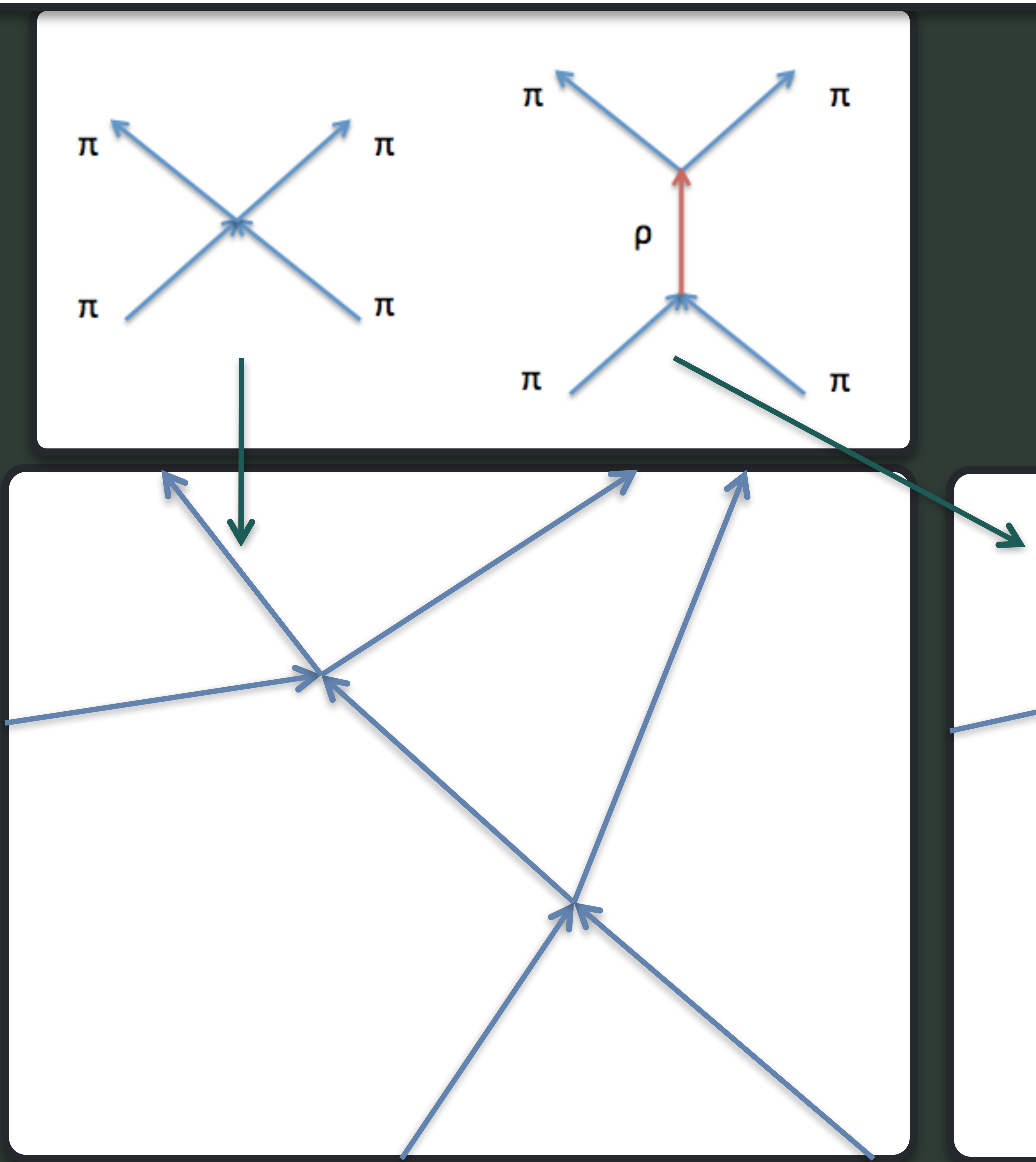

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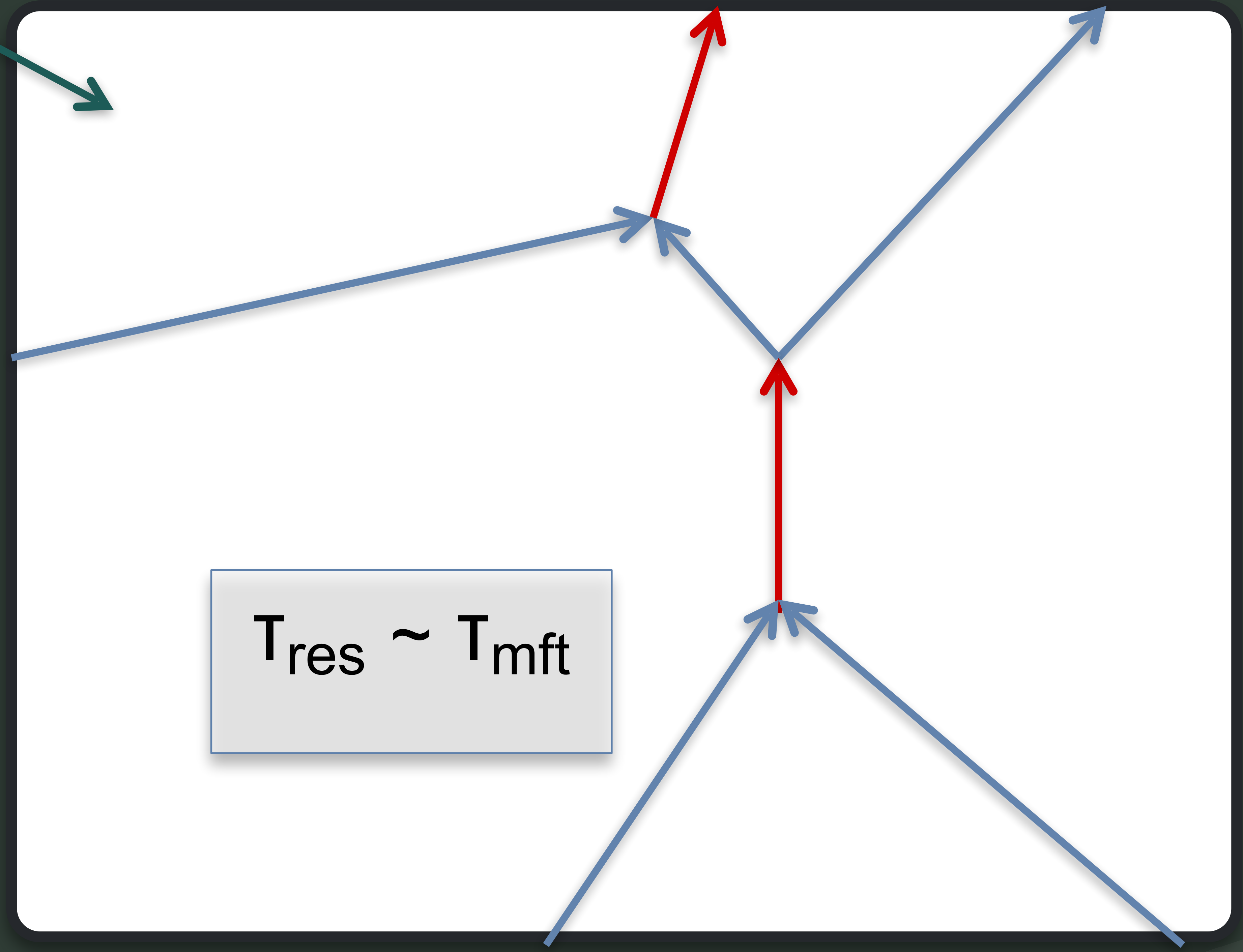

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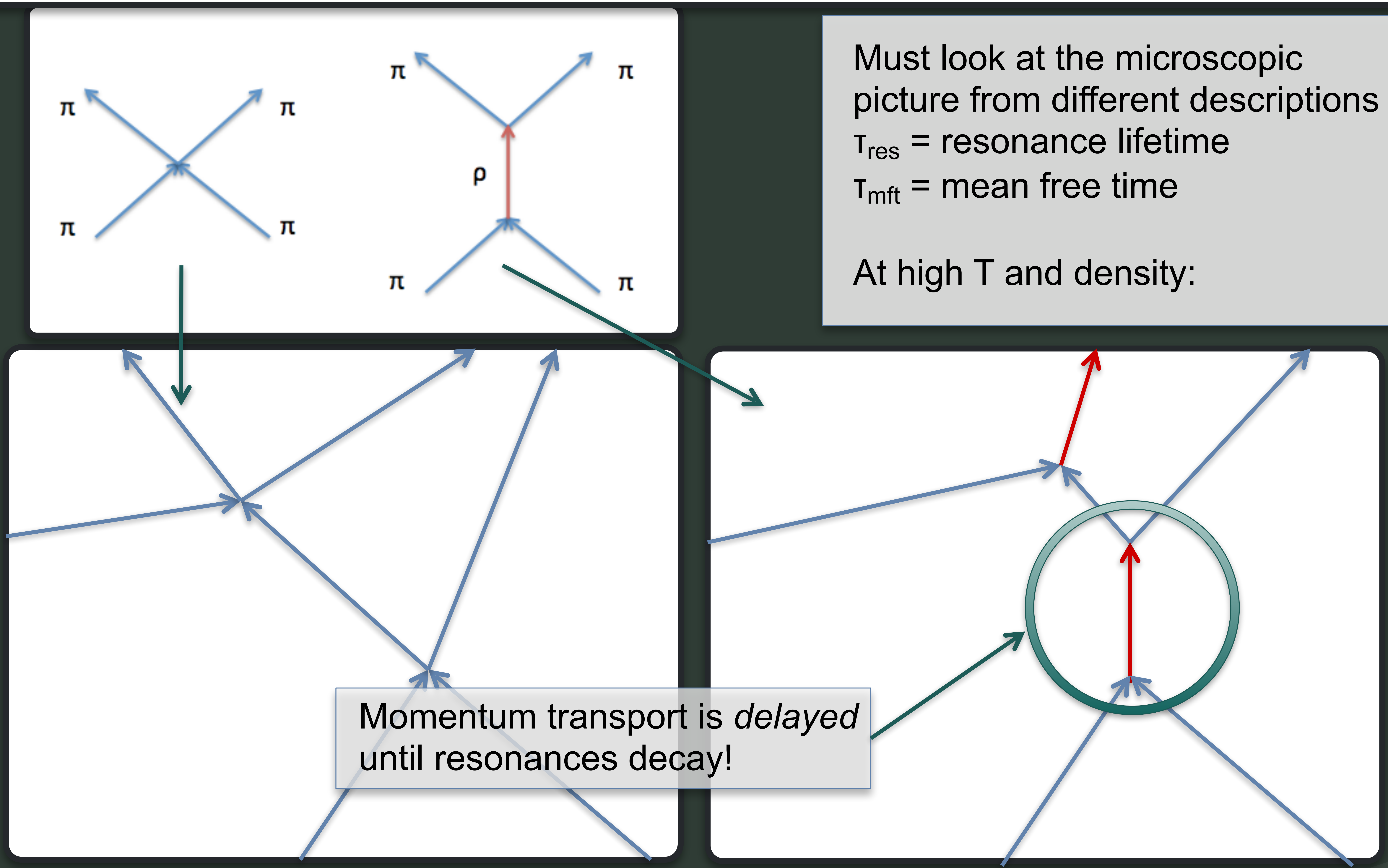
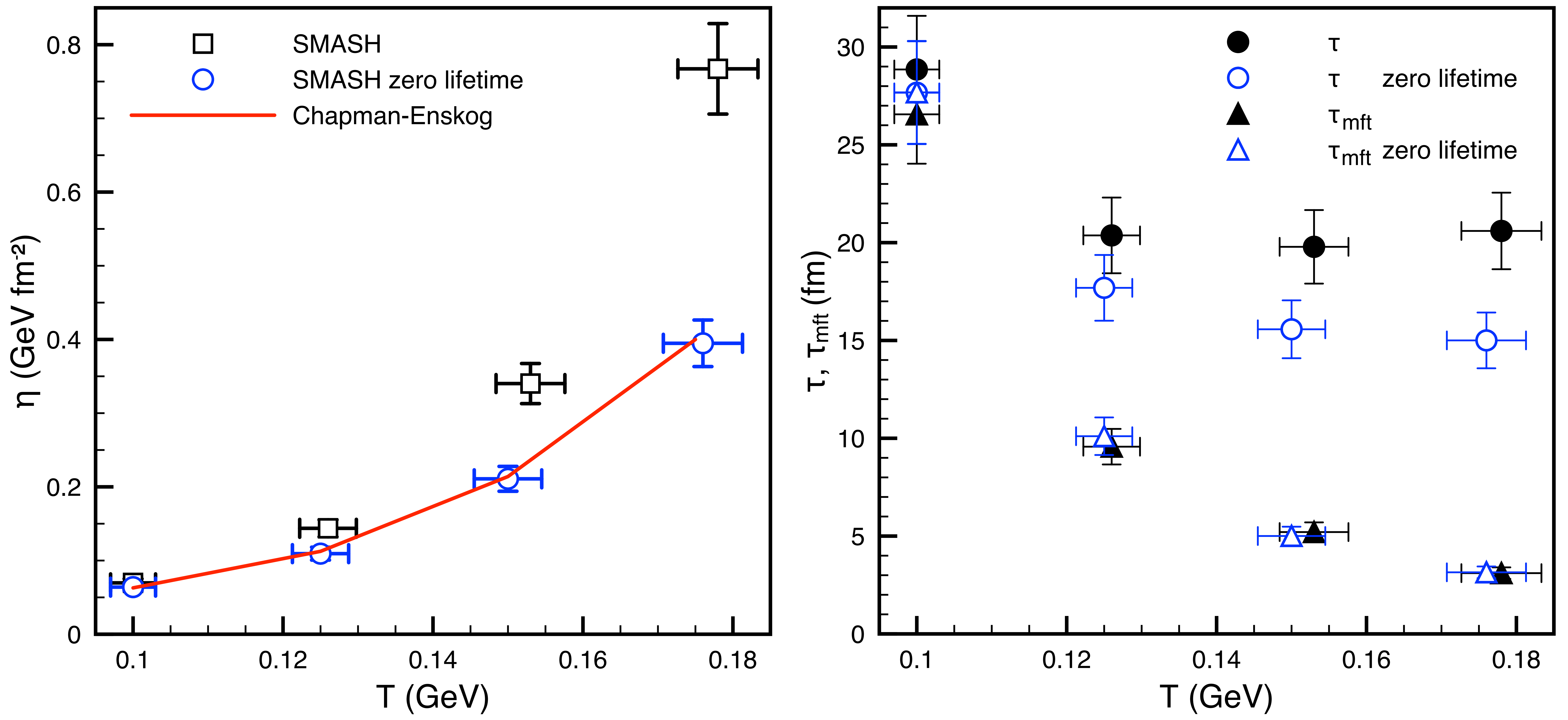


Momentum transport is *delayed* until resonances decay!

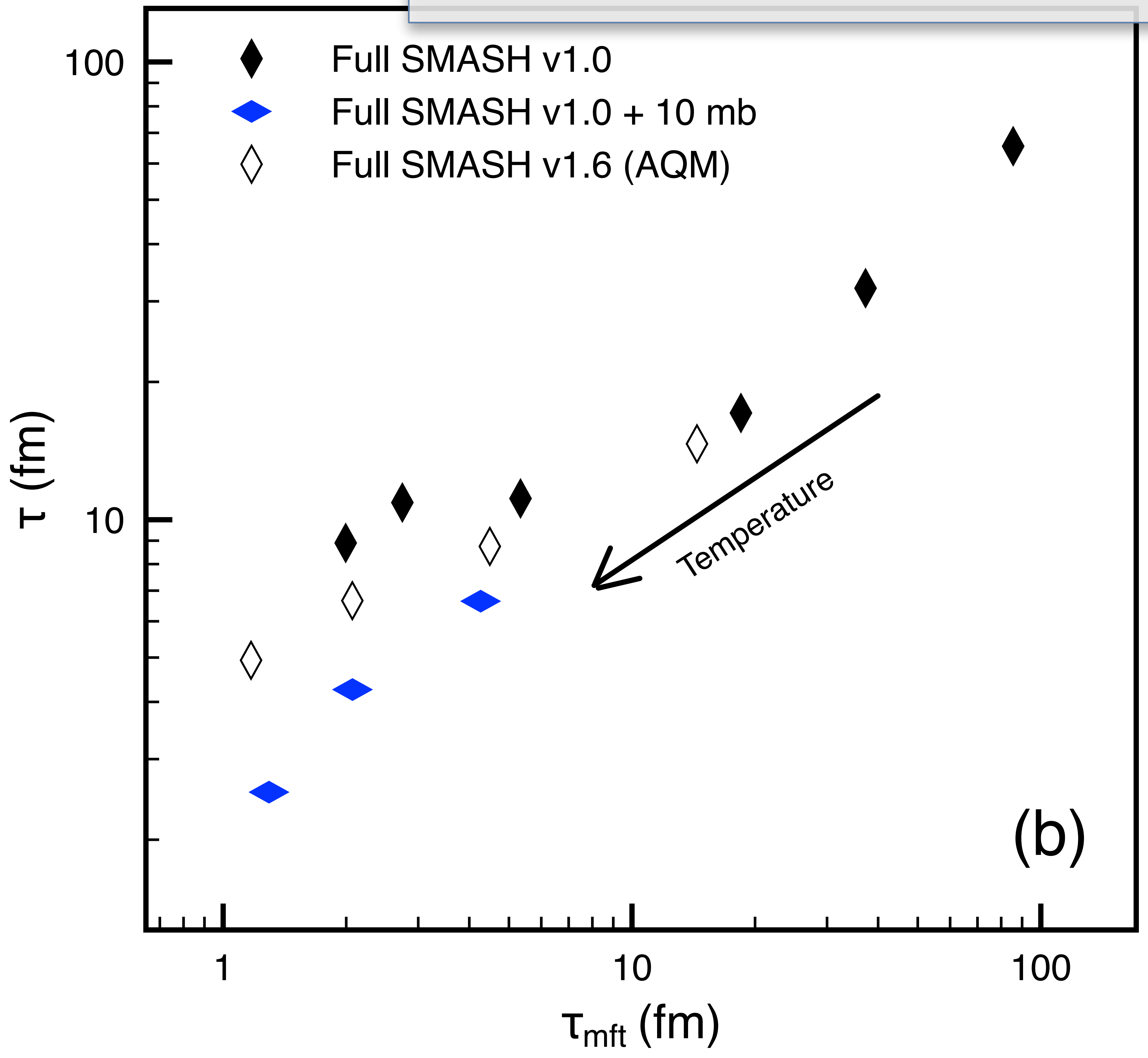
π-ρ: Zero lifetimes vs relaxation time

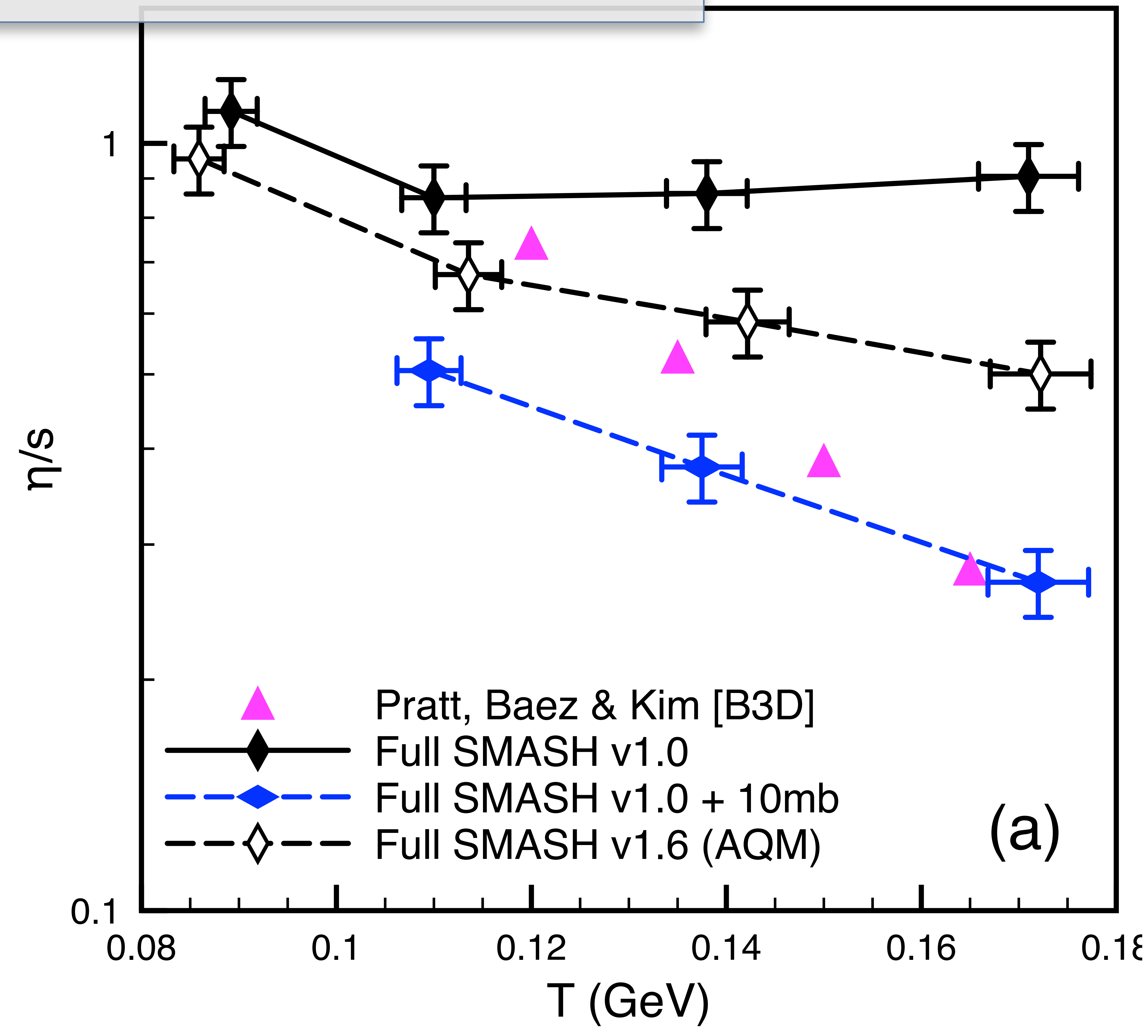
Large part of the difference explained from eliminating lifetimes



Effect of many non-resonant interactions

Introduce a constant elastic cross-section between all particles or the AQM to add many non-resonant interactions





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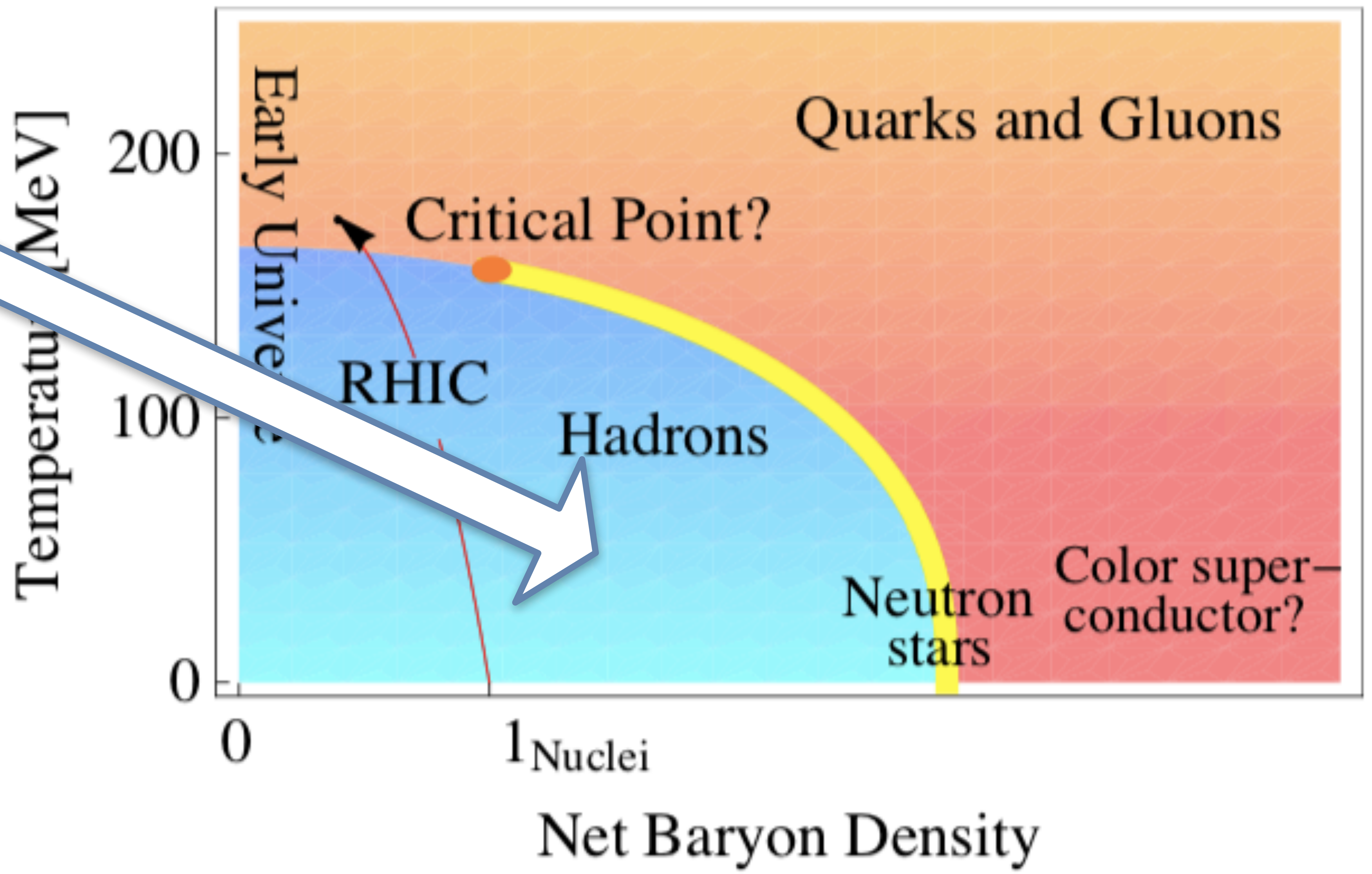
Constraining hadronic active degrees of freedom

- Composed of hadrons
- Which ones are active degrees of freedom, and do we know them all?
- How do we constrain this?
- Additional ways of constraining these properties are needed: this talk aims to provide one such new path



Edelstein et al., AIP Conf.Proc. 1116 (2009) no.1, 265-284

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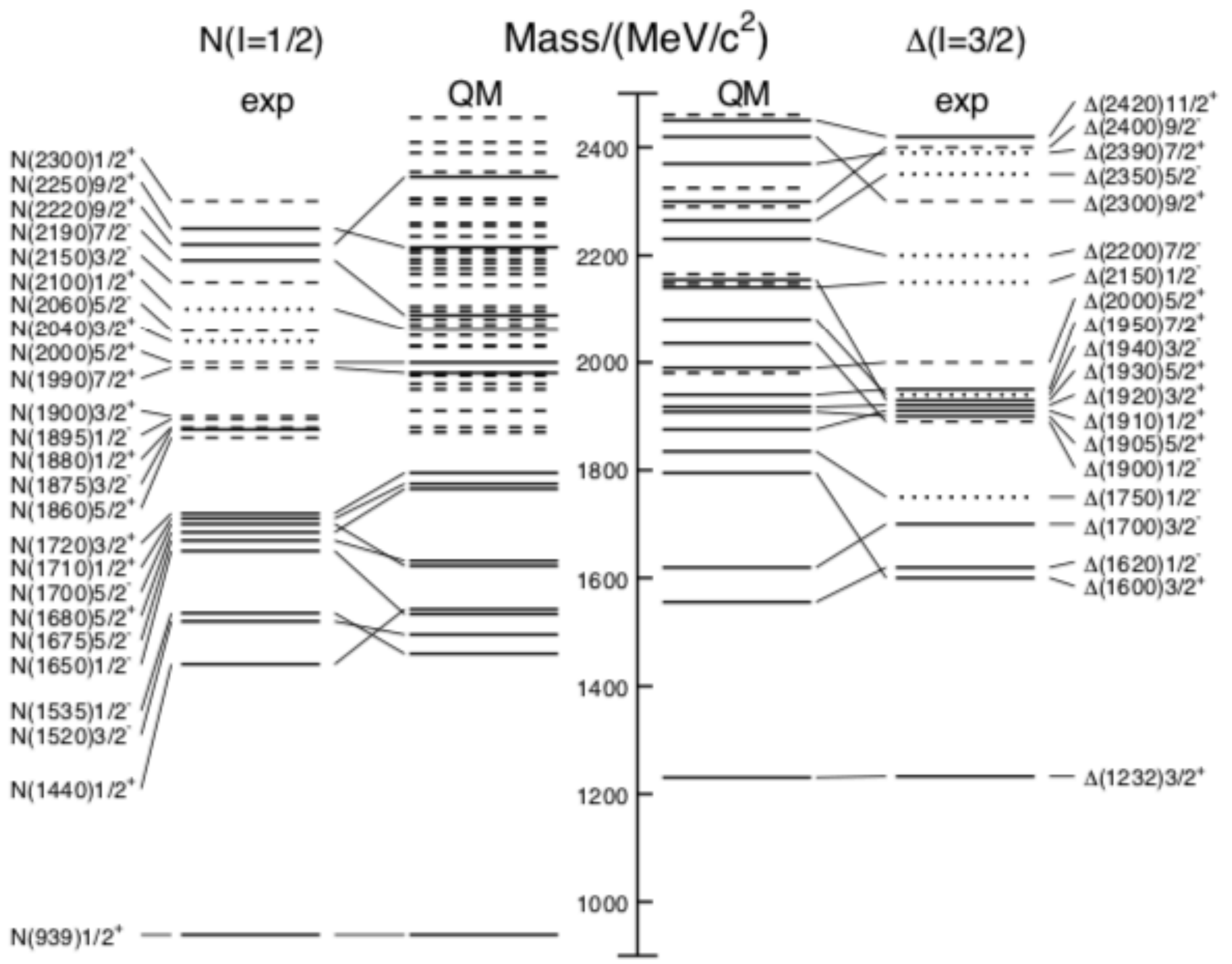

M. Tanabashi et al., Phys. Rev., vol. D98, no. 3, p. 030001, 2018

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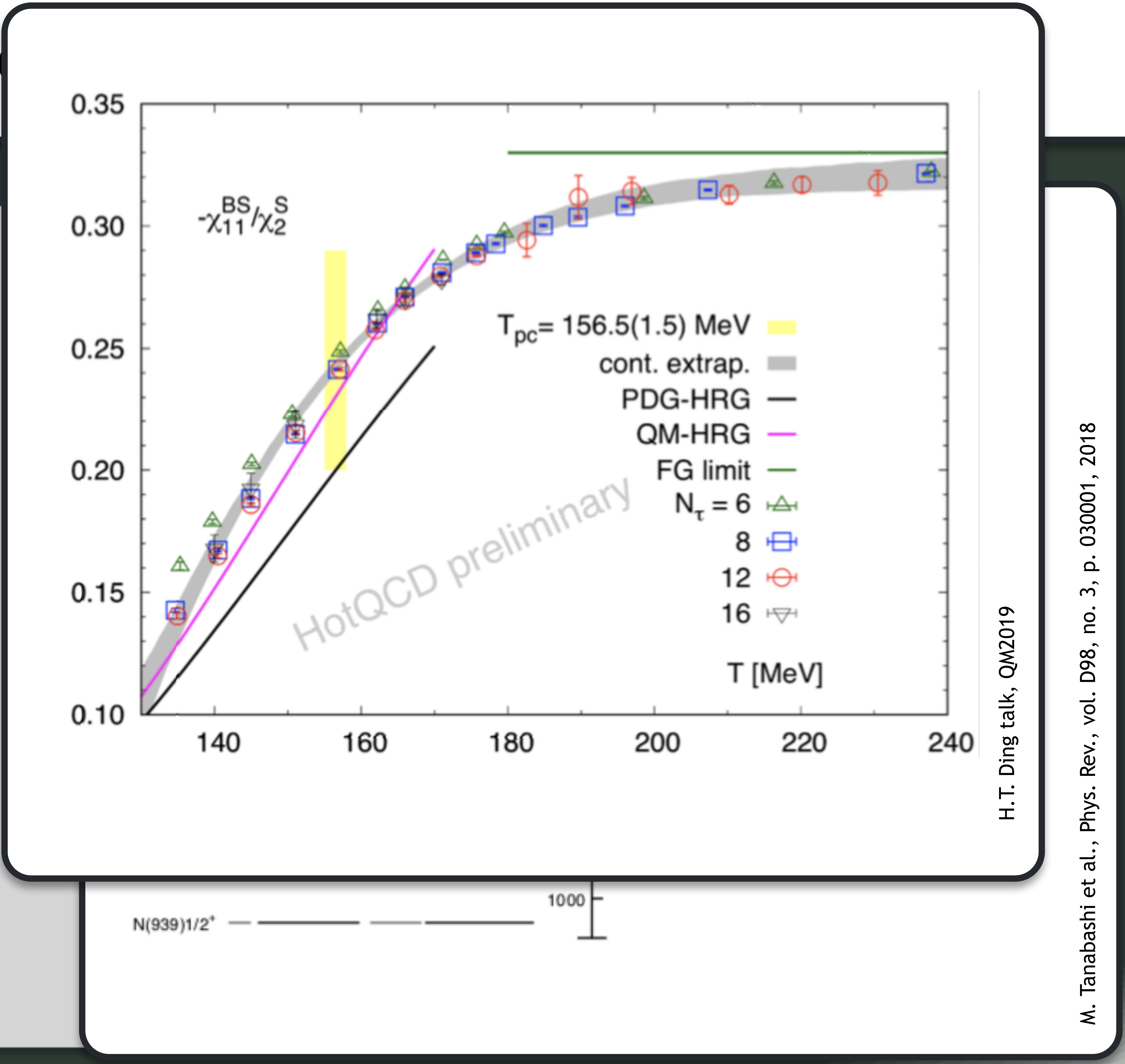

H.T. Ding talk, QM2019

M. Tanabashi et al., Phys. Rev., vol. D98, no. 3, p. 030001, 2018

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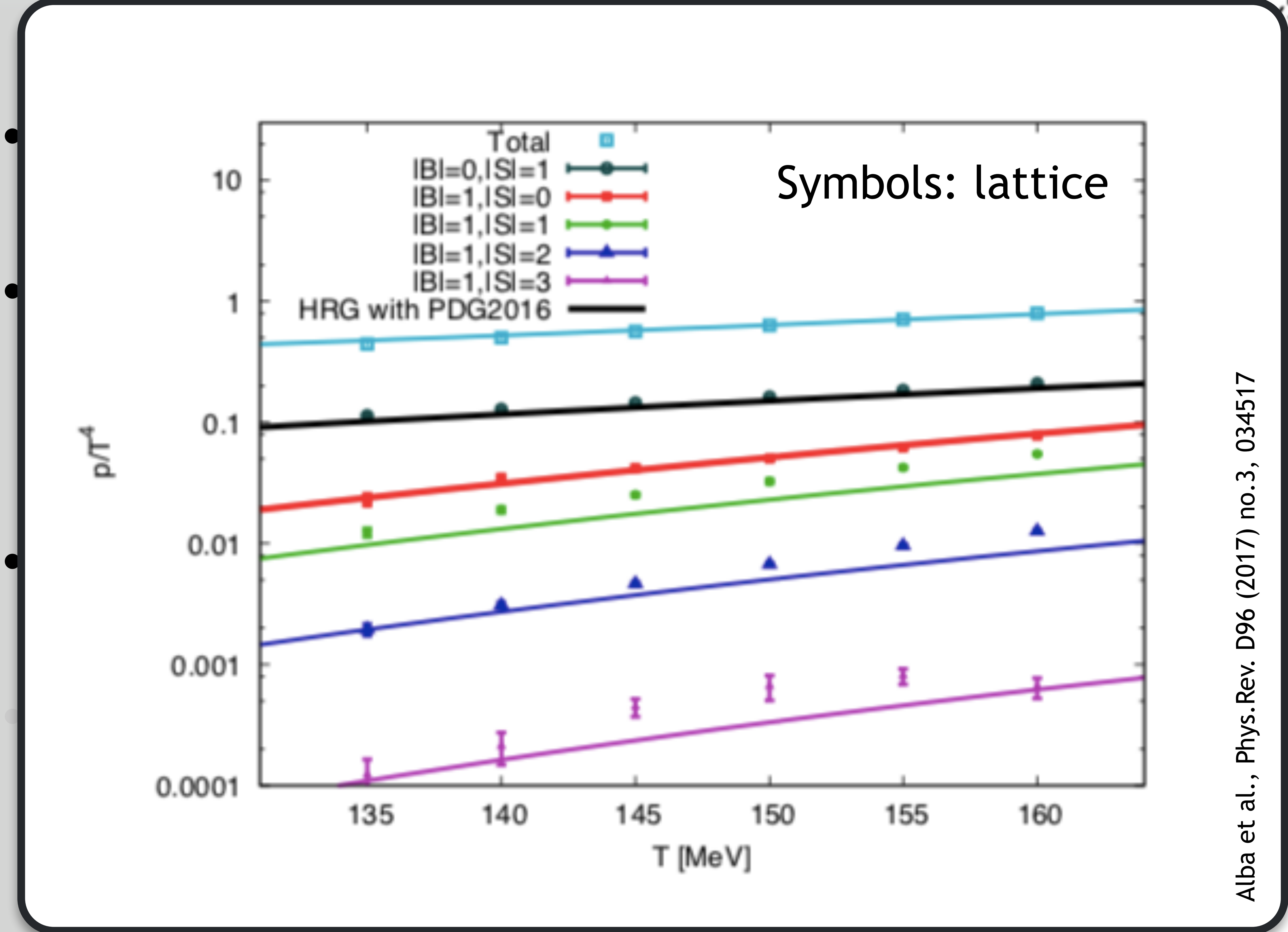
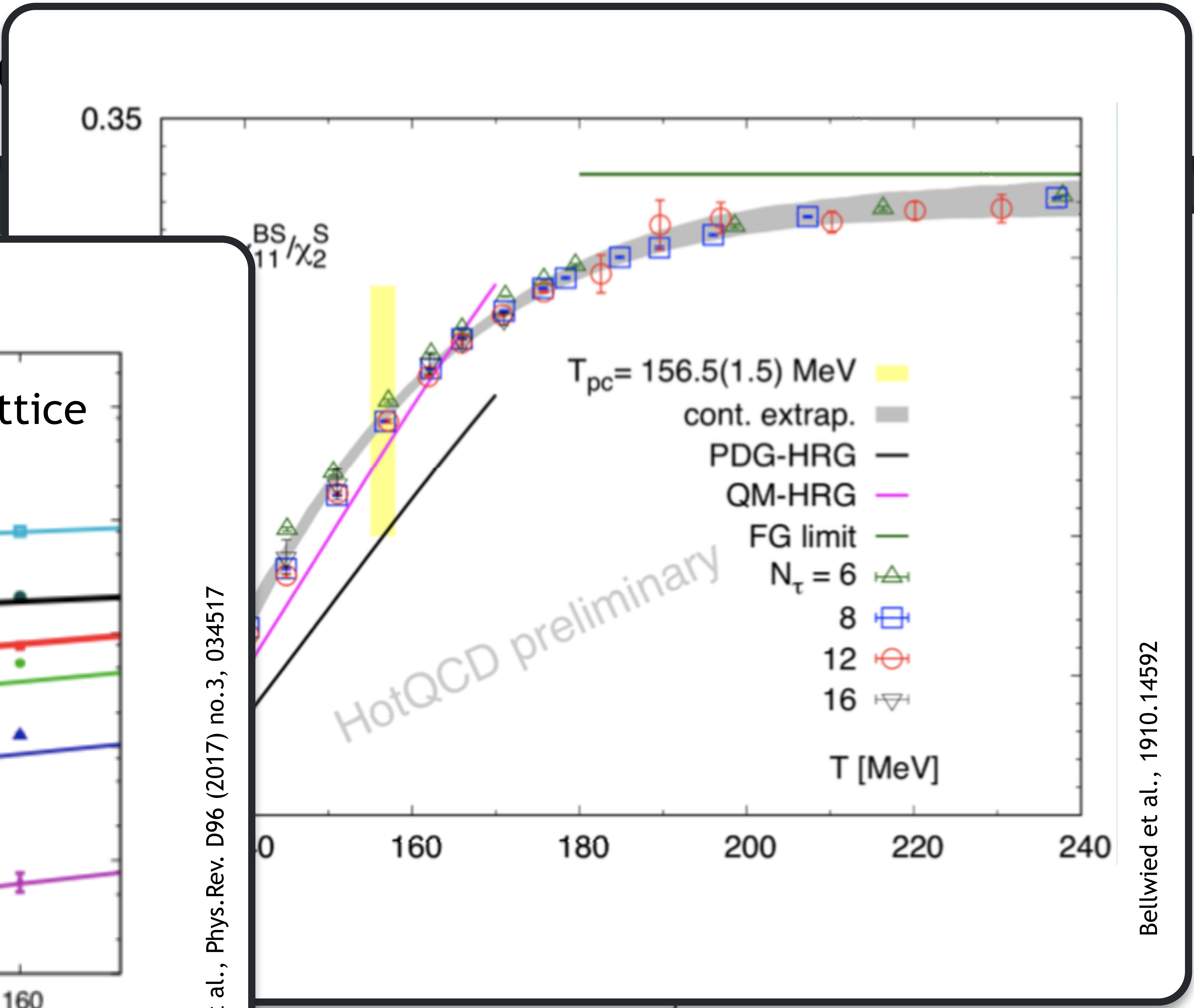
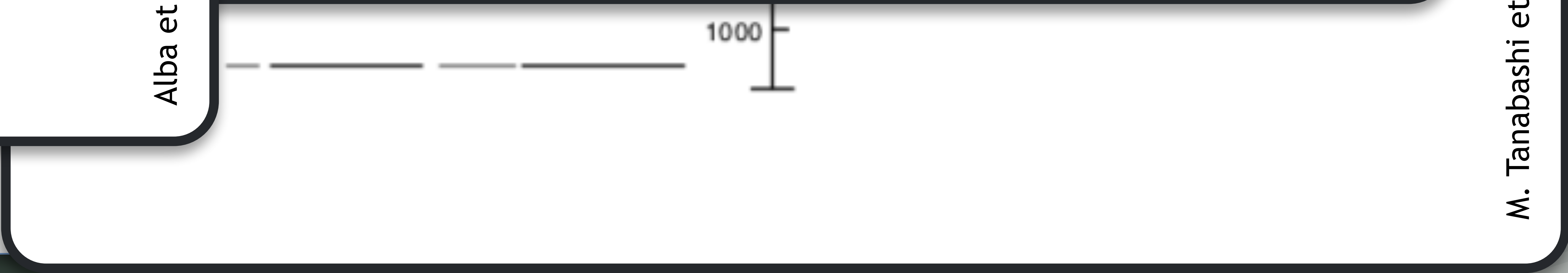


Alba et al., Phys.Rev. D96 (2017) no.3, 034517



Bellwied et al., 1910.14592



M. Tanabashi et al., Phys. Rev., vol. D98, no. 3, p. 030001, 2018

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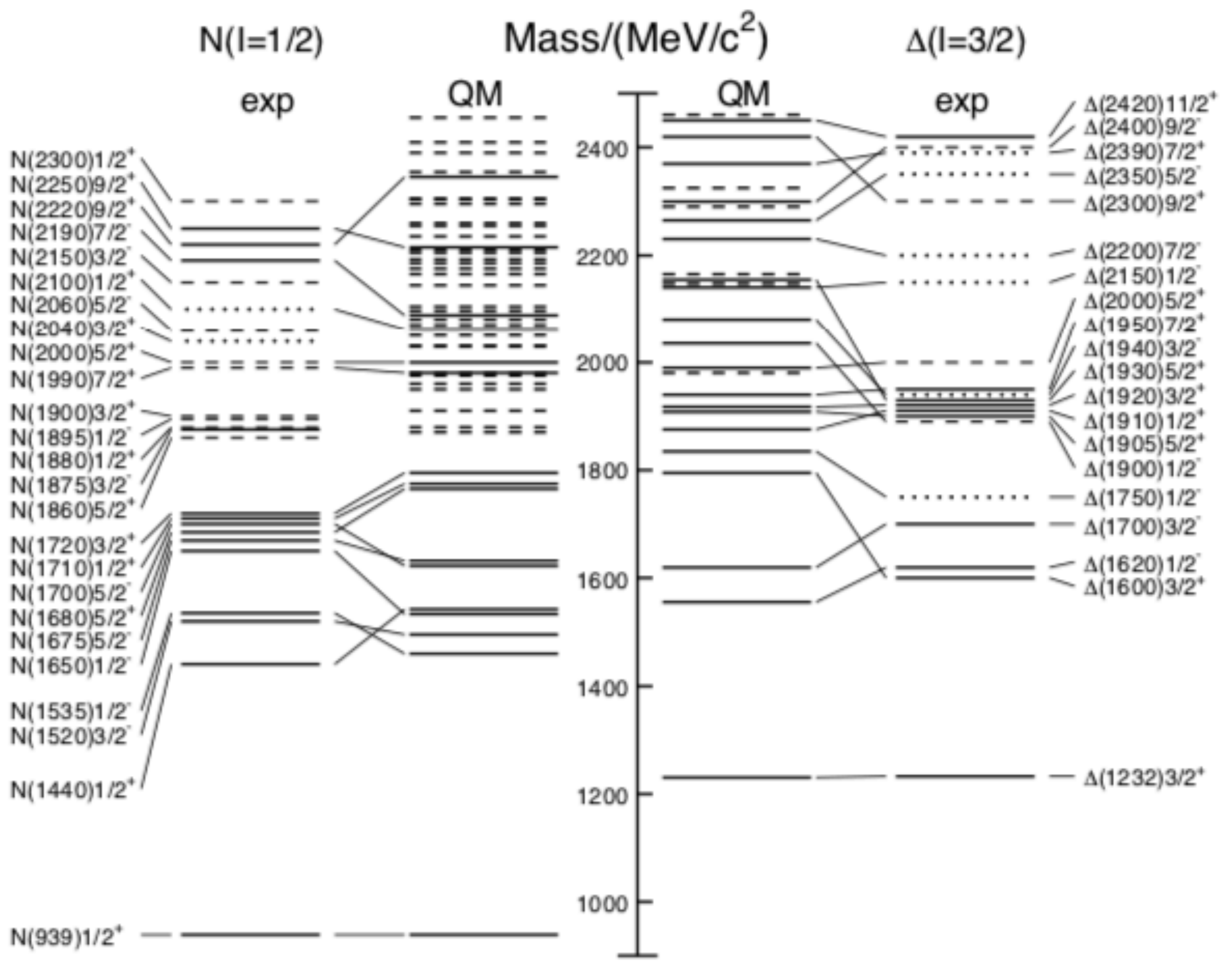
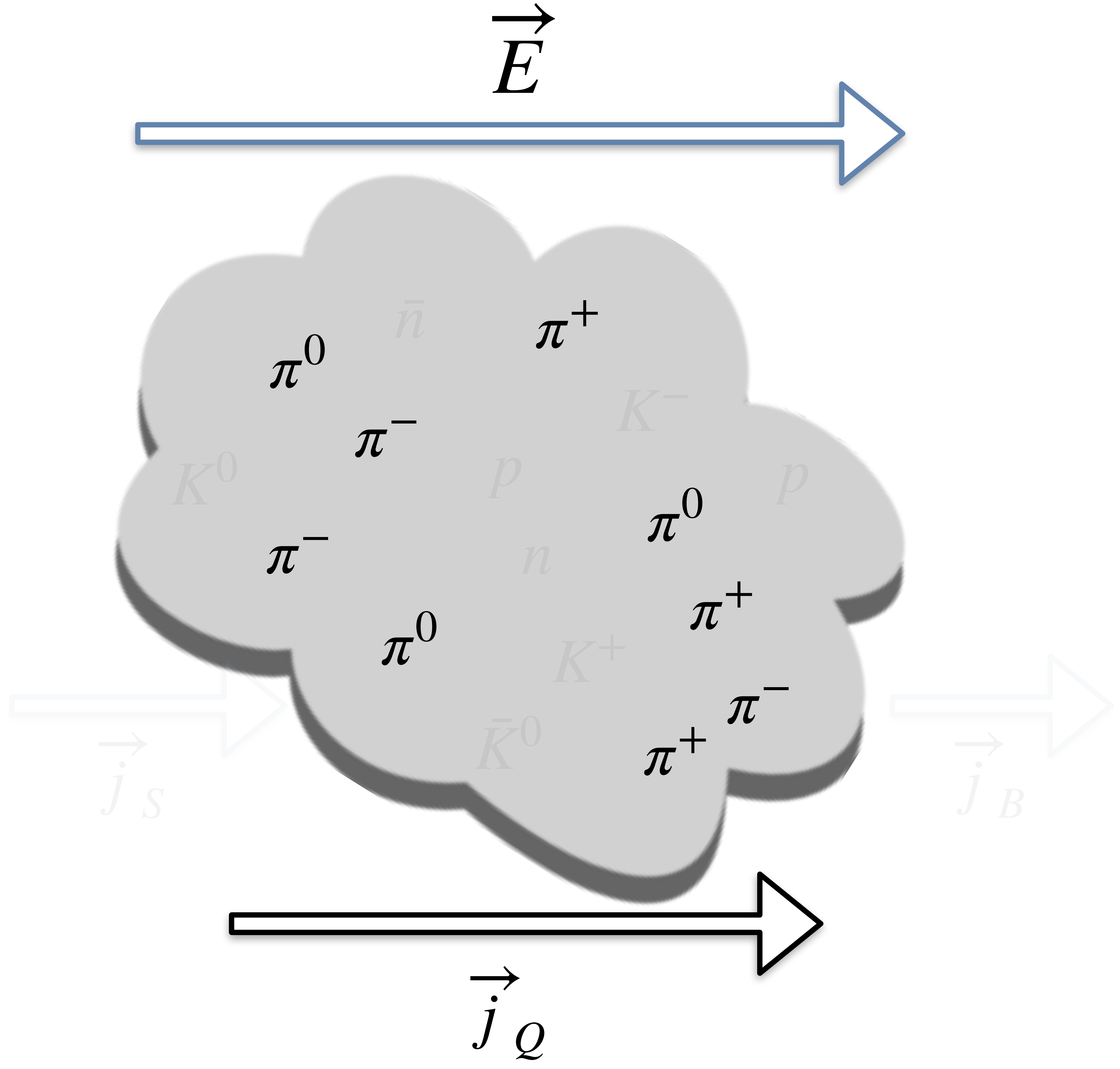


M. Tanabashi et al., Phys. Rev., vol. D98, no. 3, p. 030001, 2018

Electric cross-conductivity



- An electric field introduces an electric current:

$$\vec{j}_Q = \sigma_{QQ}\,\vec{E}_Q$$

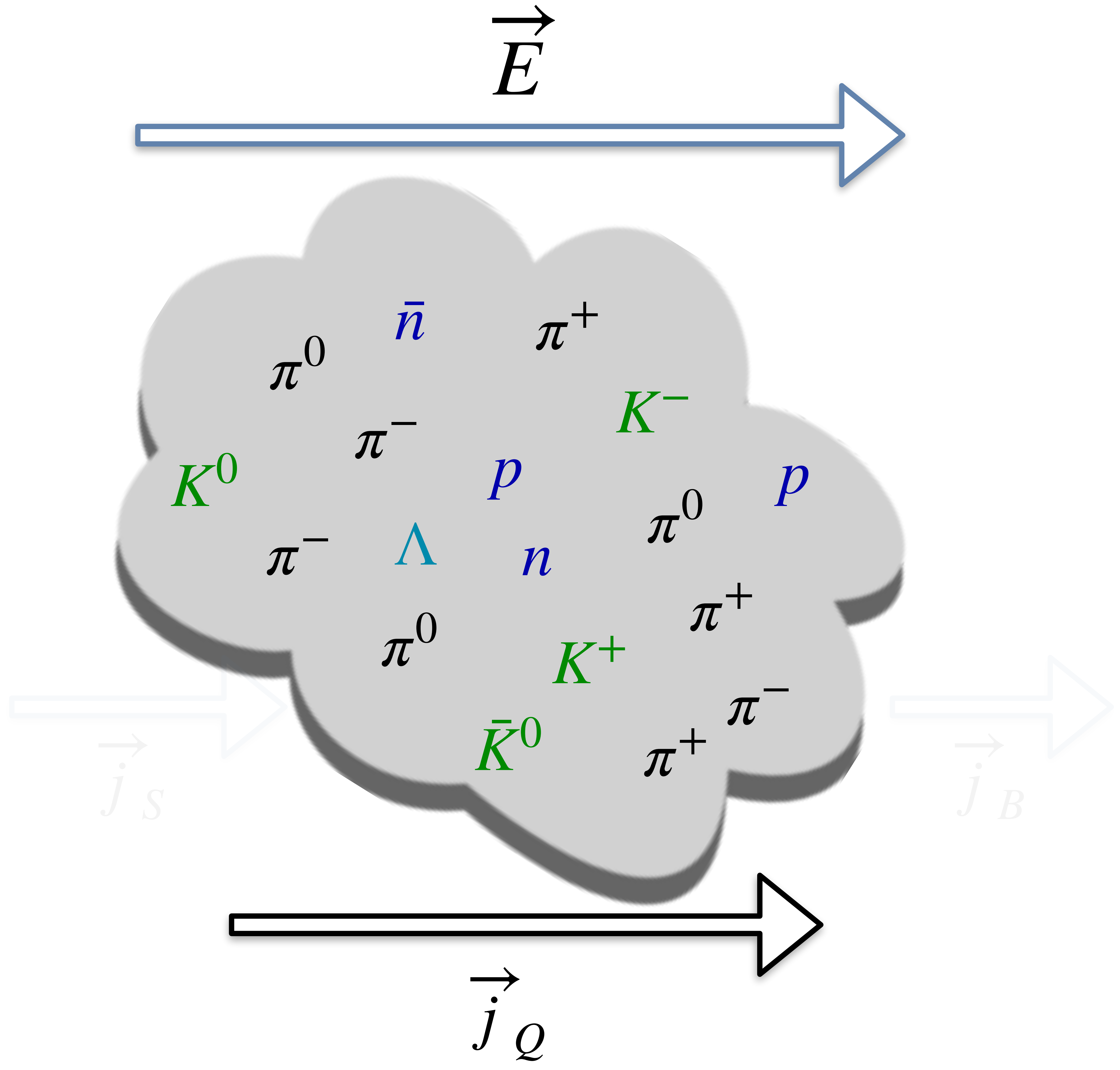
- Hadrons can have multiple charges: Q, B, S
- So the electric field also introduces other currents:

$$\vec{j}_B = \sigma_{QB}\,\vec{E}_Q$$

$$\vec{j}_S = \sigma_{QS}\,\vec{E}_Q$$

- **Can be calculated both in effective models and on the lattice!**

Electric cross-conductivity



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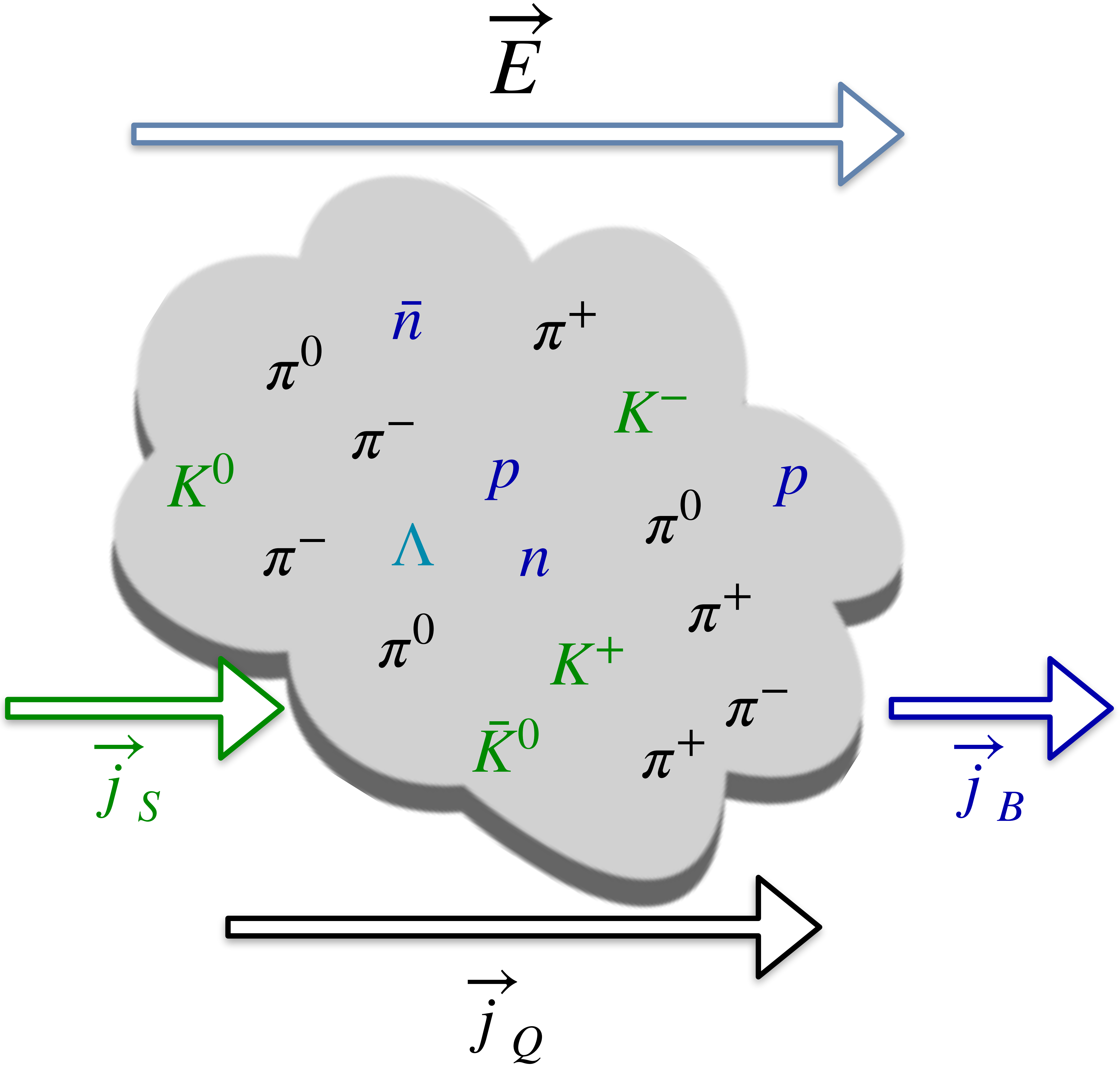

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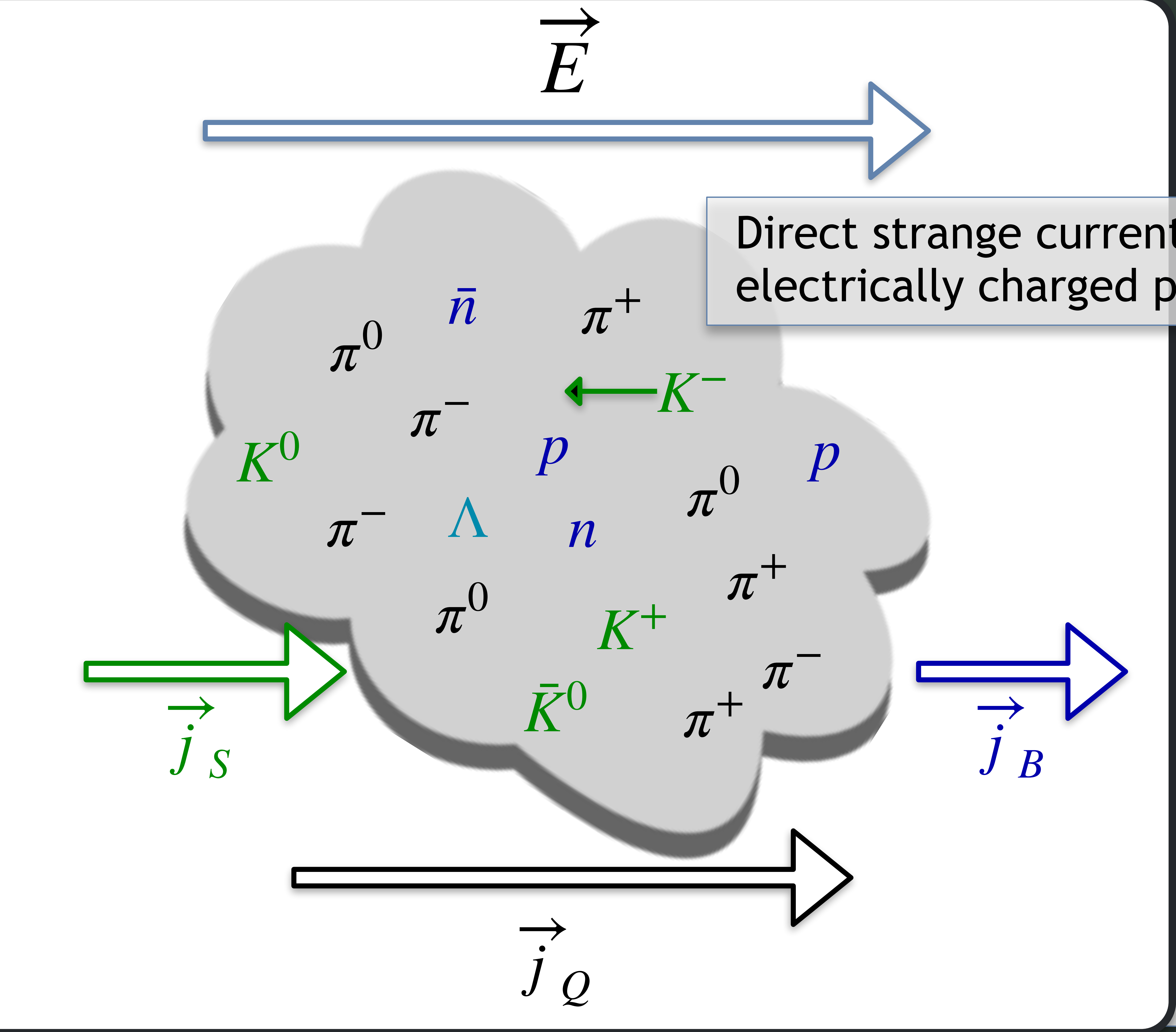

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- So the electric field also introduces other currents:

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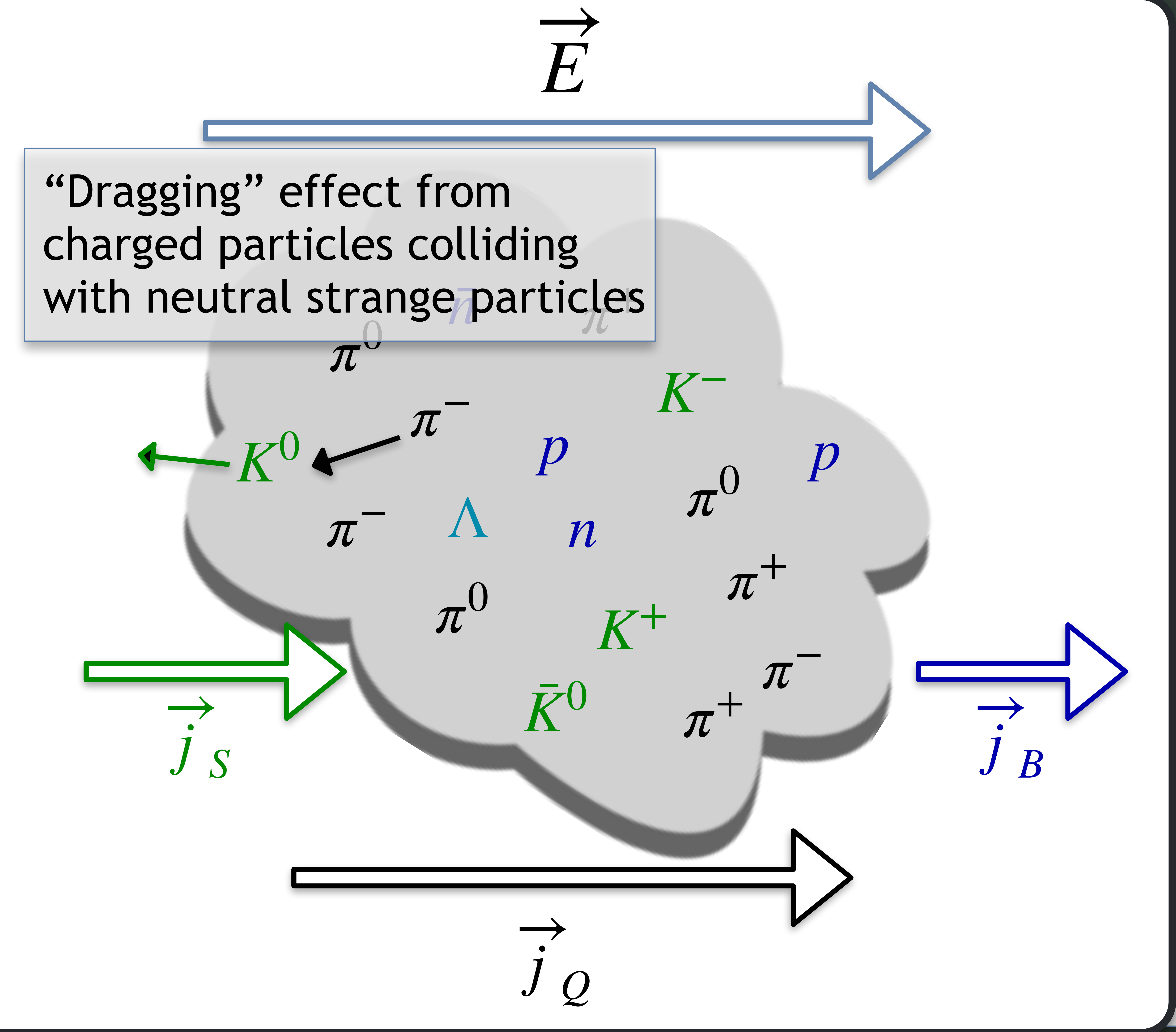
- Can be calculated both in effective models and on the lattice!

Electric cross-conductivity



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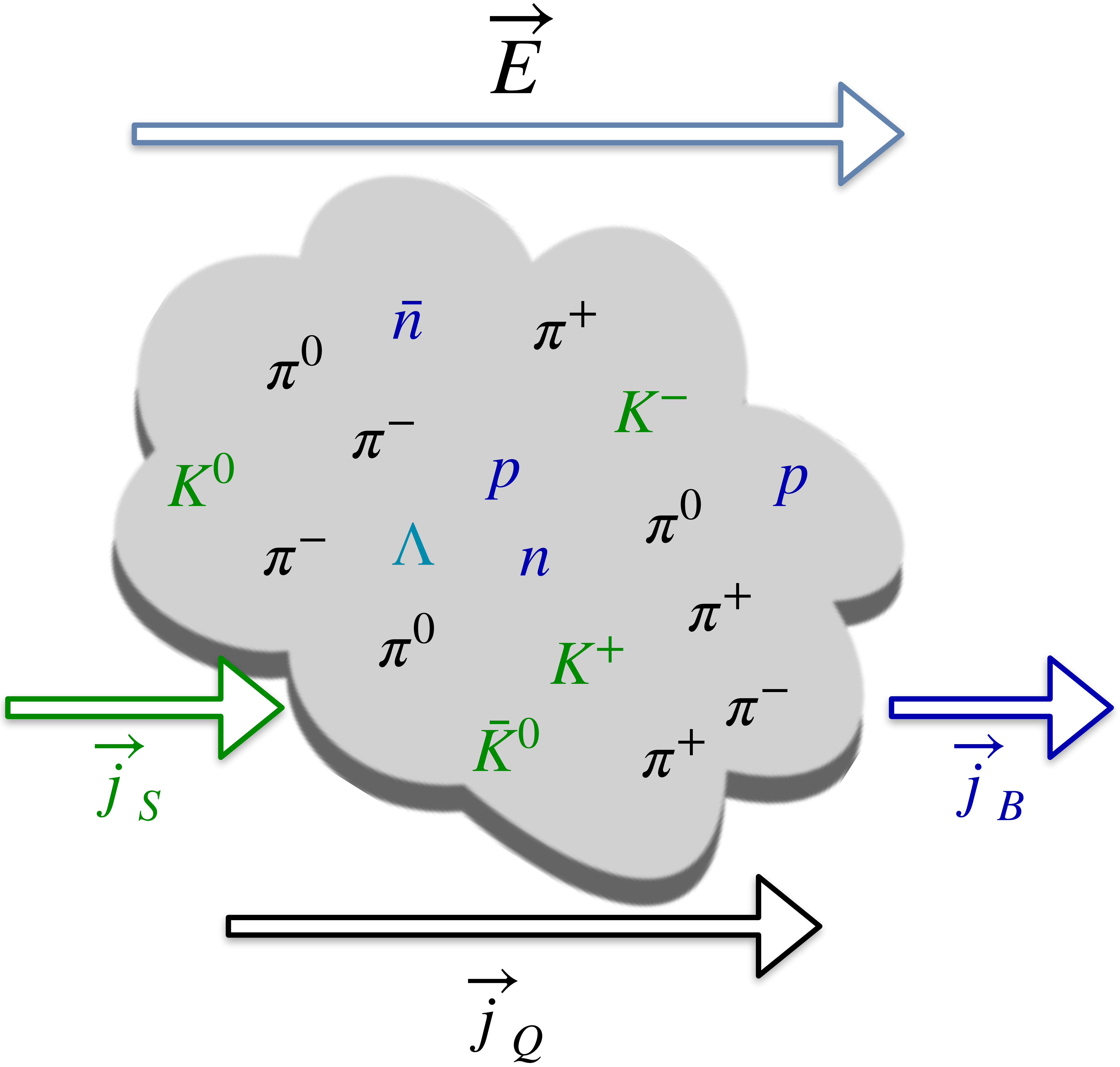

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- **Can be calculated both in effective models and on the lattice!**

Green-Kubo formalism

The cross-conductivity is calculated from

$$\sigma_{Qi} = \frac{V}{T}\int_0^\infty C_{Qi}\, dt', \quad i = Q, B, S$$

where

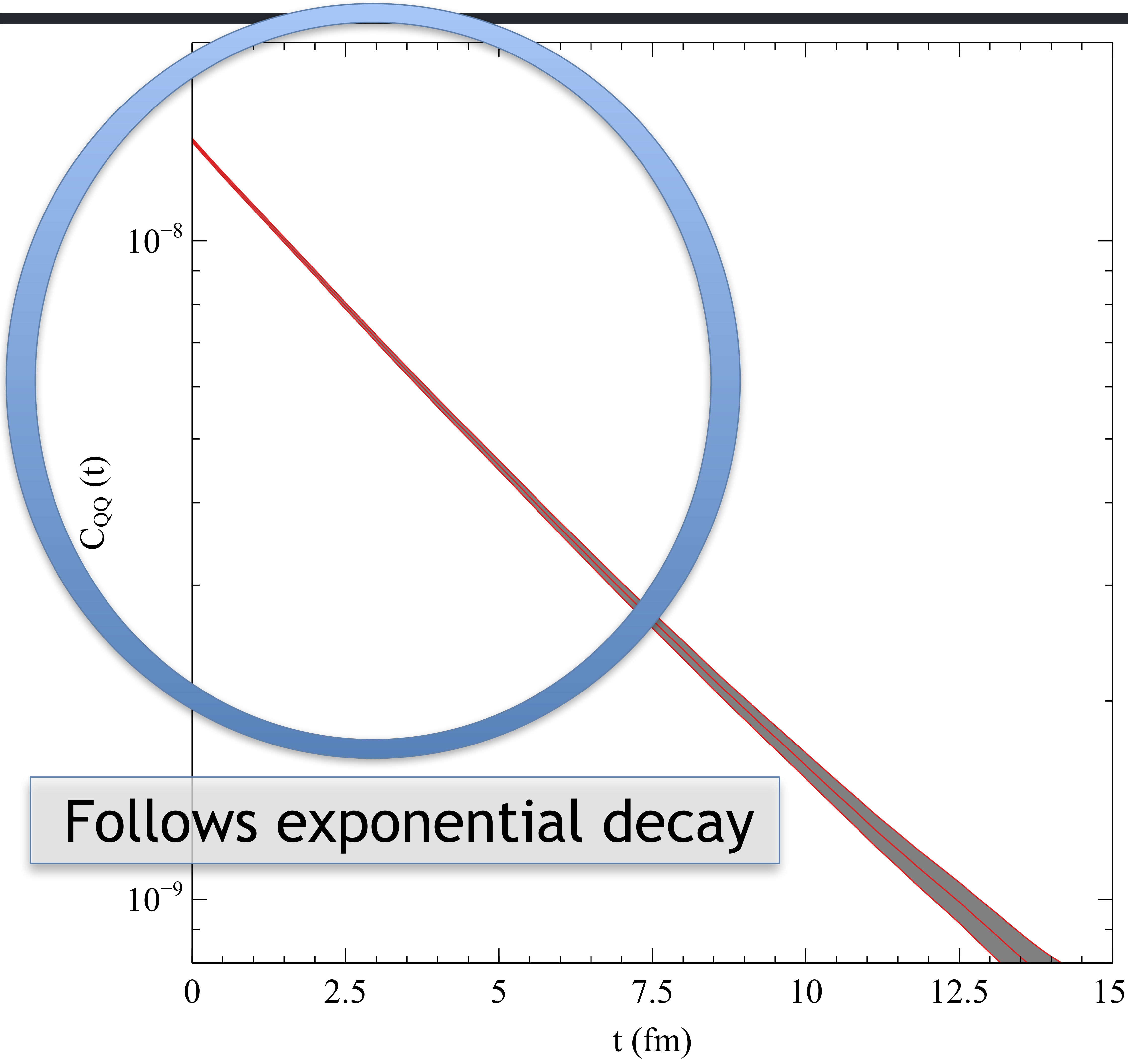
$$C_{Qi}(t) \equiv \langle (j_Q^x(t) - \langle j_Q^x \rangle_{eq}) \cdot (j_i^x(t') - \langle j_i^x \rangle_{eq}) \rangle_{eq}$$

In the dilute case, exponential *ansatz*

$$C_{Qi}(t) = C_{Qi}(0)\ e^{-\frac{t}{\tau_{Qi}}}$$

$$\sigma_{Qi} = \frac{C_{Qi}(0)\ V\ \tau_{Qi}}{T}$$

where τ_{Qi} is the relaxation time



Green-Kubo test case: π-K-N with 30 mb

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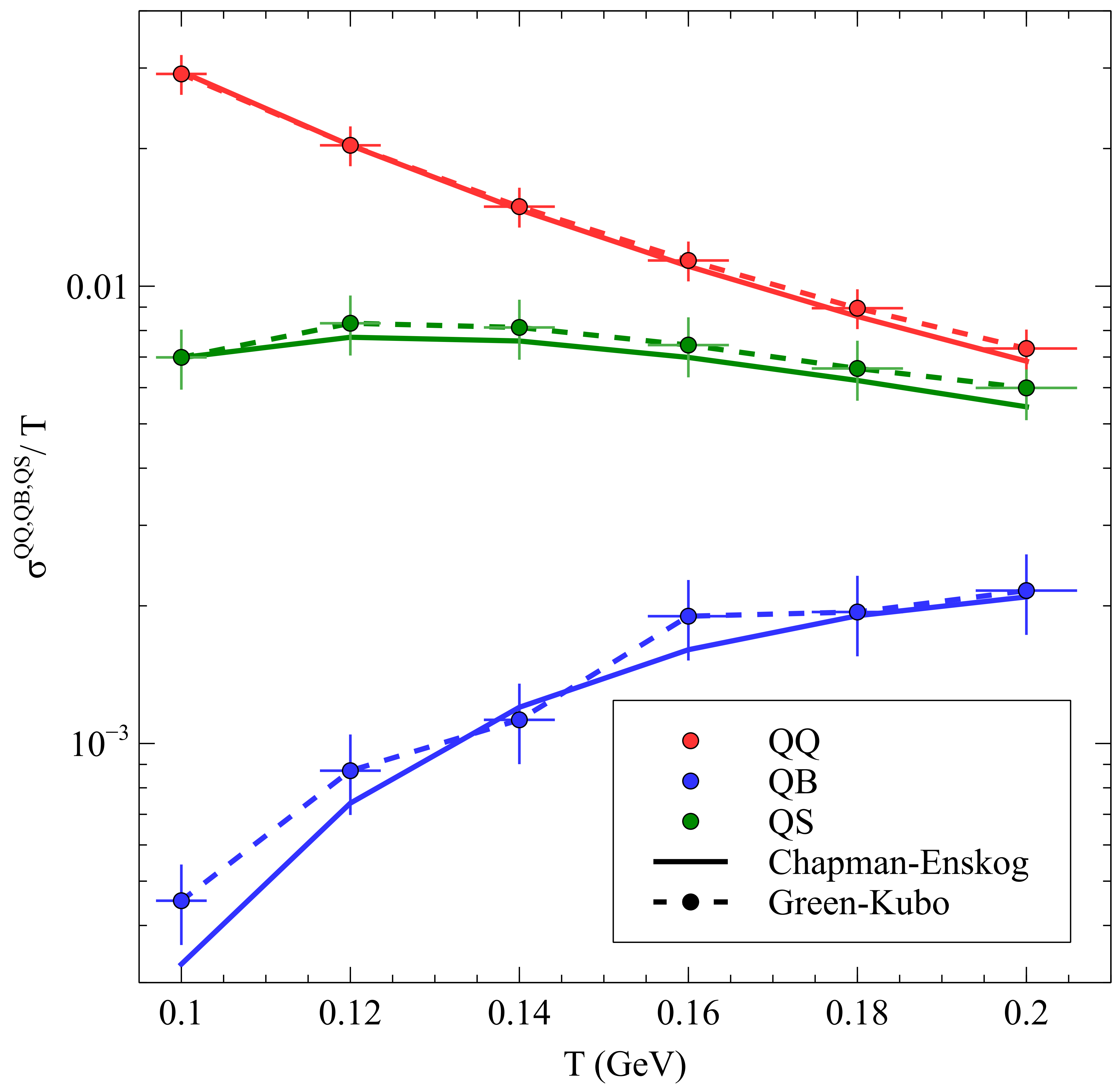
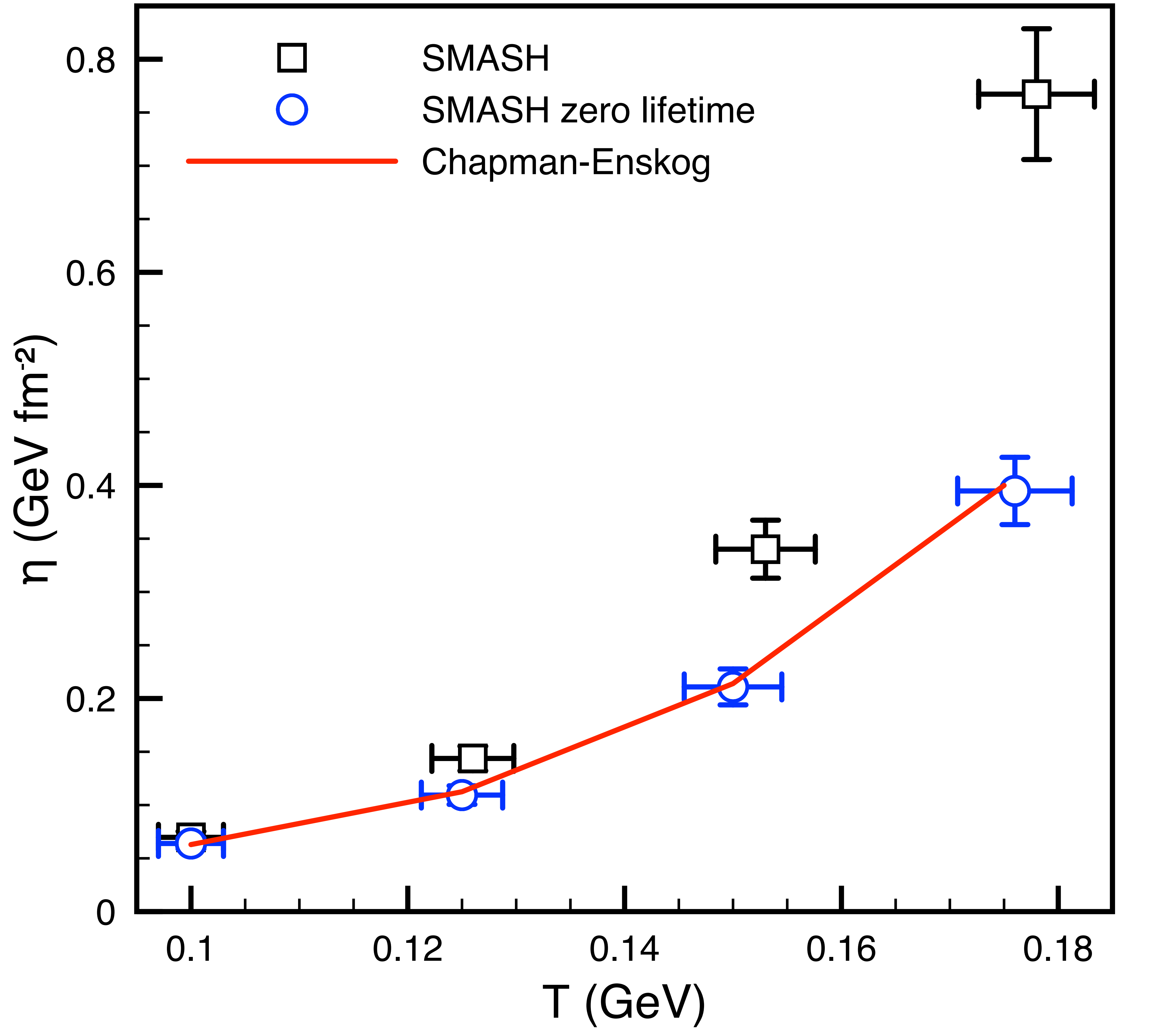
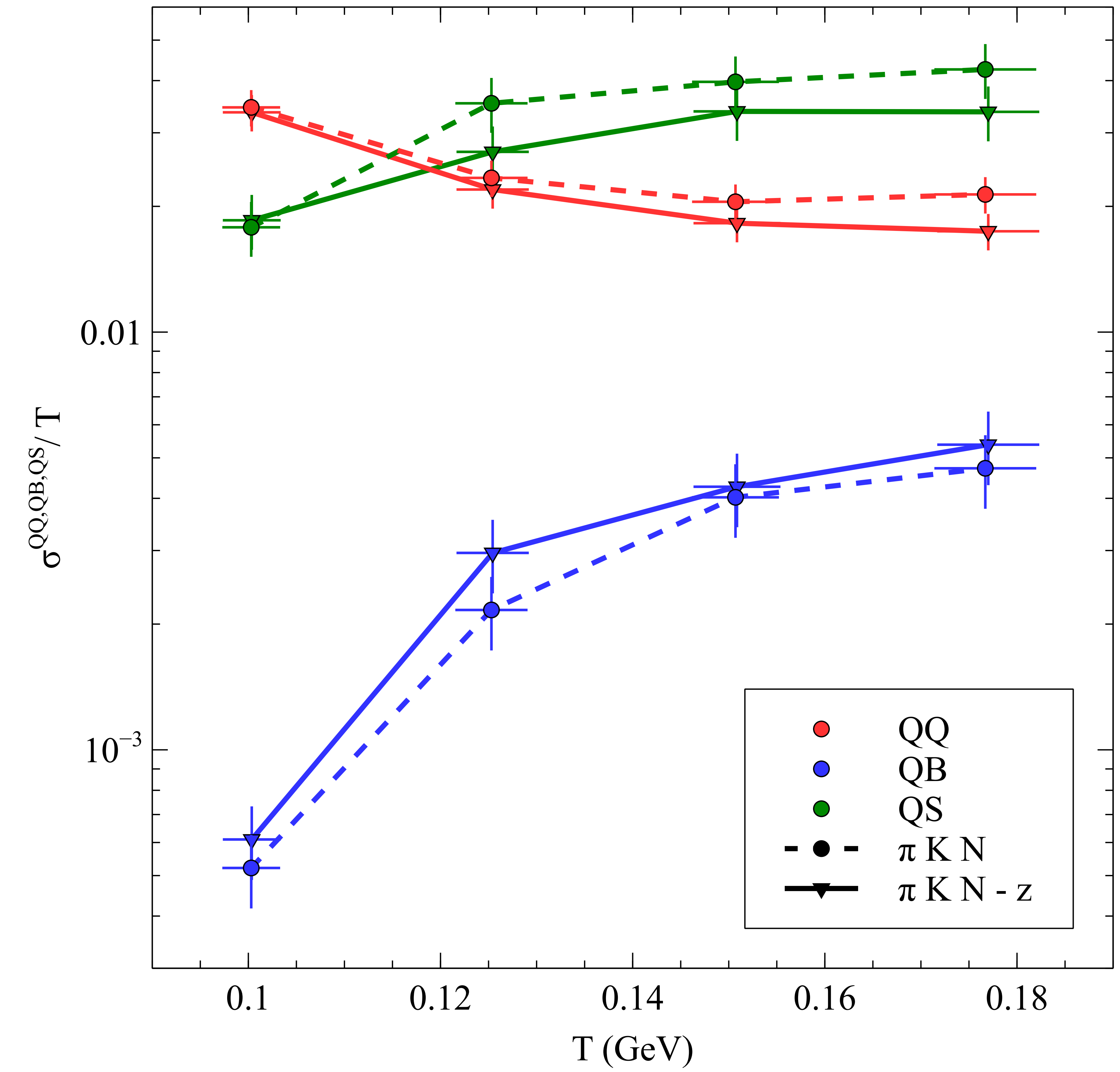


Greif et al., Phys.Rev. D93 (2016) no.9, 096012,
Erratum: Phys.Rev. D96 (2017) no.5, 059902

Resonance lifetimes: Shear vs conductivity



-Rose et al., Phys. Rev. C97 (2018) no.5, 055204



π-K-N resonant gas: Degrees of freedom

N	Δ	Λ	Σ	Ξ	Ω	Unflavored				Strange
N_{938}	Δ_{1232}	Λ_{1116}	Σ_{1189}	Ξ_{1321}	Ω^-_{1672}	π_{138}	$f_{0\ 980}$	$f_{2\ 1275}$	$\pi_{2\ 1670}$	K_{494}
N_{1440}	Δ_{1620}	Λ_{1405}	Σ_{1385}	Ξ_{1530}	Ω^-_{2250}	π_{1300}	$f_{0\ 1370}$	$f'_{2\ 1525}$		K^*_{892}
N_{1520}	Δ_{1700}	Λ_{1520}	Σ_{1660}	Ξ_{1690}		π_{1800}	$f_{0\ 1500}$	$f_{2\ 1950}$	$\rho_{3\ 1690}$	$K_{1\ 1270}$
N_{1535}	Δ_{1905}	Λ_{1600}	Σ_{1670}	Ξ_{1820}			$f_{0\ 1710}$	$f_{2\ 2010}$		$K_{1\ 1400}$
N_{1650}	Δ_{1910}	Λ_{1670}	Σ_{1750}	Ξ_{1950}		η_{548}		$f_{2\ 2300}$	$\varphi_{3\ 1850}$	K^*_{1410}
N_{1675}	Δ_{1920}	Λ_{1690}	Σ_{1775}	Ξ_{2030}		η'_{958}	$a_{0\ 980}$	$f_{2\ 2340}$		$K_0^*{}_{1430}$
N_{1680}	Δ_{1930}	Λ_{1800}	Σ_{1915}			η_{1295}	$a_{0\ 1450}$		$a_{4\ 2040}$	$K_2^*{}_{1430}$
N_{1700}	Δ_{1950}	Λ_{1810}	Σ_{1940}			η_{1405}		$f_{1\ 1285}$		K^*_{1680}
N_{1710}		Λ_{1820}	Σ_{2030}			η_{1475}	φ_{1019}	$f_{1\ 1420}$	$f_{4\ 2050}$	$K_{2\ 1770}$
N_{1720}		Λ_{1830}	Σ_{2250}				φ_{1680}			$K_3^*{}_{1780}$
N_{1875}		Λ_{1890}				σ_{800}		$a_{2\ 1320}$		$K_{2\ 1820}$
N_{1900}		Λ_{2100}					$h_{1\ 1170}$			$K_4^*{}_{2045}$
N_{1990}		Λ_{2110}				ρ_{776}		$\pi_{1\ 1400}$		
N_{2080}		Λ_{2350}				ρ_{1450}	$b_{1\ 1235}$	$\pi_{1\ 1600}$		
N_{2190}						ρ_{1700}				
N_{2220}							$a_{1\ 1260}$	$\eta_{2\ 1645}$		
N_{2250}						ω_{783}				
						ω_{1420}		$\omega_{3\ 1670}$		
						ω_{1650}				

- + anti-particles
- Isospin symmetry

SMASH v1.6, smash-transport.github.io, DOI:10.5281/zenodo.3485108

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N_{1520}	Δ_{1700}	Λ_{1520}	Σ_{1660}	Ξ_{1690}	
N_{1535}	Δ_{1905}	Λ_{1600}	Σ_{1670}	Ξ_{1820}	
N_{1650}	Δ_{1910}	Λ_{1670}	Σ_{1750}	Ξ_{1950}	
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	$f_{0\ 1710}$	$f_{2\ 2010}$		$K_{1\ 1400}$
η_{548}		$f_{2\ 2300}$	$\varphi_{3\ 1850}$	K^*_{1410}
η'_{958}	$a_{0\ 980}$	$f_{2\ 2340}$		$K_0^*{}_{1430}$
η_{1295}	$a_{0\ 1450}$		$a_{4\ 2040}$	$K_2^*{}_{1430}$
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σ_{800}		$a_{2\ 1320}$		$K_{2\ 1820}$
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SMASH v1.6, smash-transport.github.io, DOI:10.5281/zenodo.3485108

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N_{1650}	Δ_{1910}	Λ_{1670}	Σ_{1750}	Ξ_{1950}	
N_{1675}	Δ_{1920}	Λ_{1690}	Σ_{1775}	Ξ_{2030}	
N_{1680}	Δ_{1930}	Λ_{1800}	Σ_{1915}		
N_{1700}	Δ_{1950}	Λ_{1810}	Σ_{1940}		
N_{1710}		Λ_{1820}	Σ_{2030}		
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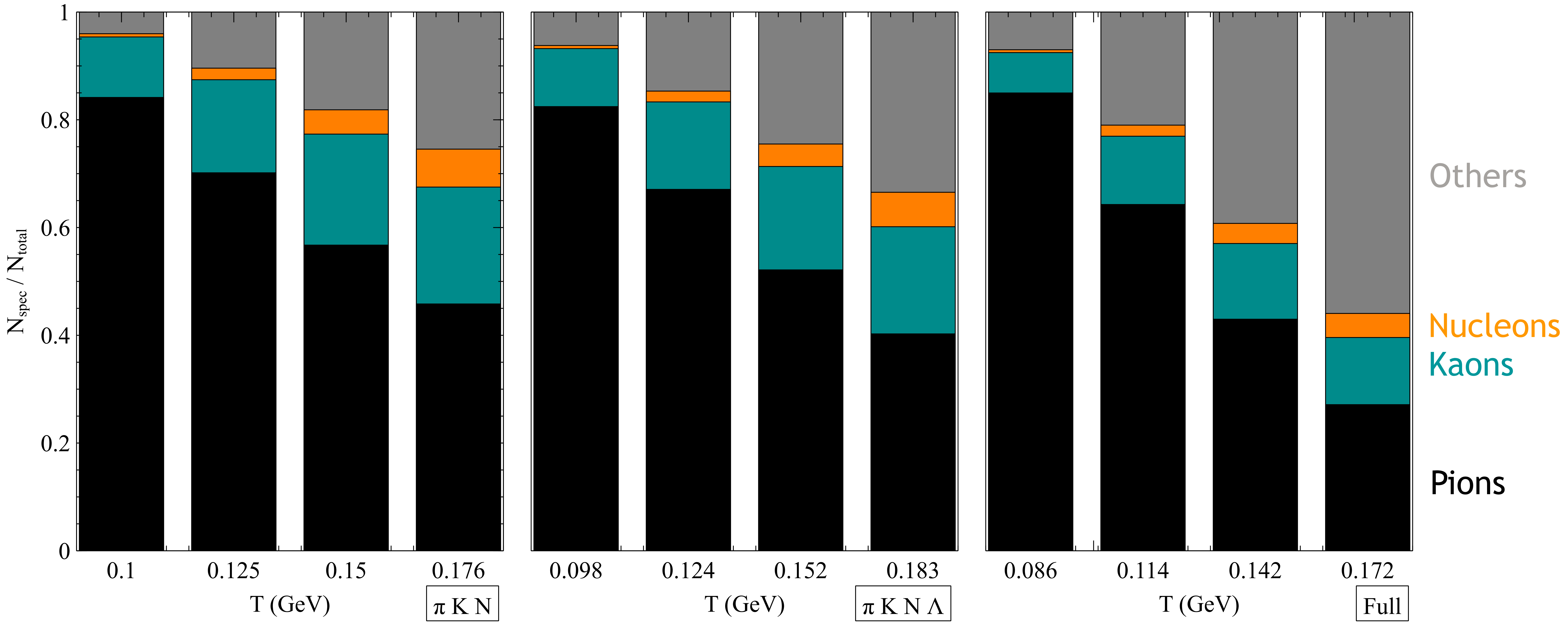
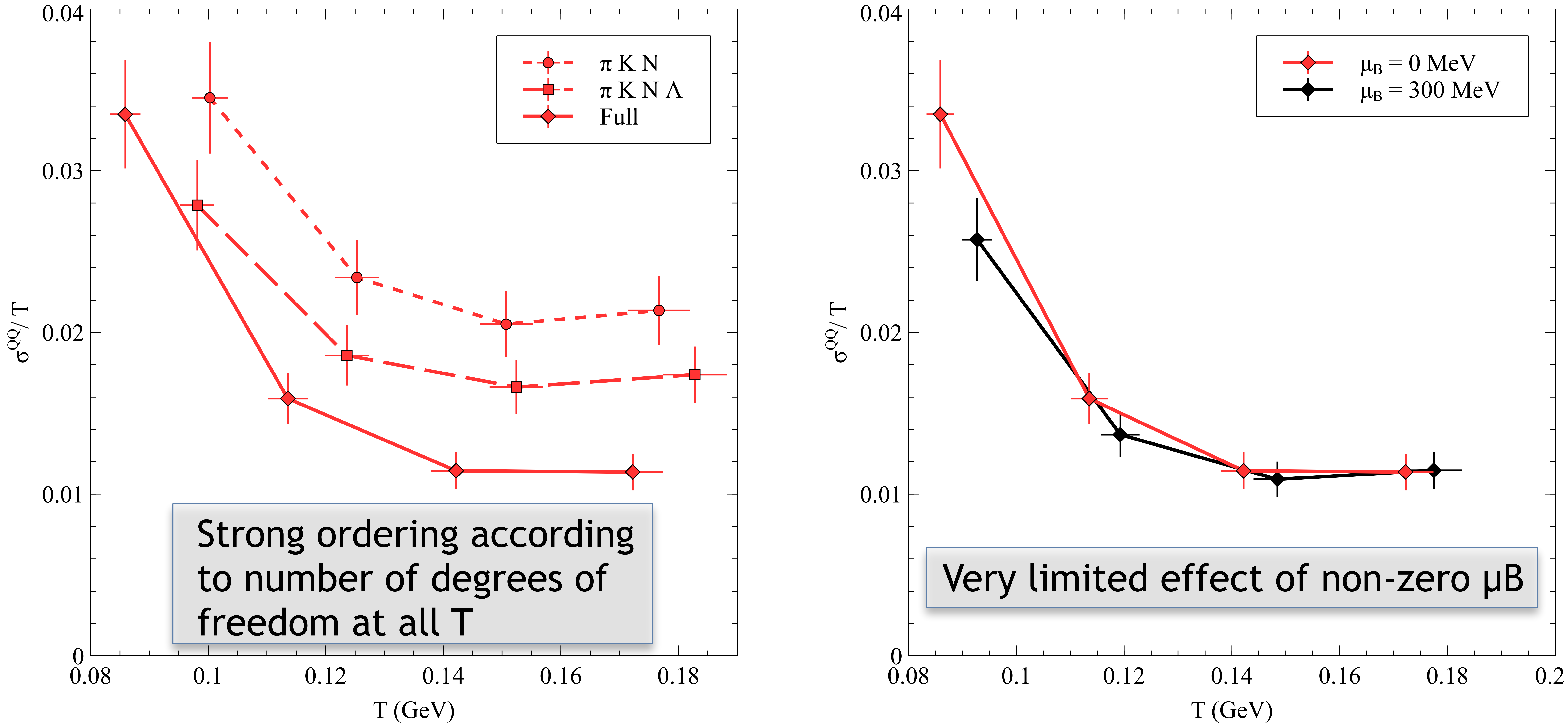
SMASH v1.6, smash-transport.github.io, DOI:10.5281/zenodo.3485108

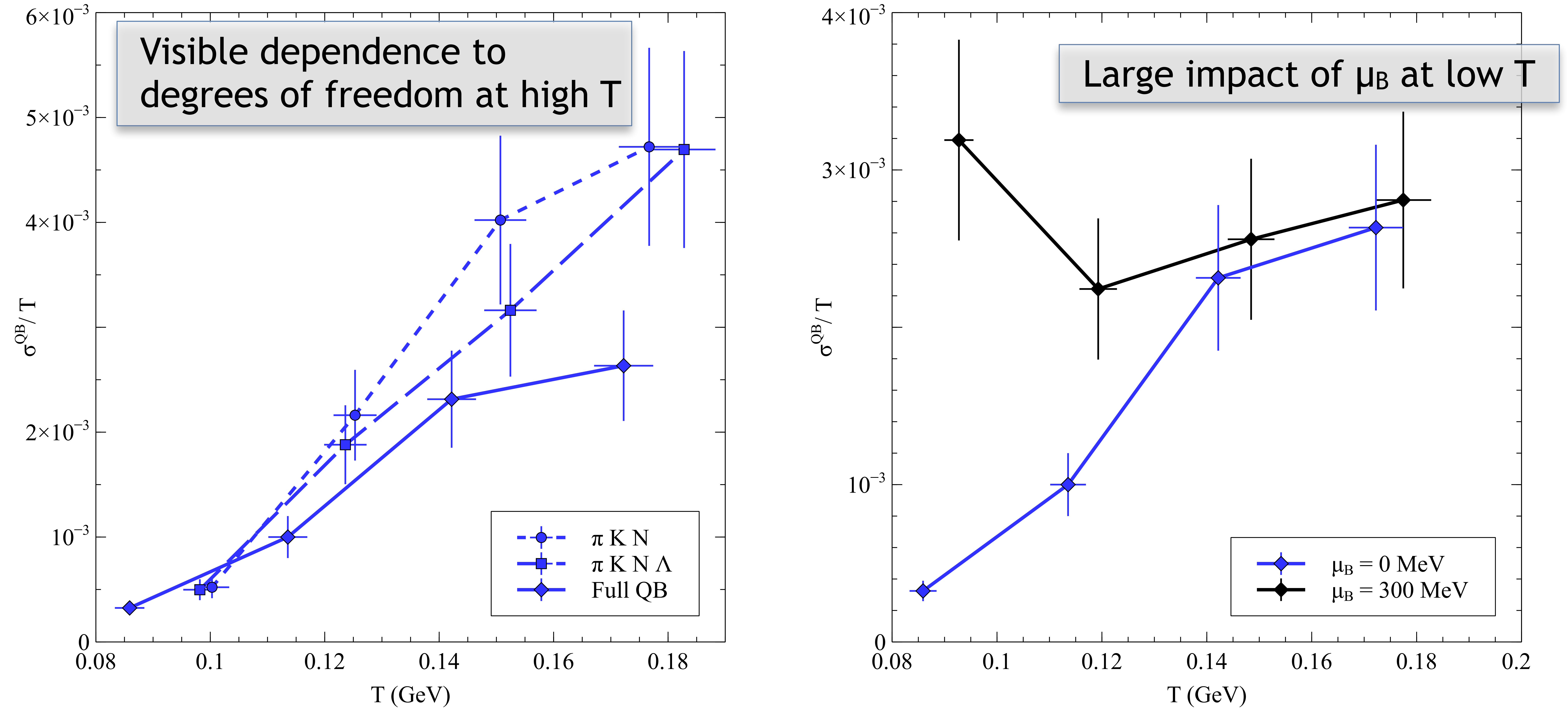
Chemical composition



Electric conductivity

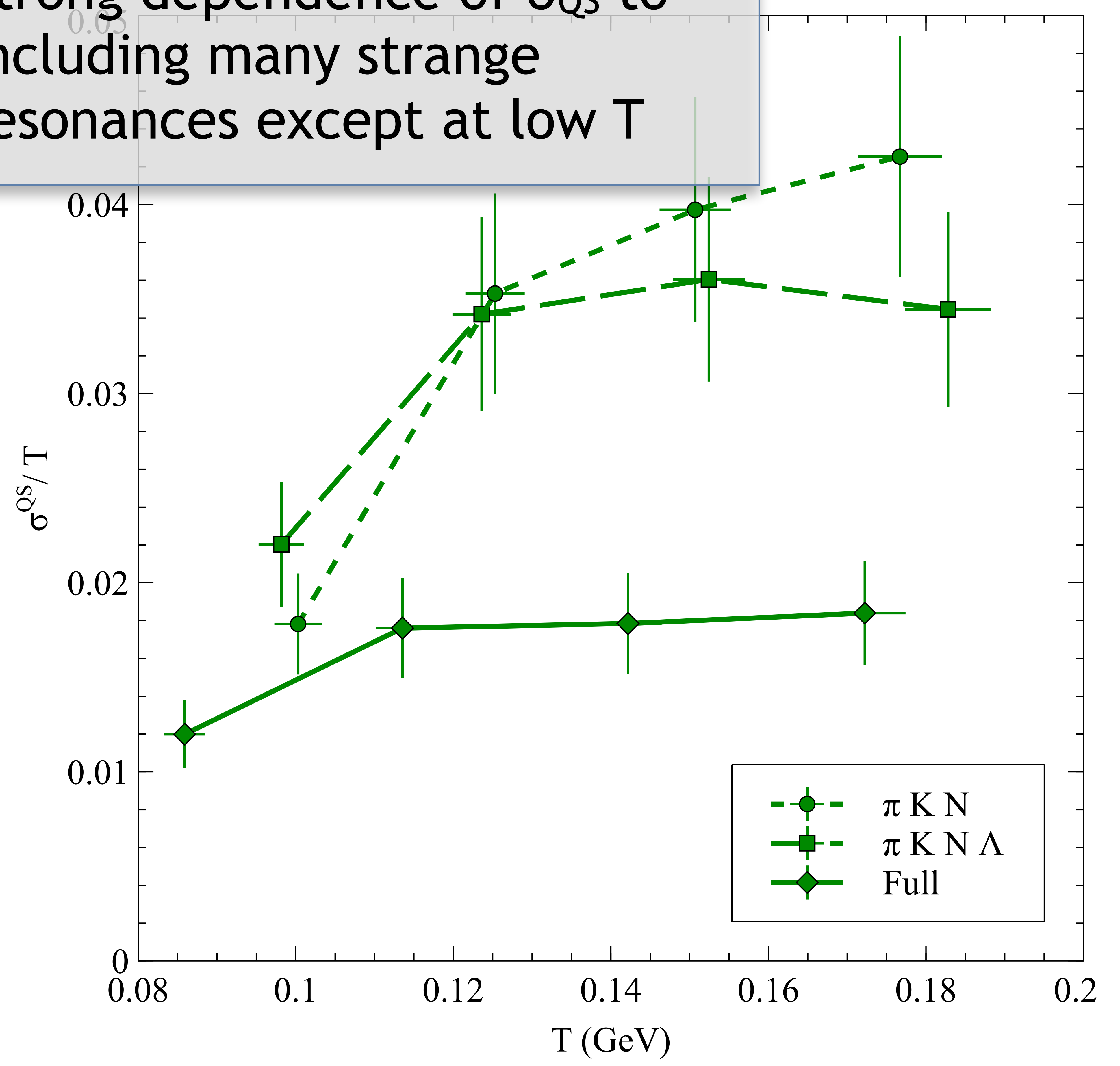


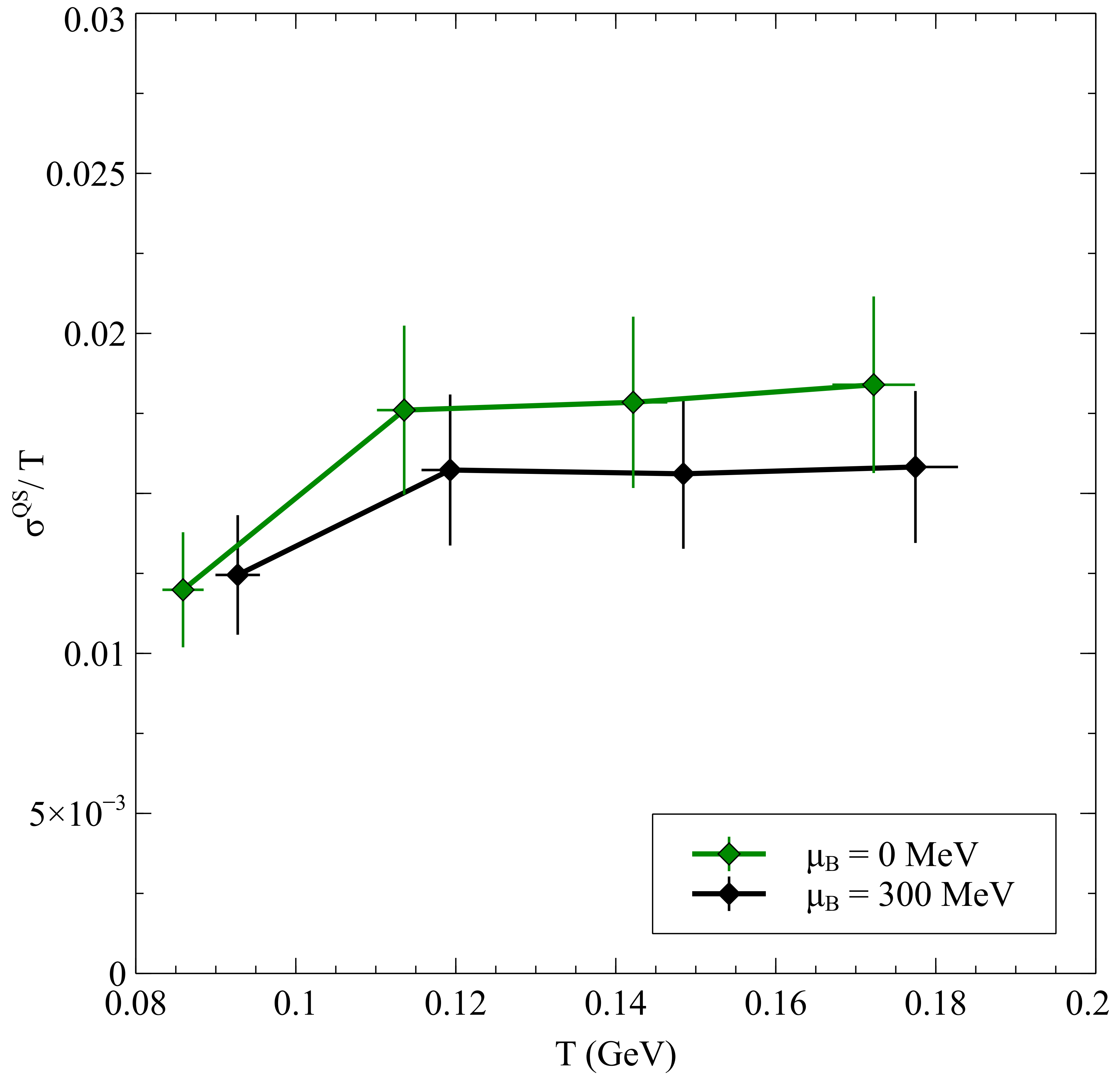
Baryonic-electric conductivity



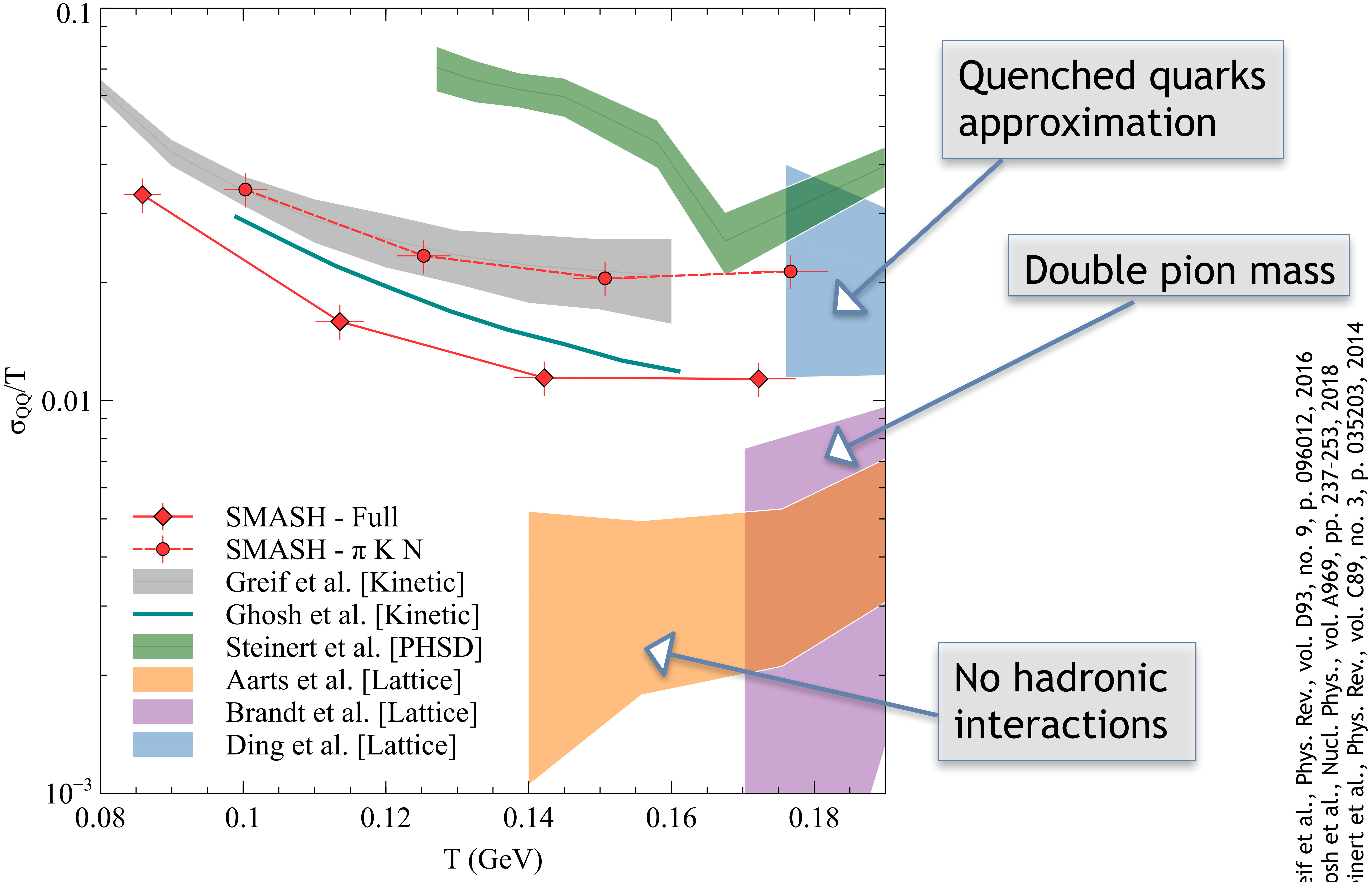
Strange-electric conductivity

Strong dependence of σ_{QS} to including many strange resonances except at low T





Electric conductivity comparison



-Greif et al., Phys. Rev., vol. D93, no. 9, p. 096012, 2016
-Ghosh et al., Nucl. Phys., vol. A969, pp. 237-253, 2018
-Steinert et al., Phys. Rev., vol. C89, no. 3, p. 035203, 2014
-Aarts et al., JHEP, vol. 02, p. 186, 2015
-Brandt et al., Phys. Rev., vol. D93, no. 5, p. 054510, 2016
-Ding et al., Phys. Rev., vol. D94, no. 3, p. 034504, 2016

Multiple lattice calculations exist, but they have large errors and systematic uncertainties

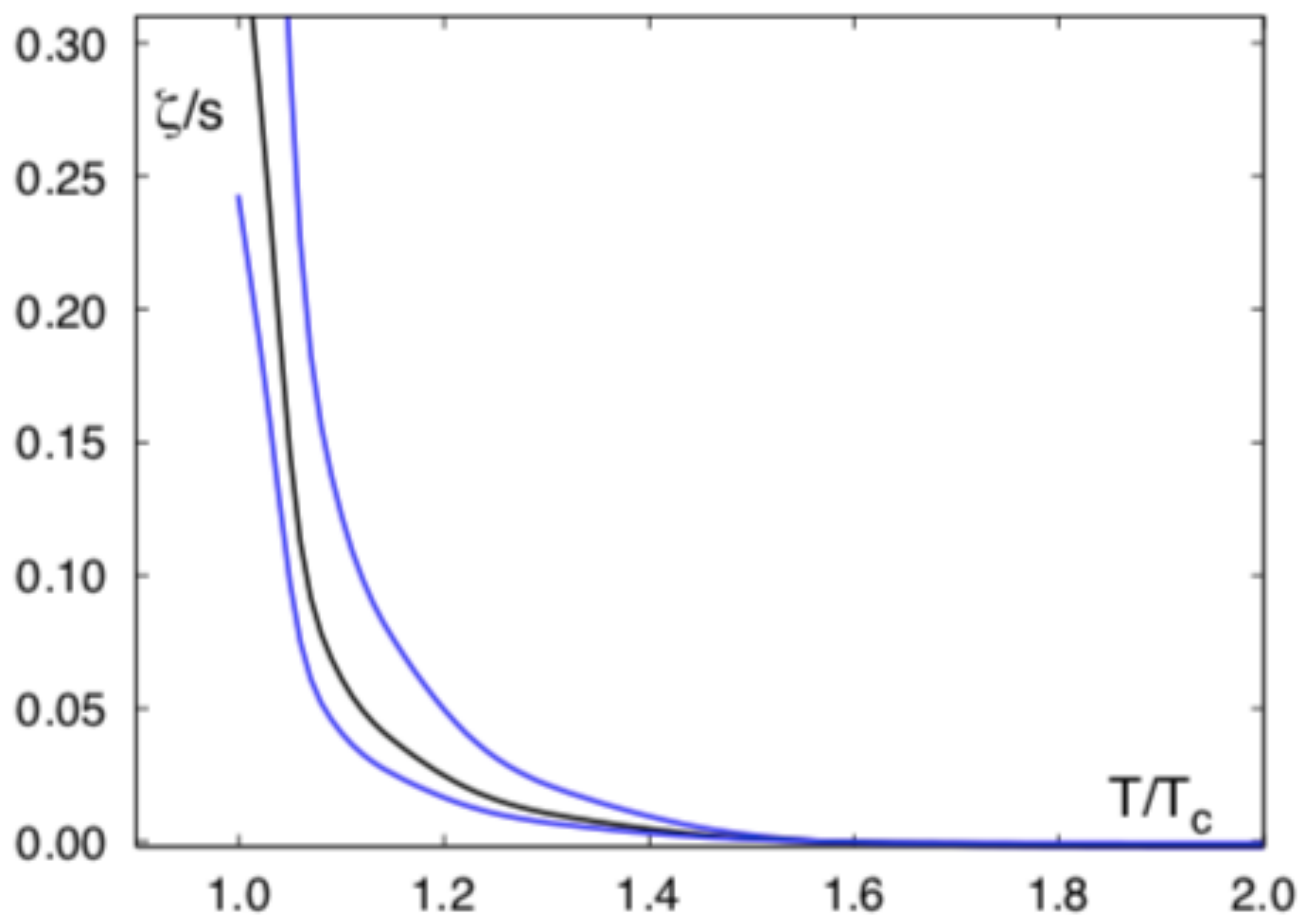
Need consistent results in a larger T range

Three talks for the price of one!

1. Shear Viscosity
2. Cross-Conductivity
3. Bulk Viscosity

Why study bulk viscosity?

- Isn't it zero according to AdS-CFT? Not a conformal fluid, especially at phase transition



Karsch, Kharzeev and Tuchin, Phys. Lett., vol. B663, pp. 217-221, 2008

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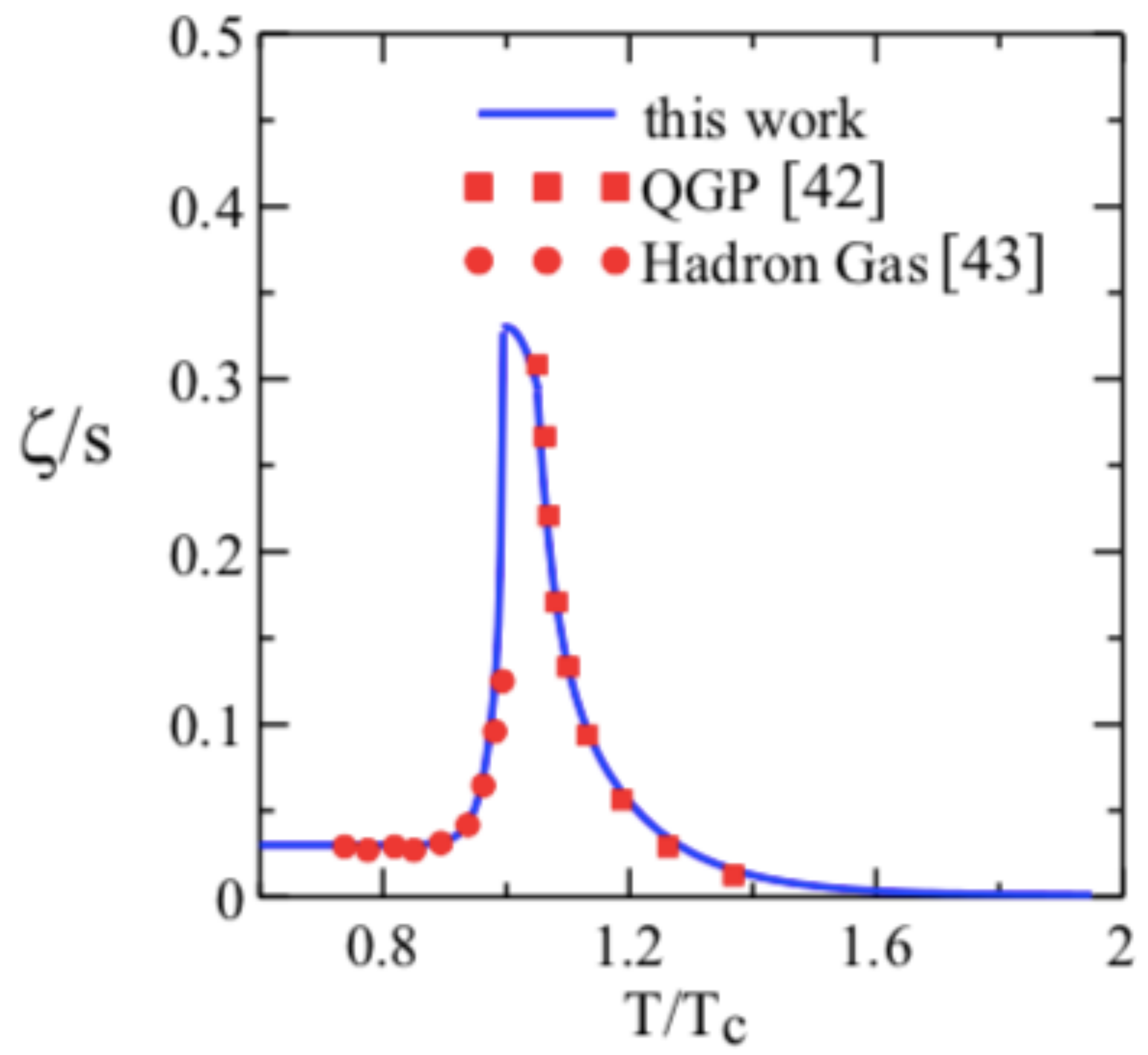
- Was shown to be important to simultaneously fit multiplicities, mean p_T, and elliptic flow



Ryu, Paquet, Shen, Denicol, Schenke, Jeon, and Gale, Phys. Rev. Lett., vol. 115, no. 13, p. 132301, 2015

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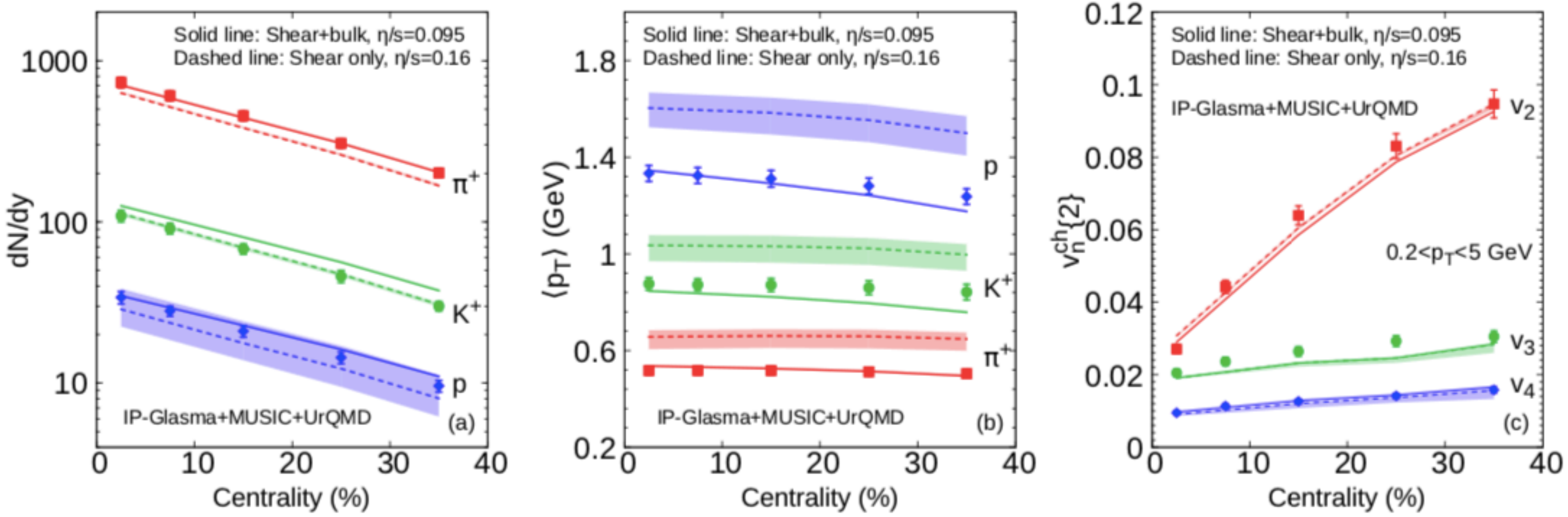

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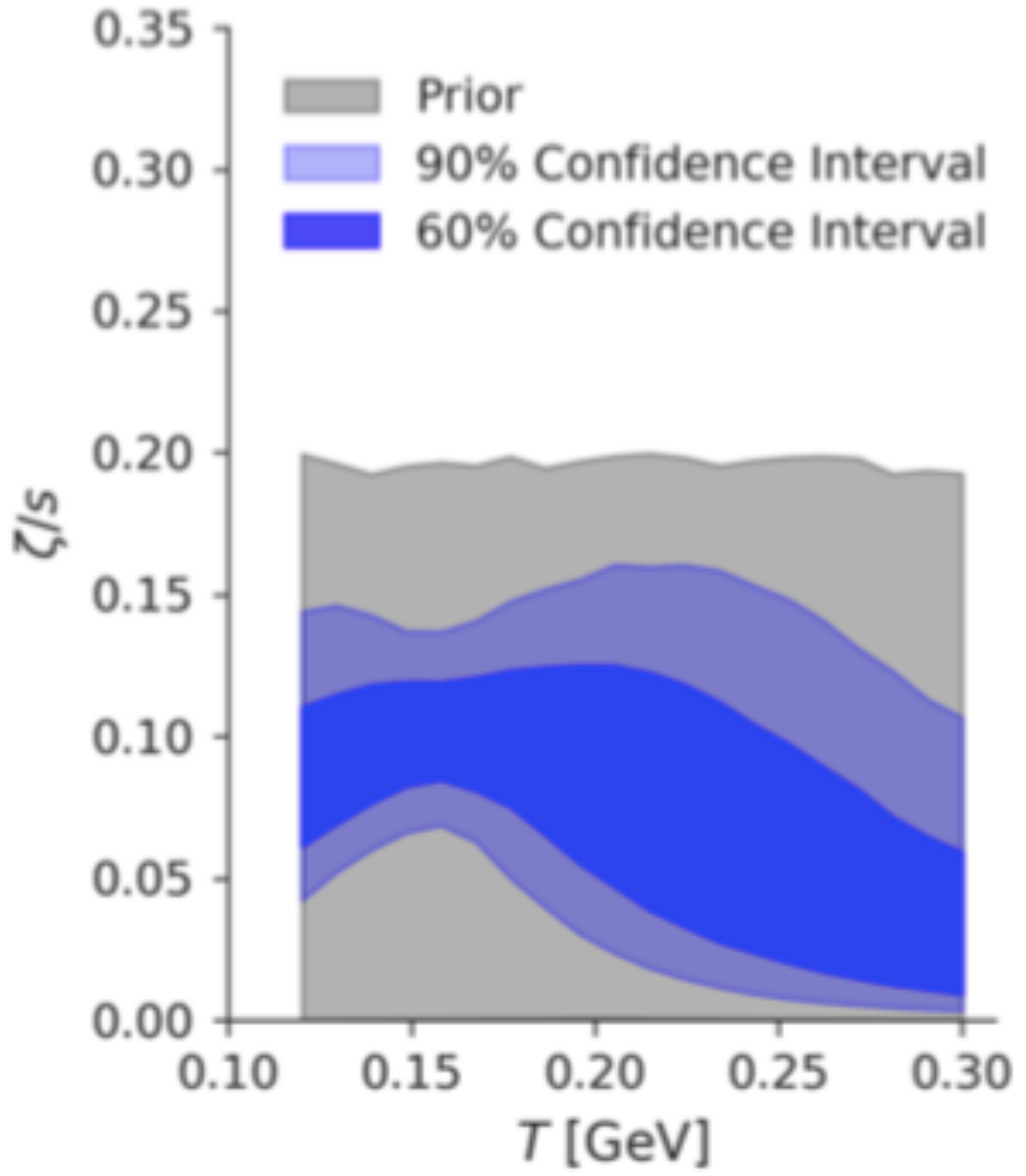
Why study bulk viscosity?

- Isn't it zero according to AdS-CFT? Not a conformal fluid, especially at phase transition
- Was shown to be important to simultaneously fit multiplicities, mean p_T, and elliptic flow
- More recently, was shown using Bayesian techniques in hybrid models that bulk viscosity has a large structure around T_c



Paquet, QM2019 talk

A little reminder on Green-Kubo: Bulk

The shear viscosity is calculated from

$$\zeta = \frac{V}{T}\int_0^\infty C^\Pi(t)dt$$

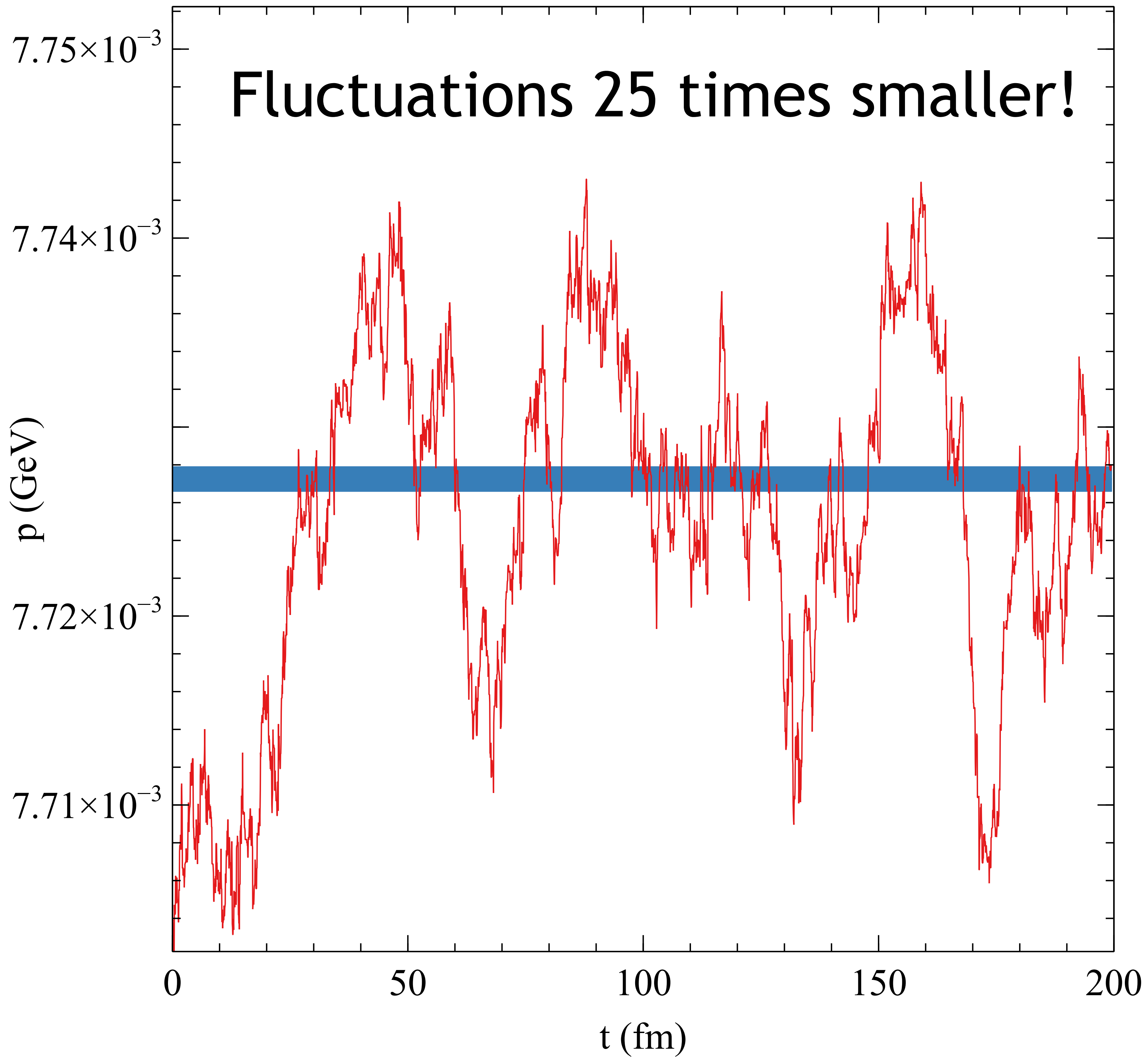
where

$$C^\Pi(t) \equiv \langle (p(0) - \langle p \rangle_{eq}) \cdot (p(t) - \langle p \rangle_{eq}) \rangle_{eq}$$

<p> is NOT zero!!

In a pion constant cross-section system:

A little reminder on Green-Kubo: Bulk



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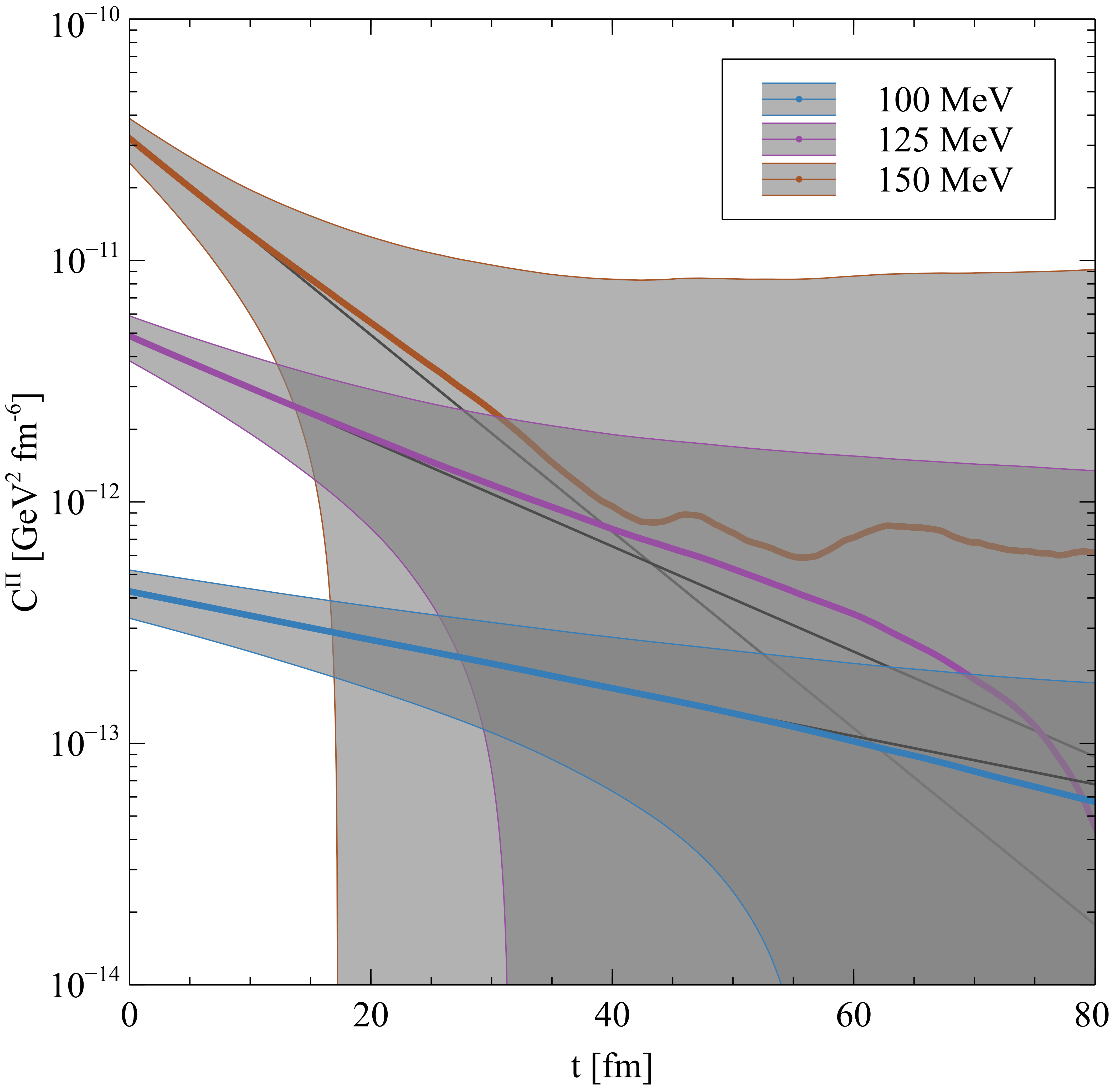

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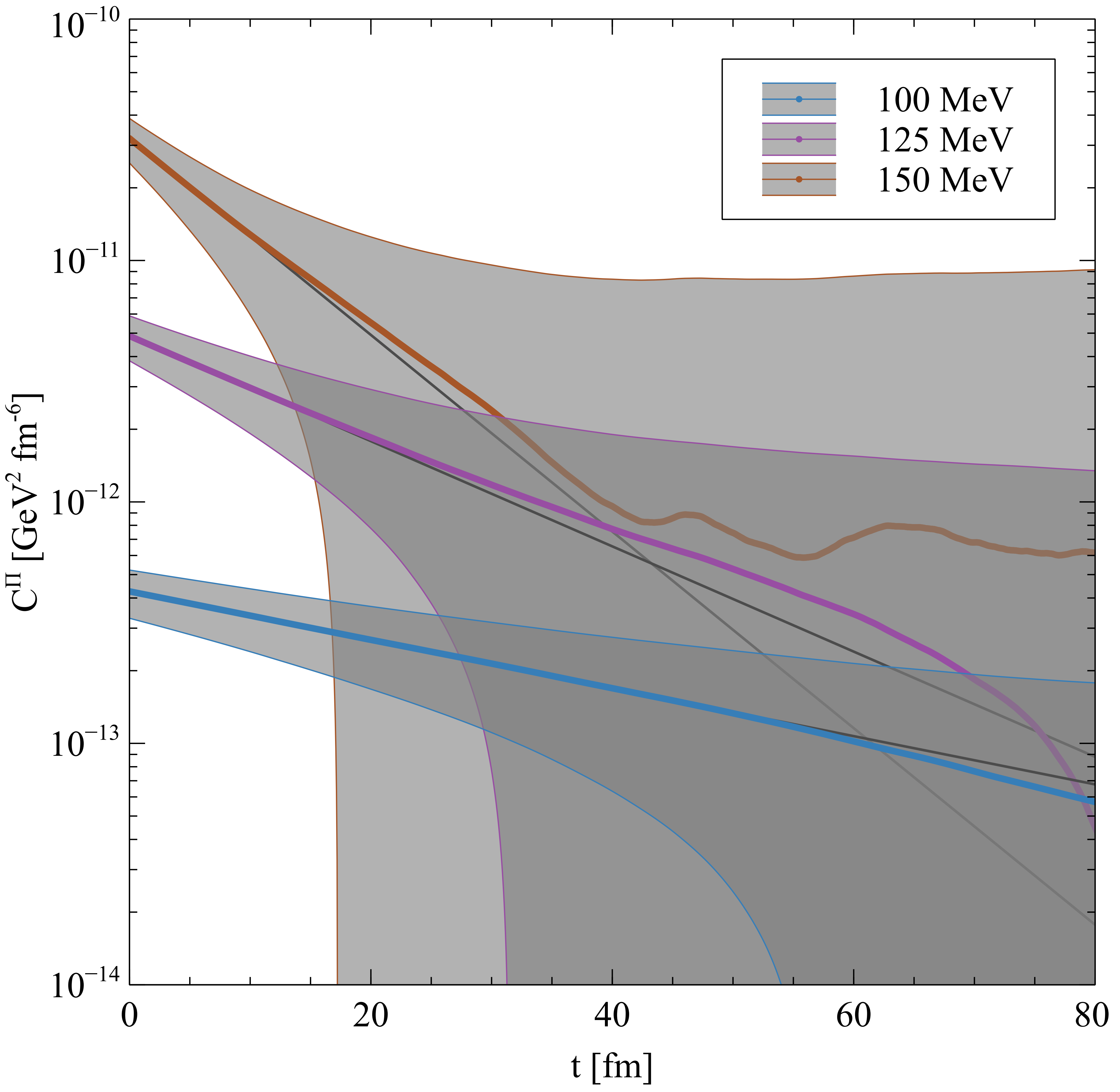
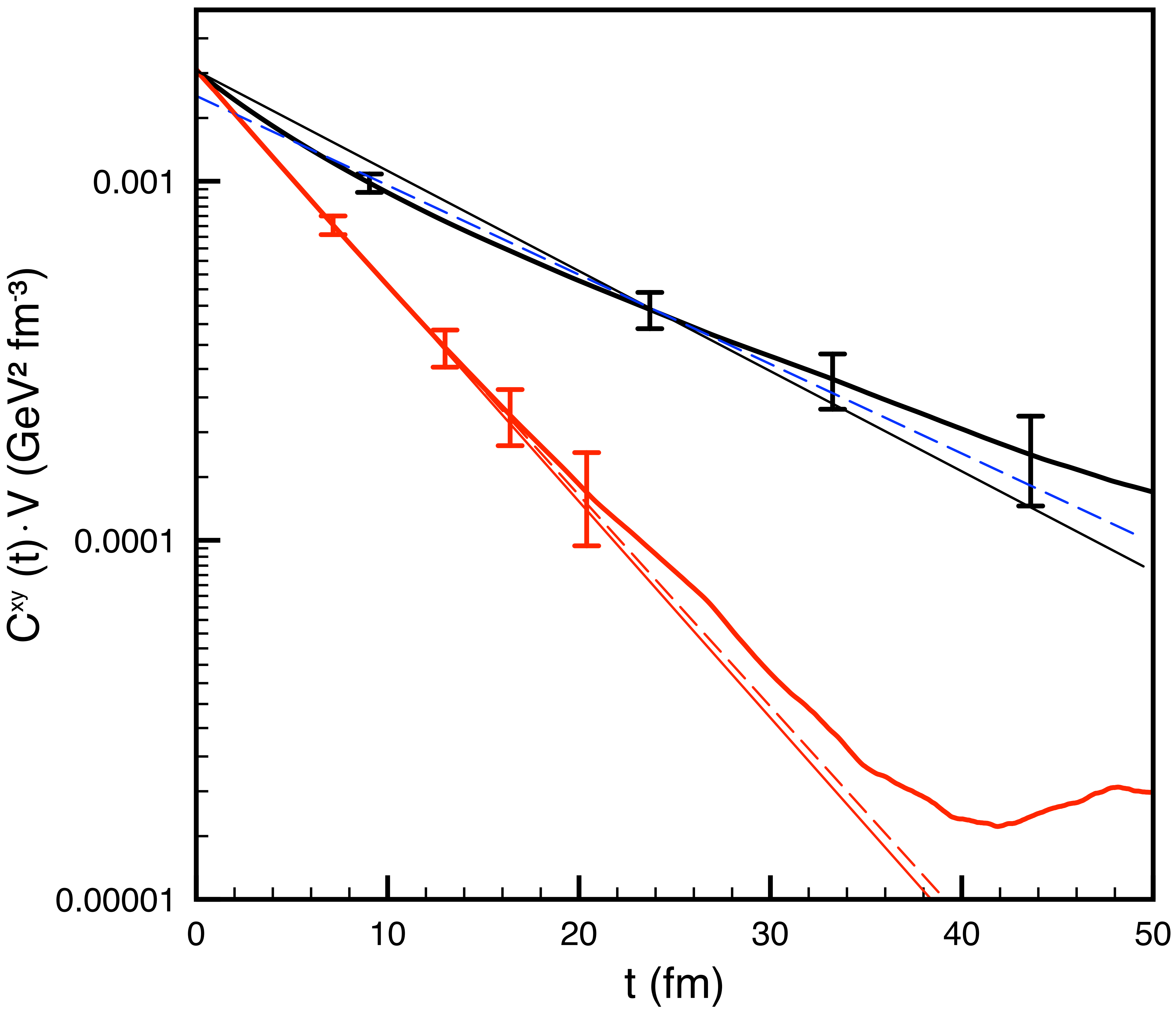
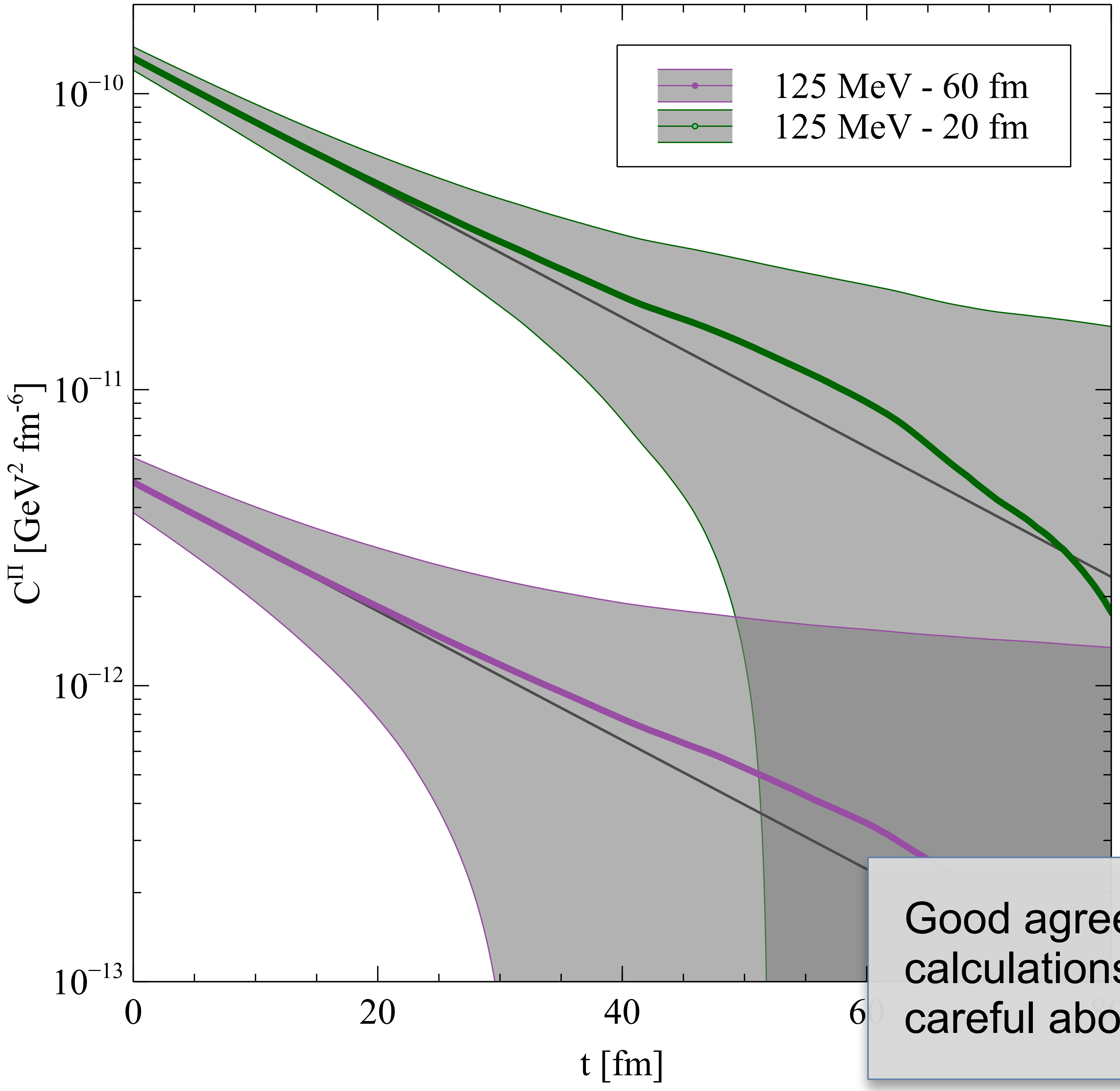
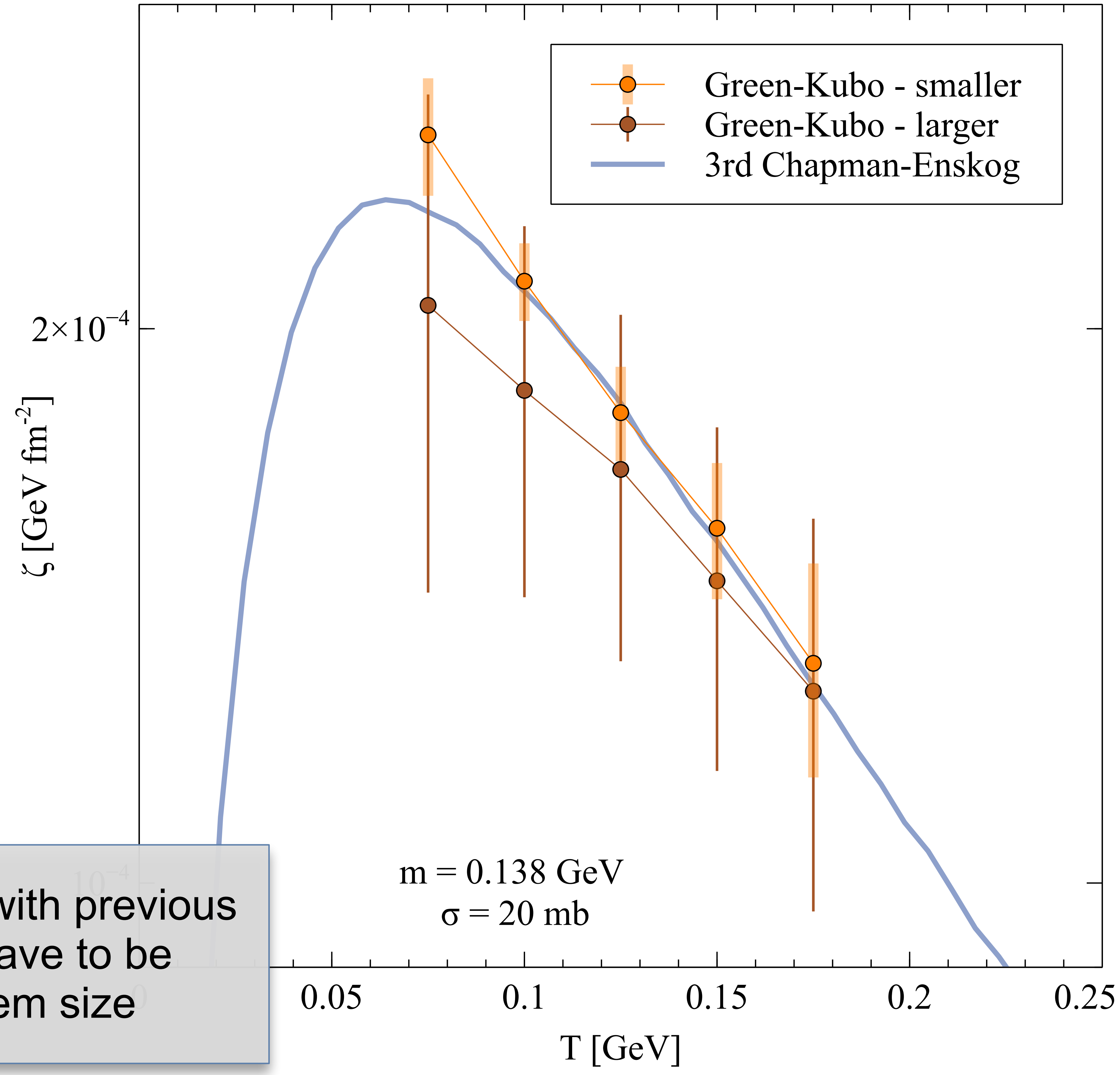




Test case #1: π with constant σ

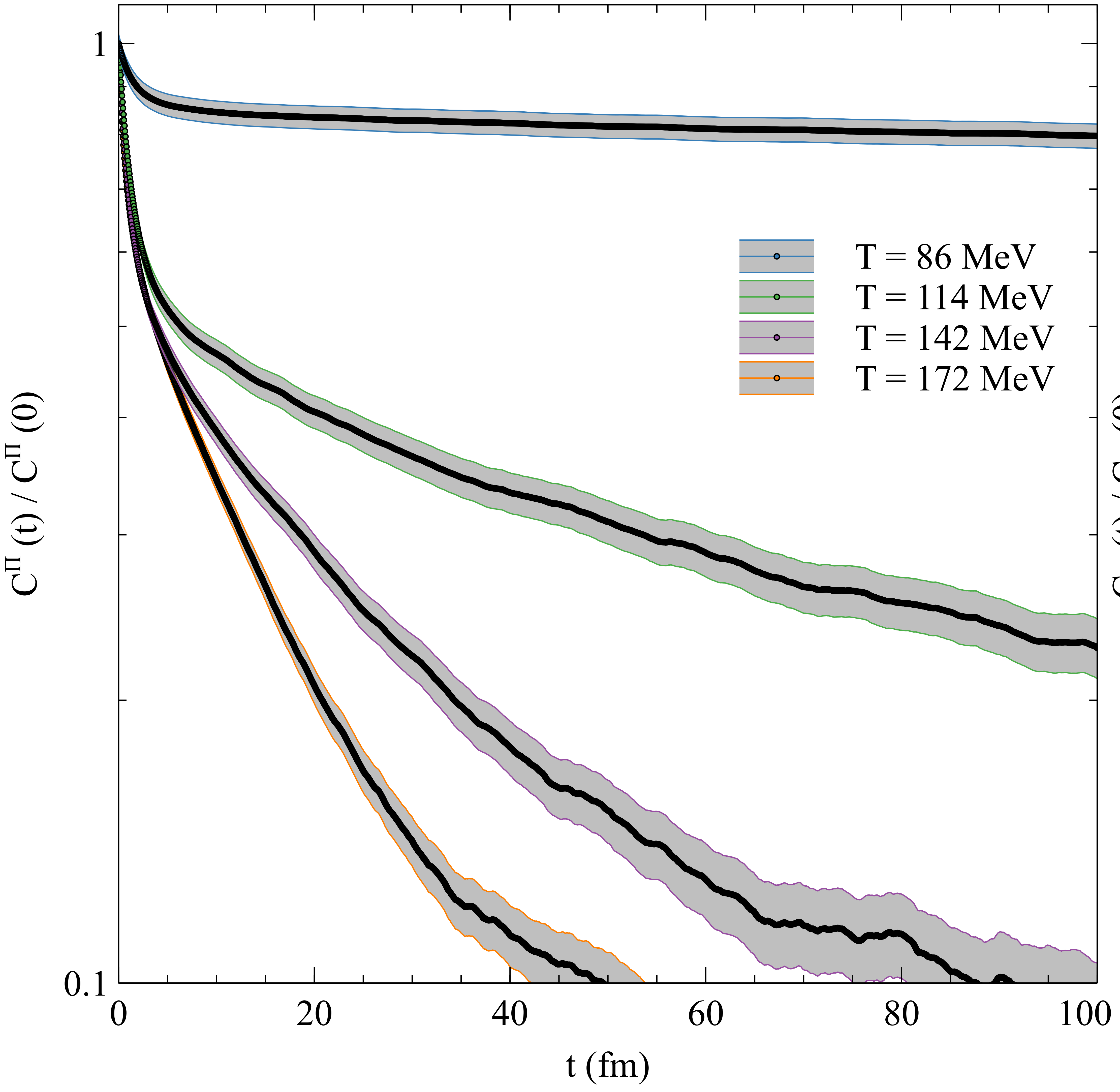


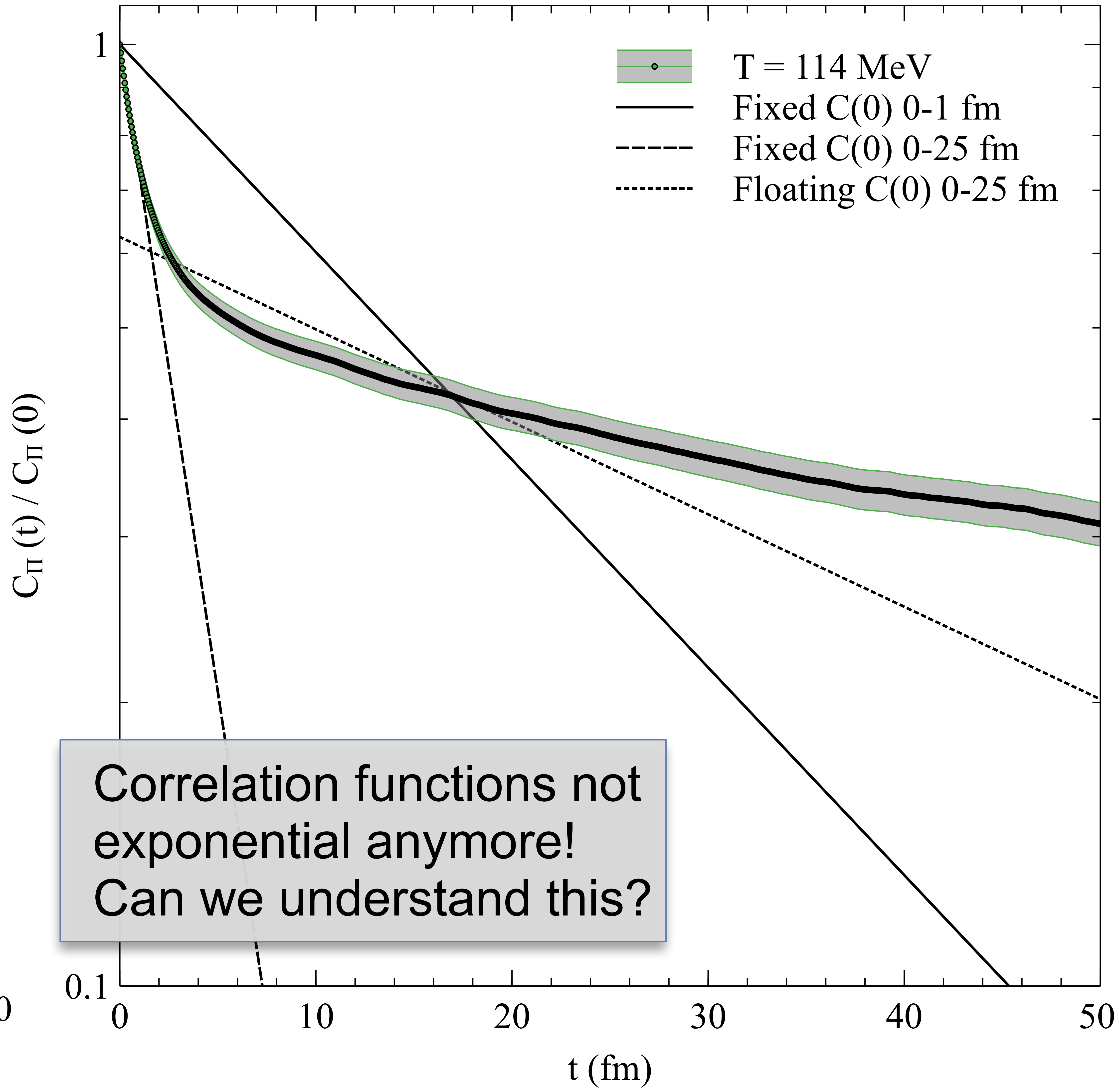


Good agreement with previous calculations, but have to be careful about system size

J. Torres-Rincon, PhD dissertation (2012), *Hadronic Transport Coefficients from Effective Field Theories*

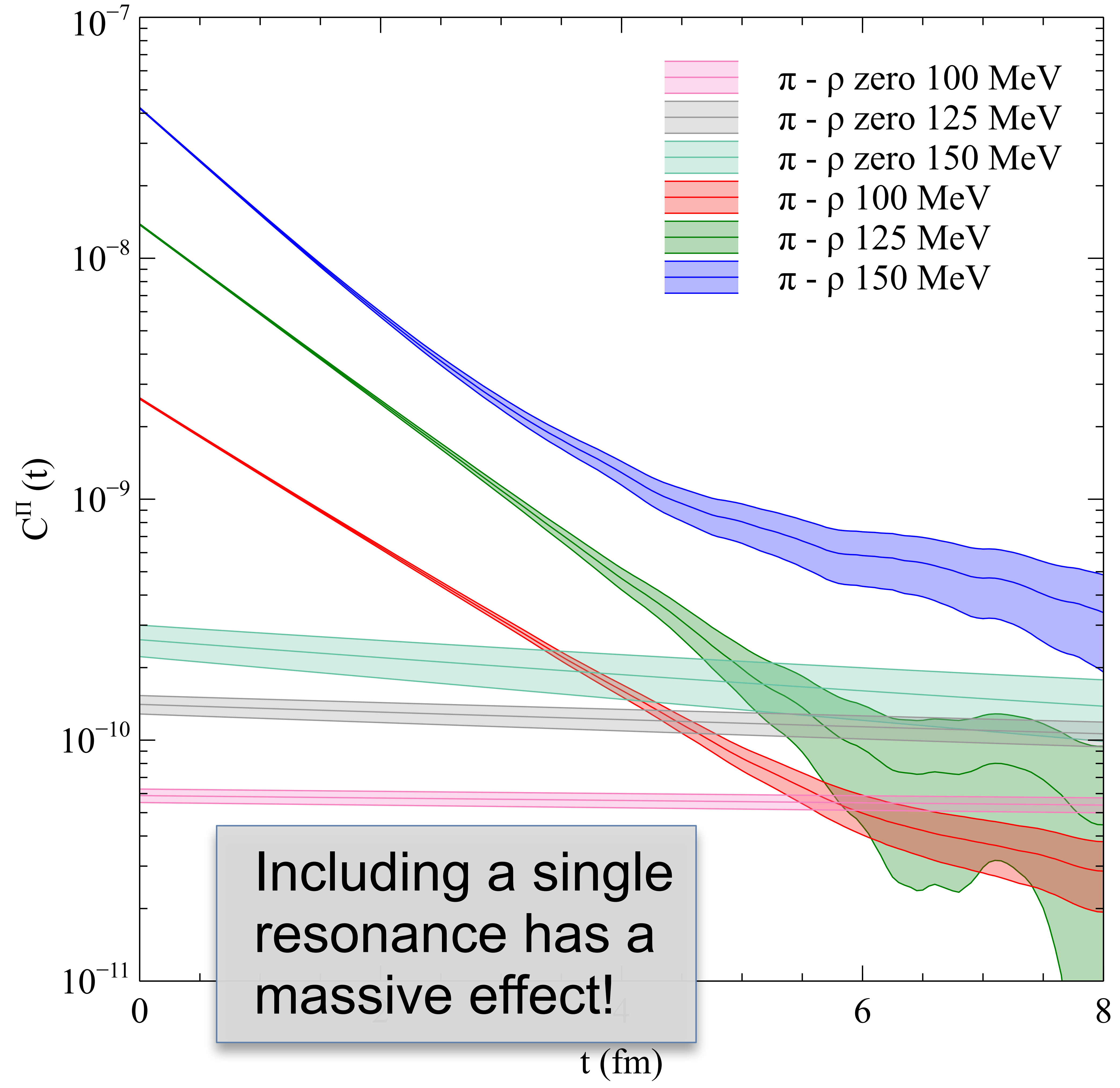
Full hadron gas: Correlations



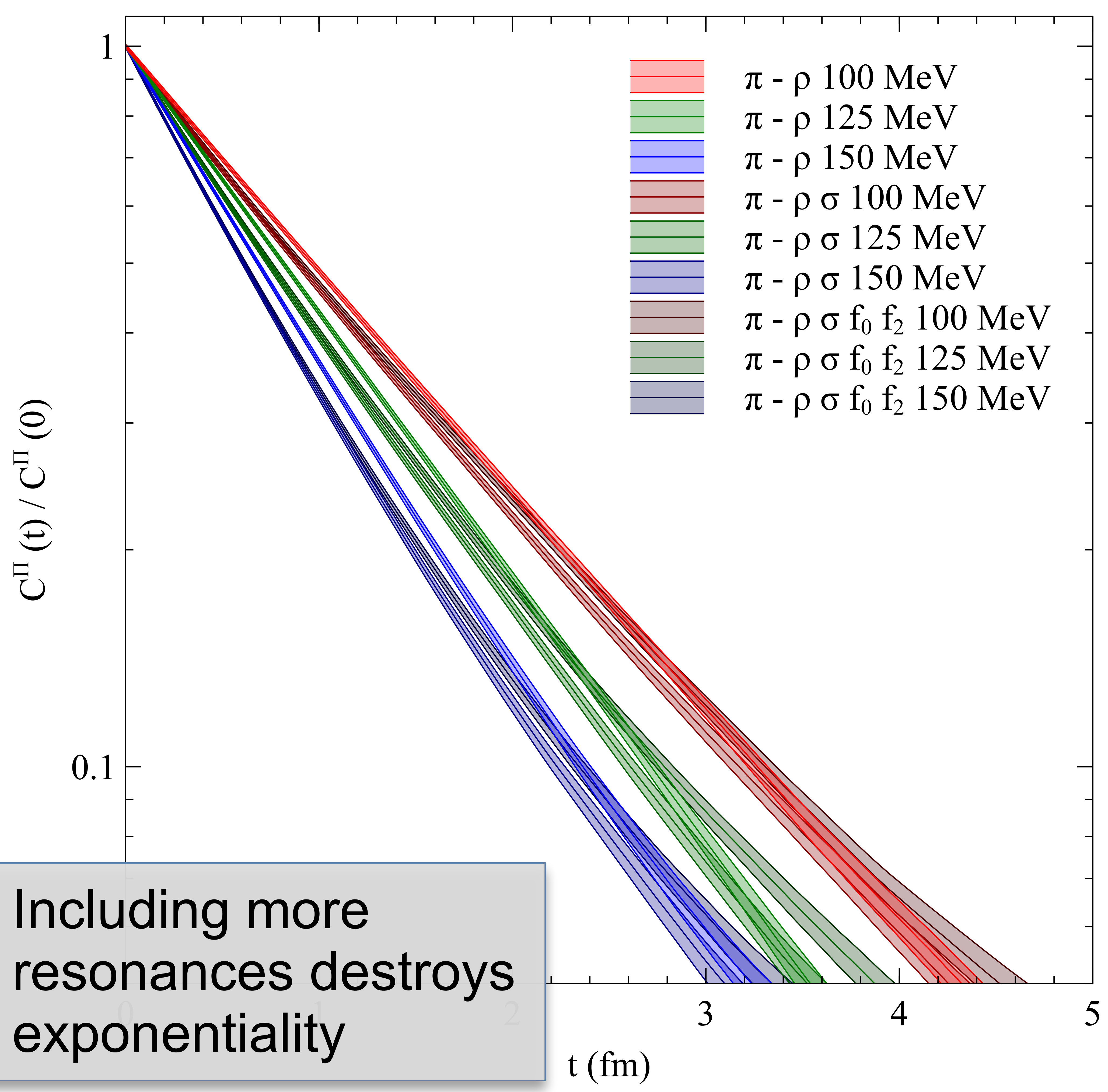


Correlation functions not exponential anymore! Can we understand this?

Test case #2: π with resonances

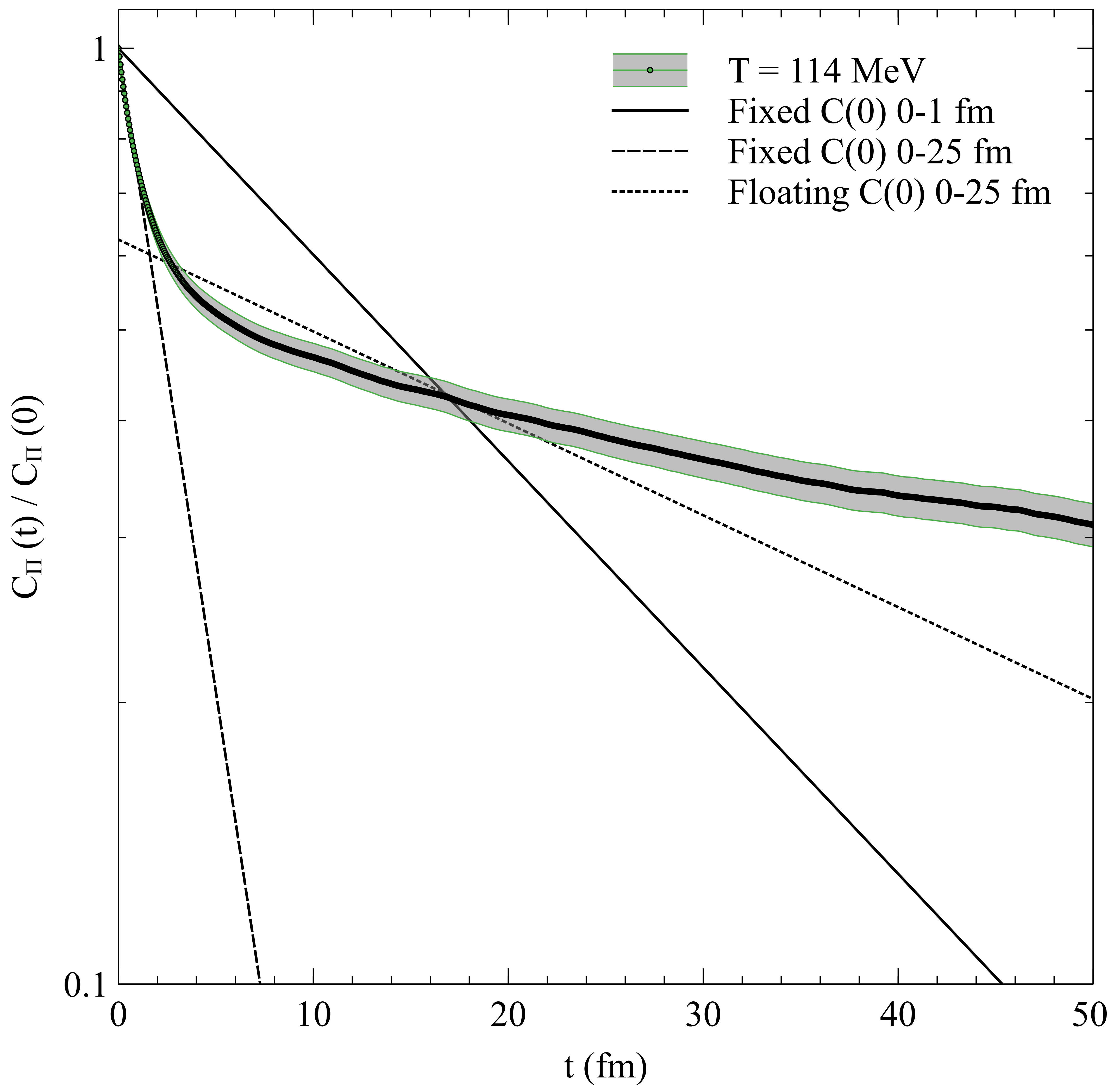


Including a single resonance has a massive effect!



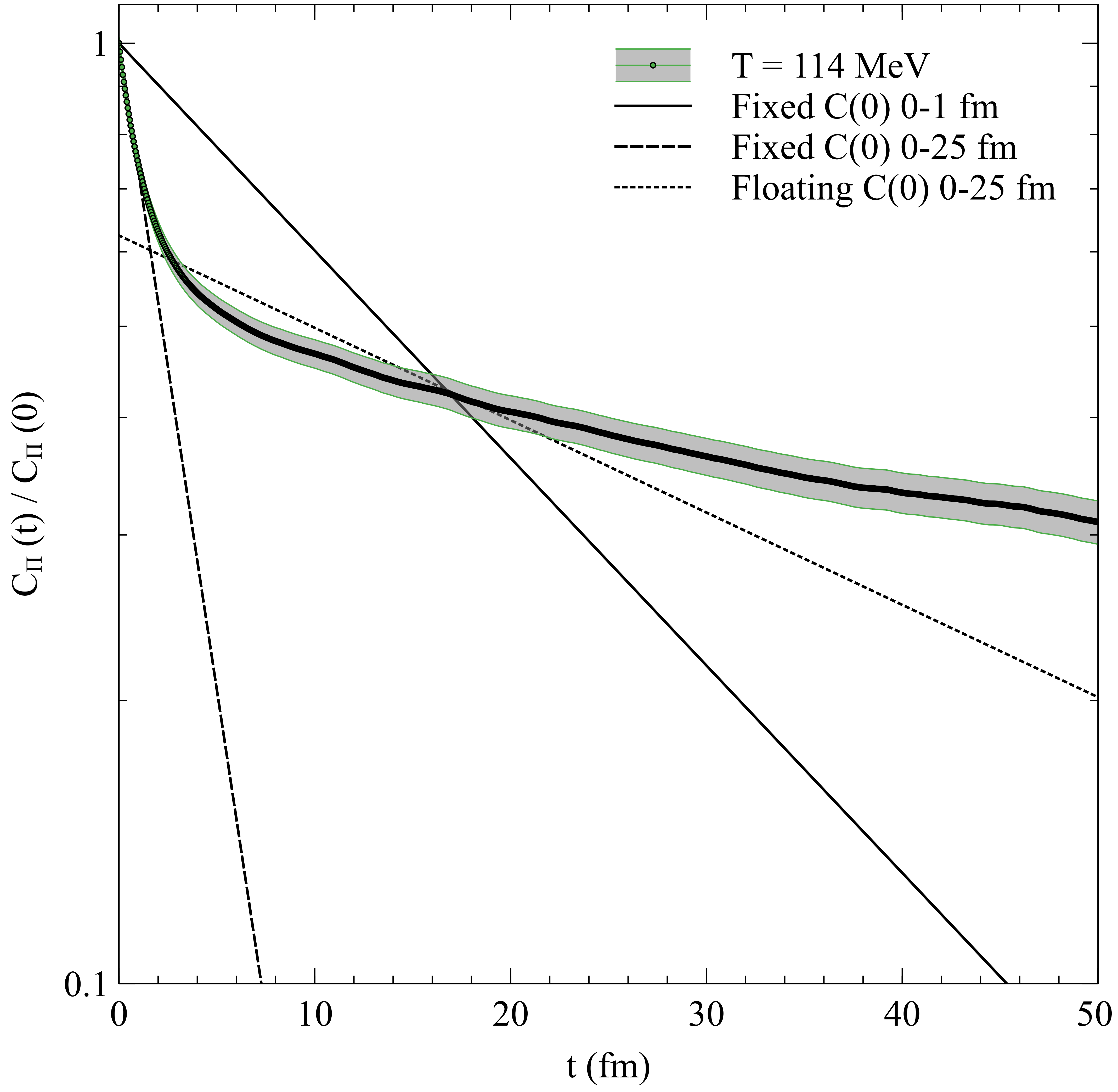
Including more resonances destroys exponentiality

Full hadron gas: Bulk

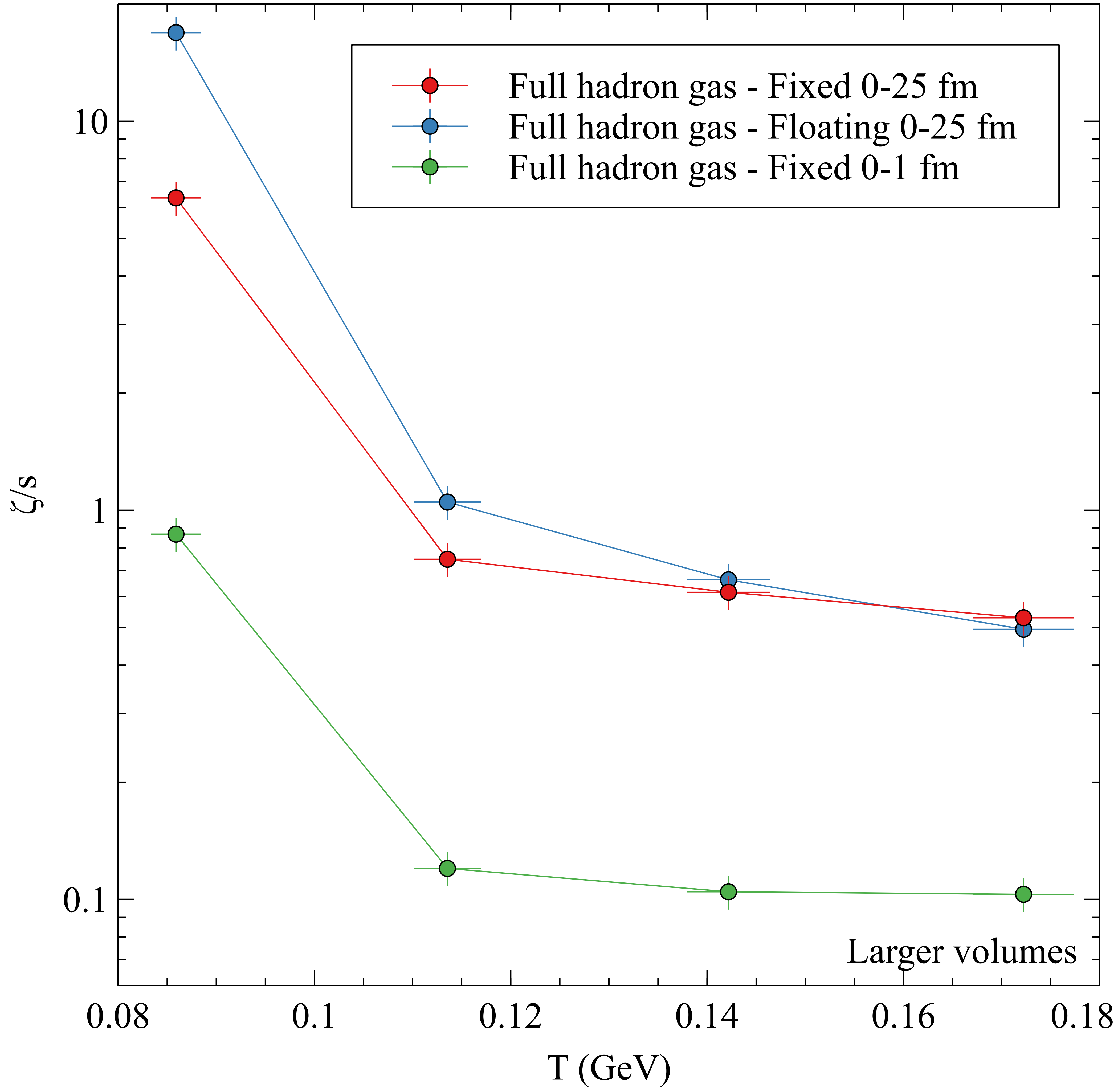


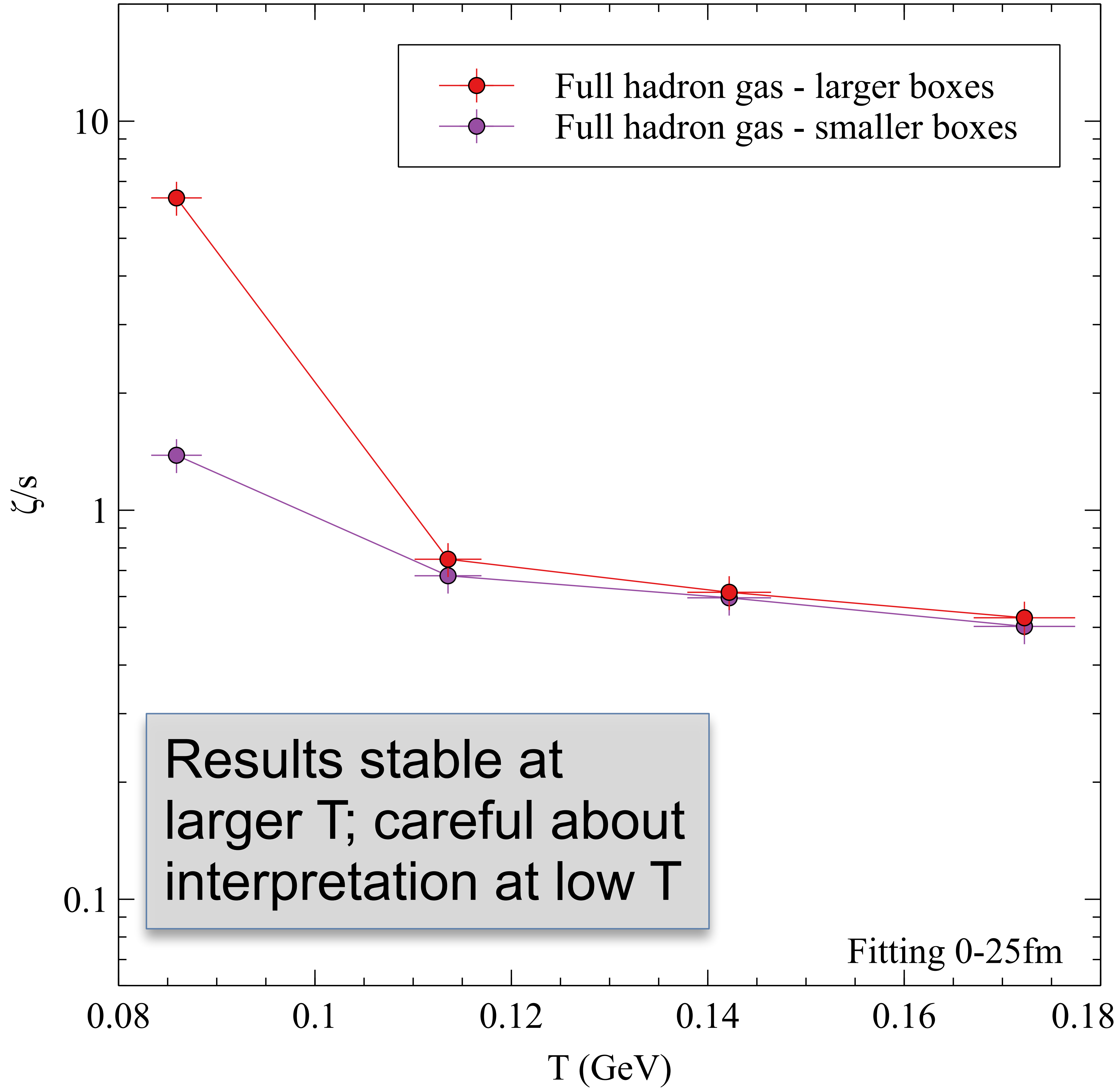
You really shouldn't do this.

Full hadron gas: Bulk



Full hadron gas: Bulk





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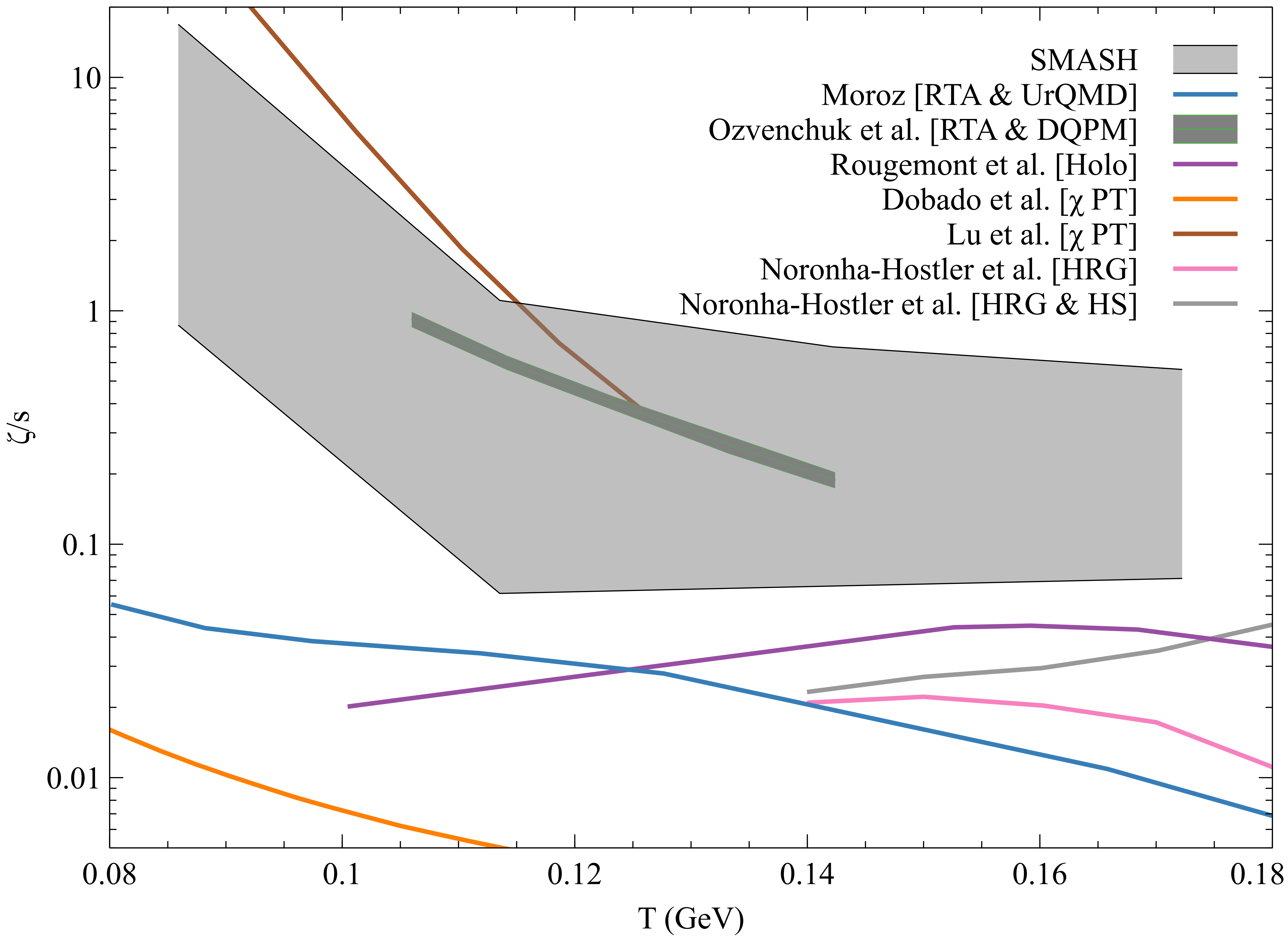
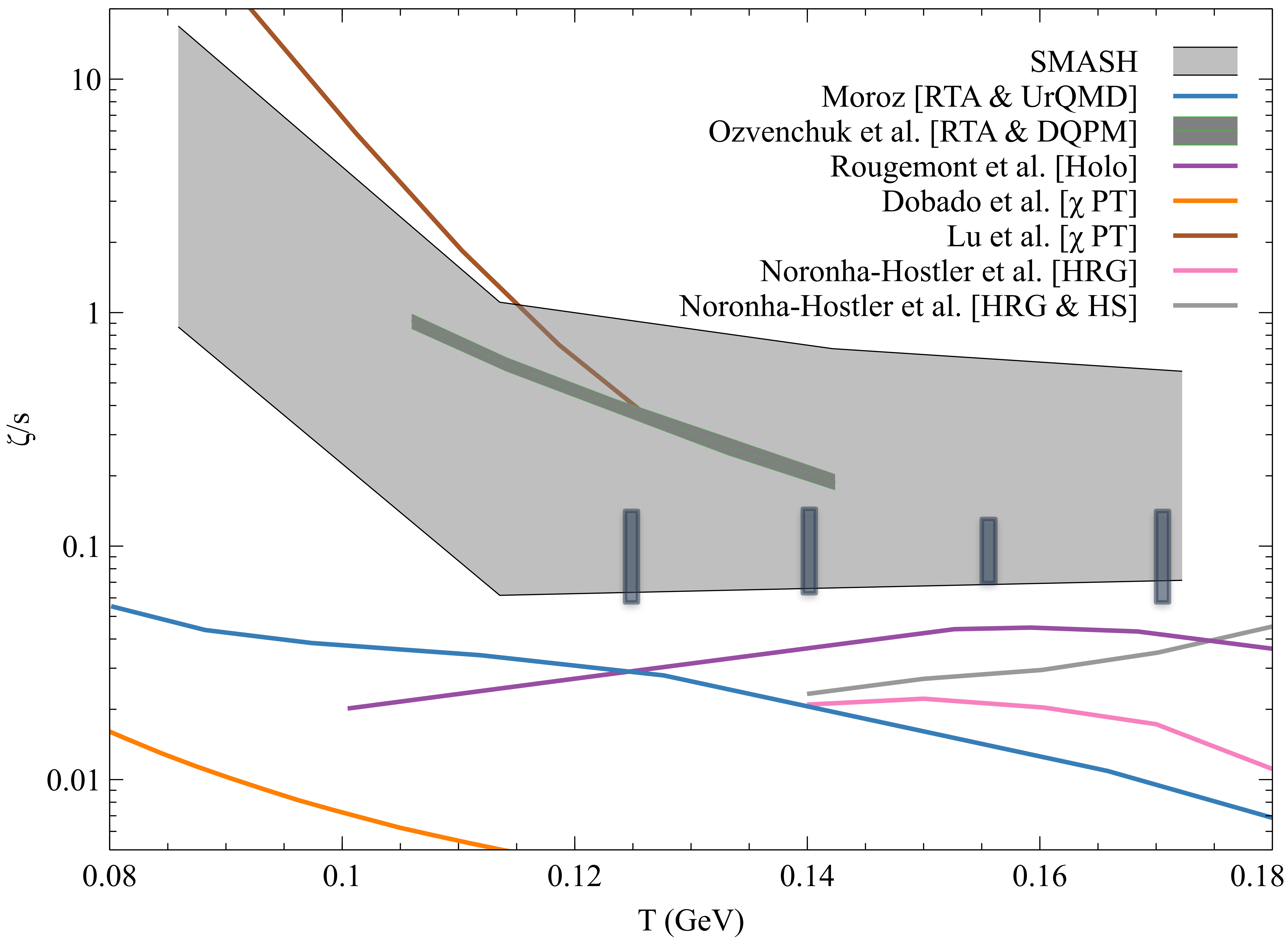
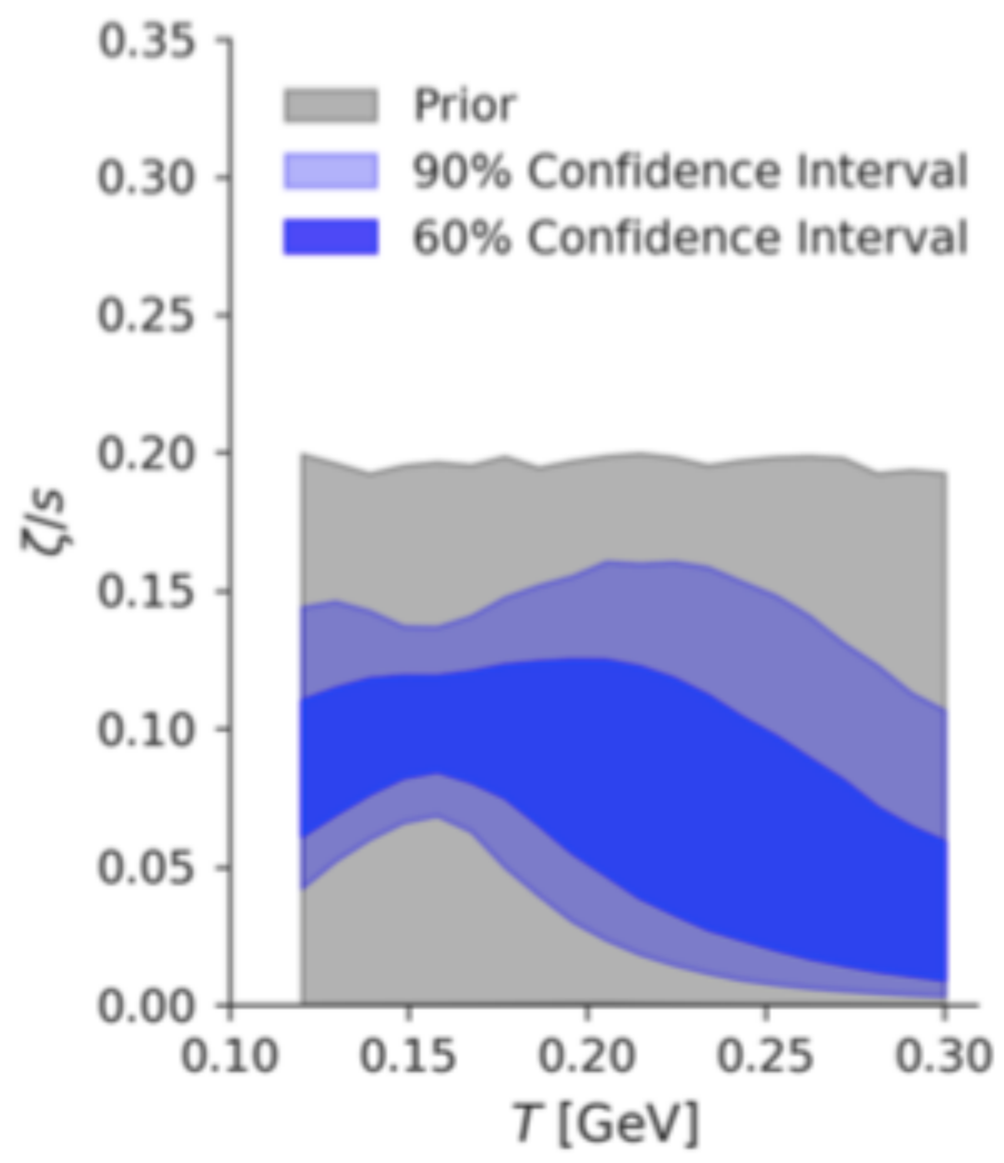


Full hadron gas: Bulk





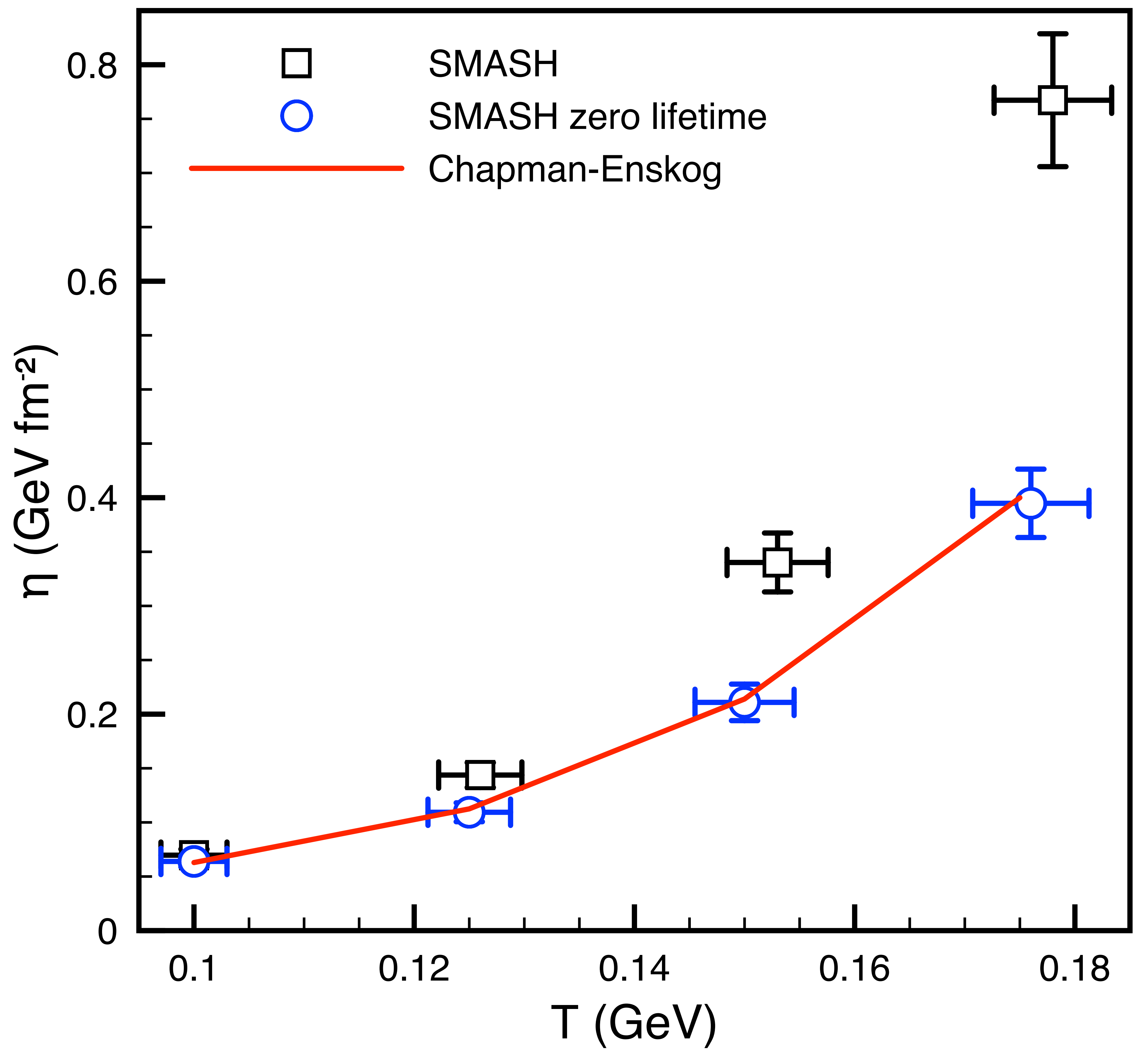
Summary & Outlook

- **Investigated temperature and μ_B dependence of shear viscosity, cross-conductivity and bulk viscosity in various systems**
  - Very good agreement with Chapman-Enskog approximation in all cases
  - Shear viscosity strongly affected by resonance lifetimes
  - Cross-conductivity sensitive to increasing number of degrees of freedom; comparison with current lattice data inconclusive
  - Bulk viscosity requires presence of mass-changing processes such as resonances to not be negligible; massive uncertainty remains due to breakdown of exponential ansatz

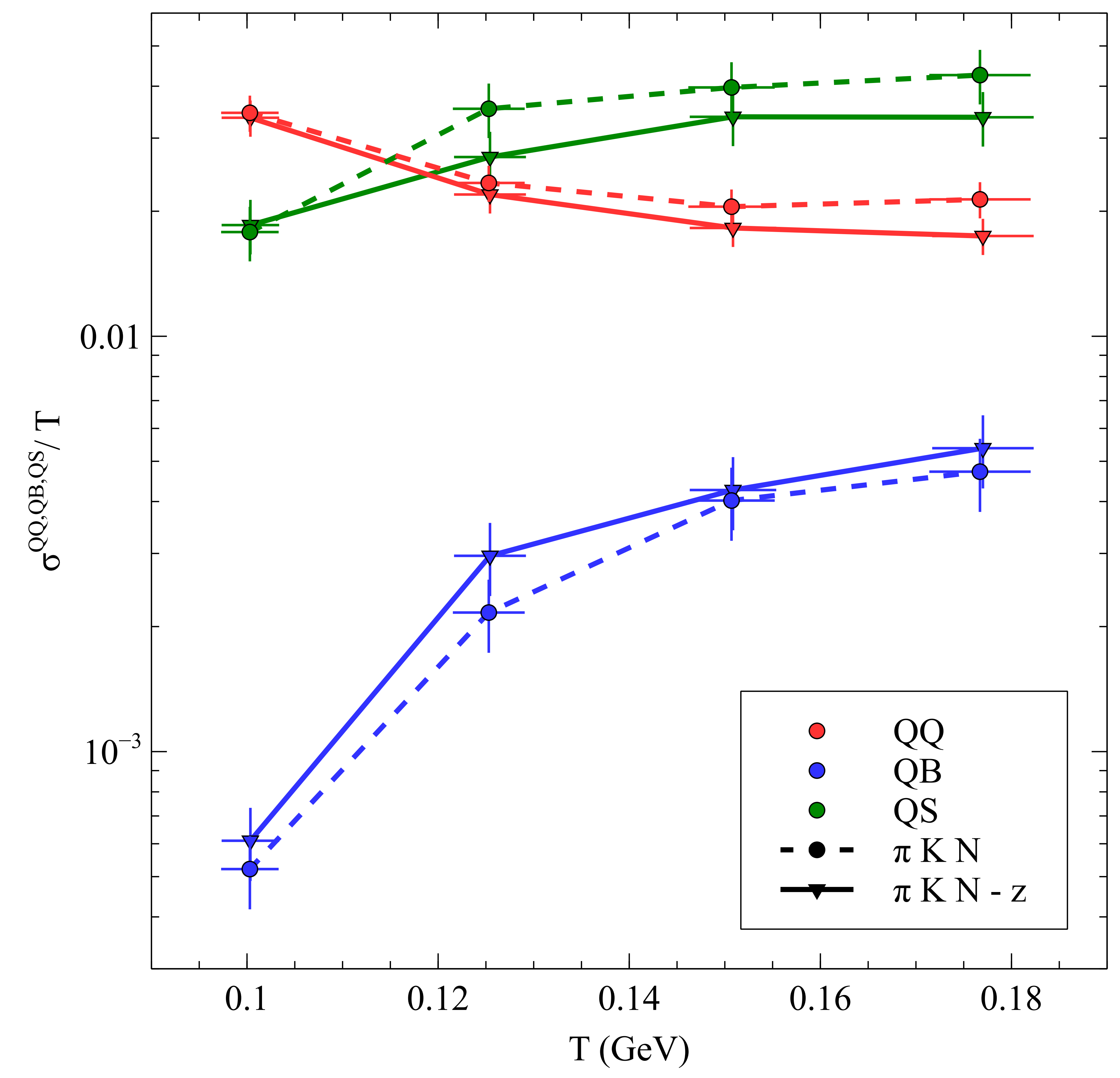
- **Outlook:**
  - Investigation of angular dependent interactions
  - Inclusion of multi-particle interaction will play a role at phase transition
  - Precise low temperature conductivity calculations on the lattice are needed
  - Diffusion of multiple charges (stay tuned: P. Karan)

Backup slides

Resonance lifetimes: Shear vs conductivity

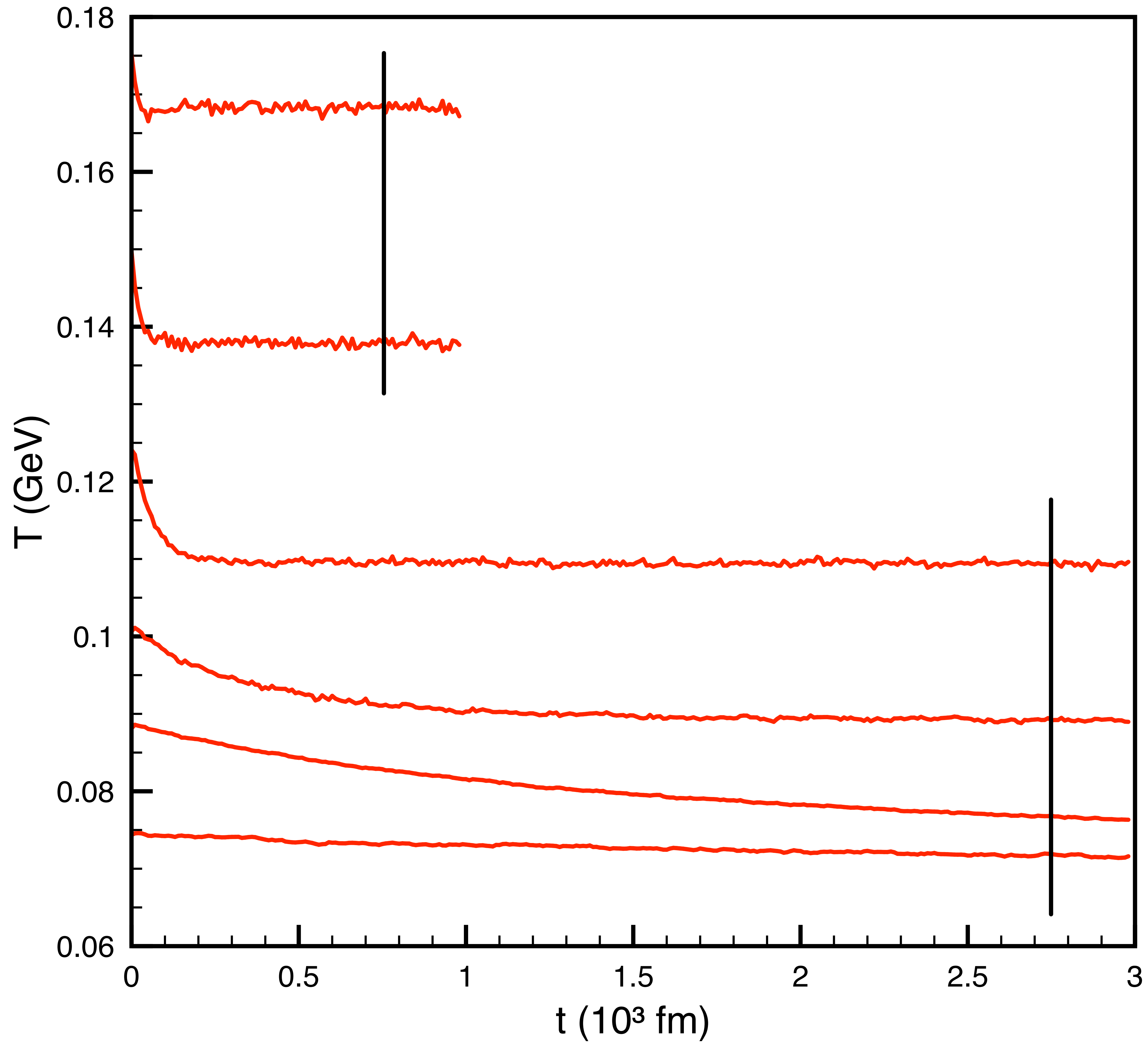


-Rose et al., Phys. Rev. C97 (2018) no.5, 055204



Equilibrium in SMASH

- All particles and resonances initialized to thermal multiplicities (at the pole mass)
- Must wait for equilibration and compute *T*, μ once in equilibrium from most abundant particles
  - T fitted from weighted momentum spectra of π, *K* & *N*
  - μ_B obtained from *N* / anti-*N* ratio



What about entropy?

The entropy density can be calculated from the Gibbs formula:

$$s = \frac{e + p - \mu n}{T} = \frac{w - \mu n}{T}$$

where the energy density and pressure can be taken from the average shear-stress tensor according to:

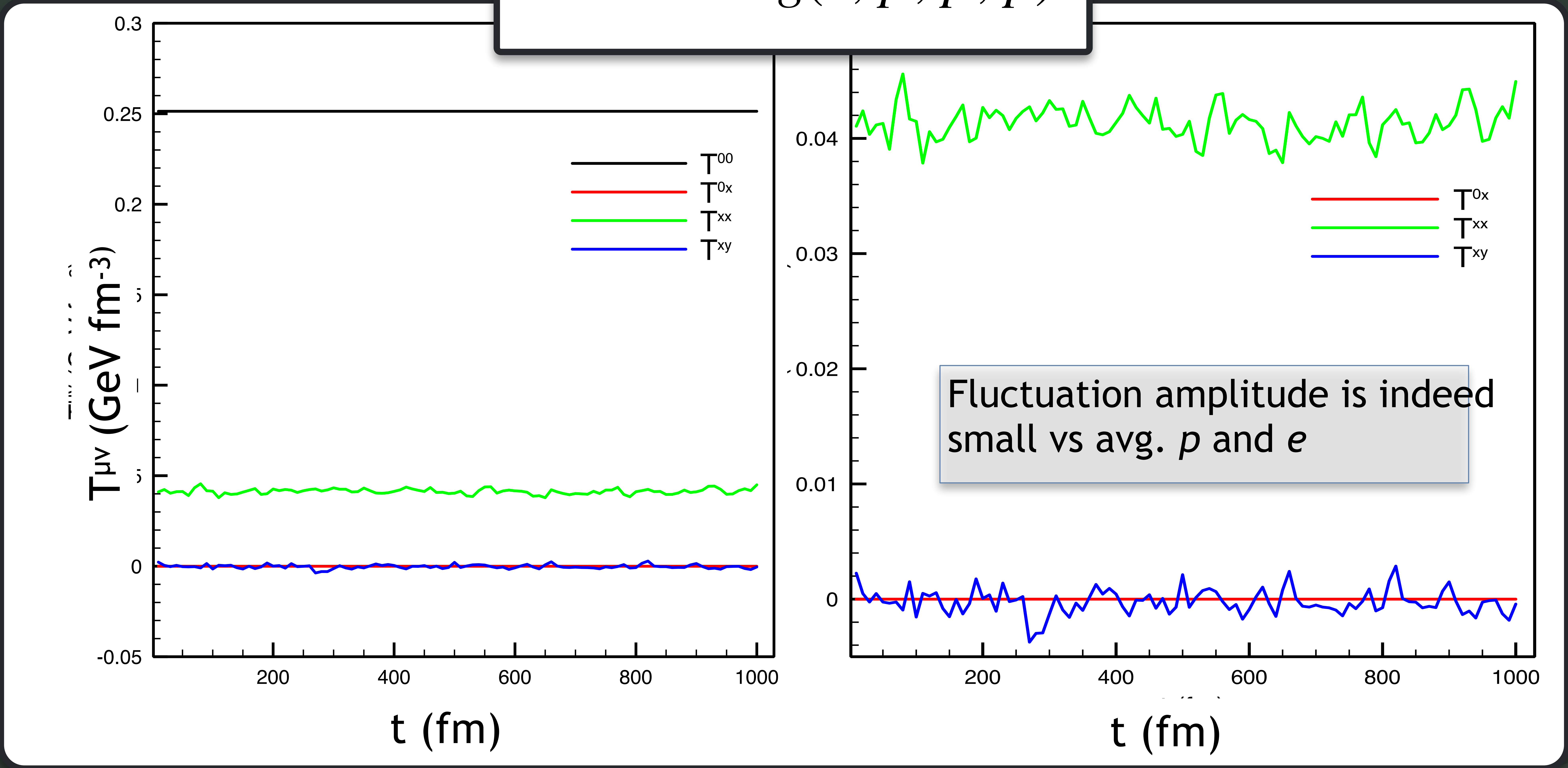
$$T^{\mu\nu} = diag(e, p, p, p)$$

Assuming a nearly ideal gas, one can fit the temperature and chemical potential with momentum distributions:

$$\frac{dN}{dp} = \frac{g}{2\pi^2} V p^2 \exp\left(-\frac{\sqrt{p^2 + m^2} - \mu}{T}\right)$$

Energy density and pressure

$$T^{\mu\nu} = diag(e, p, p, p)$$



Fluctuation amplitude is indeed small vs avg. *p* and *e*