(not only) Transport effects due to conformal anomaly

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A motivation:

Fermions and axial anomaly

Massless Dirac fermions

Covariant formulation

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} i \not{D} \psi$$

$$\not{D} = \gamma^\mu D_\mu \qquad D_\mu = \partial_\mu + ieA_\mu$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad \tilde{F}^{\mu\nu} = \tfrac{1}{2} \varepsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}$$

Semimetals (solid state):

$$\bar{\psi} \left[i\gamma^0 \hbar \frac{\partial}{\partial t} + v_F \boldsymbol{\gamma} (i\hbar \boldsymbol{\nabla} - e\boldsymbol{A}) \right] \psi$$

Currents

$$j_V^\mu = \bar{\psi}\gamma^\mu\psi, \qquad j_A^\mu = \bar{\psi}\gamma^5\gamma^\mu\psi$$

Vector Axial

Axial anomaly

$$\partial_\mu j_A^\mu = \frac{e^2}{16\pi^2} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

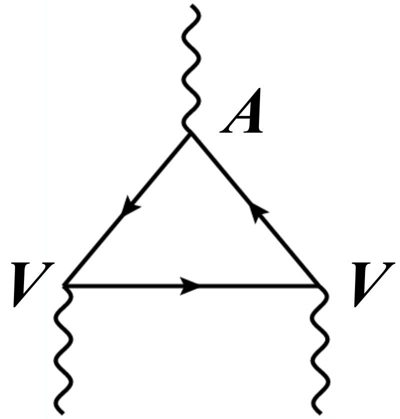
Usual (vector) chemical potential

Chiral (axial) chemical potential

Transport:

$$\boldsymbol{j}_A = \frac{\mu_V}{2\pi^2} e\boldsymbol{B}, \qquad \boldsymbol{j}_V = \frac{\mu_A}{2\pi^2} e\boldsymbol{B}$$

Chiral separation effect Chiral magnetic effect



AVV diagram

Other anomalies / diagrams / anomalous transport effects?

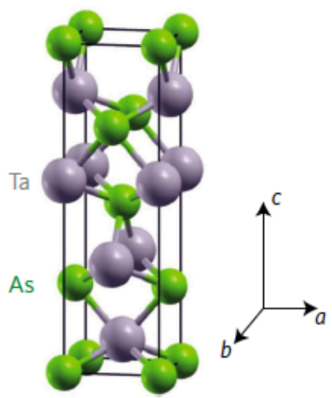
Massless Dirac fermions

A generic system in particle physics, cosmology, solid state …

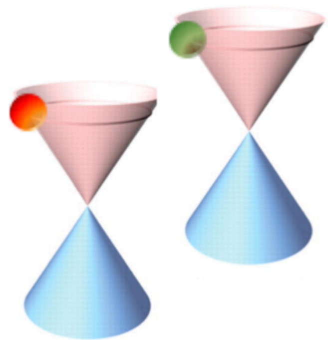
Covariant formulation (quantum field theory)

Dirac semimetals (solid state):

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \boxed{\bar{\psi} i \not{D} \psi} \longrightarrow \boxed{\bar{\psi}\left[i\gamma^0 \hbar \frac{\partial}{\partial t} + v_F \boldsymbol{\gamma}(i\hbar\boldsymbol{\nabla} - e\boldsymbol{A})\right]\psi}$$

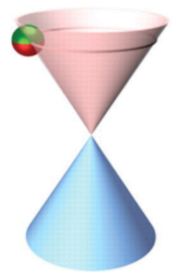


Weyl semimetal
(non-degenerated bands)



TaAs
NbAs
NbP
TaP

Dirac semimetal
(doubly degenerated bands)



$ZrTe_5$
Na_3Bi,
Cd_3As_2

nature › nature materials › volumes › volume 15 › issue 11

Effective low energy description around band crossings in 3D crystals.

Classical symmetries

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi} i \not{D} \psi$$

Vector

$$\psi \to e^{i\omega_V}\psi$$

local/gauge symmetry

vector current is classically conserved

$$j_V^\mu = \bar{\psi}\gamma^\mu\psi \qquad \partial_\mu j_V^\mu = 0$$

Axial

$$\psi \to e^{i\omega_5\gamma^5}\psi$$

global symmetry (no axial gauge field)

axial current is classically conserved

$$j_A^\mu = \bar{\psi}\gamma^5\gamma^\mu\psi \qquad \partial_\mu j_A^\mu = 0$$

Conformal

global scale transformations

$$x \to \lambda^{-1}x, \qquad A_\mu \to \lambda A_\mu, \qquad \psi \to \lambda^{3/2}\psi$$

Dilatation current is classically conserved

$$j_D^\mu = T^{\mu\nu}x_\nu \qquad \partial_\mu j_D^\mu \equiv T^\mu_{\ \mu} \equiv 0$$

$$(T^\mu_\mu)_{\rm cl} \equiv 0$$

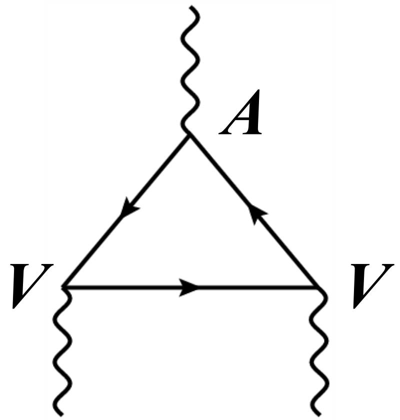
Energy-Momentum tensor

$$T^{\mu\nu} = -F^{\mu\alpha}F^\nu_{\ \alpha} + \frac{1}{4}\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} + \frac{i}{2}\bar{\psi}(\gamma^\mu D^\nu + \gamma^\nu D^\mu)\psi - \eta^{\mu\nu}\bar{\psi}i\not{D}\psi$$

Zoo of anomalies

(three out of six triangular vertices)

Axial

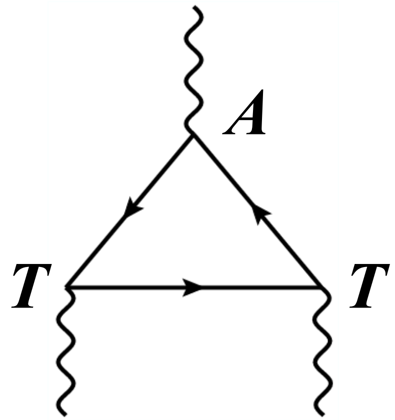


$$\partial_\mu j^\mu_A = \frac{e^2}{16\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

$$\tilde{F}^{\mu\nu} = \tfrac{1}{2}\varepsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$$

Mixed

(axial-gravitational anomaly)

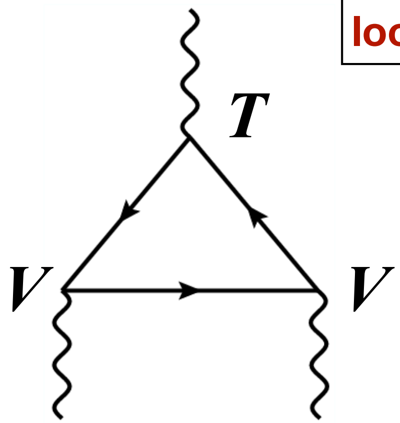


$$\partial_\mu j^\mu_A = -\frac{1}{384\pi^2} R_{\mu\nu\alpha\beta}\tilde{R}^{\mu\nu\alpha\beta}$$

$$\tilde{R}^{\mu\nu\alpha\beta} = \tfrac{1}{2}\varepsilon^{\mu\nu\gamma\lambda}R_{\gamma\lambda}{}^{\alpha\beta}$$

Conformal

not one-loop exact



$$\partial_\mu j^\mu_D = T^\alpha_\alpha$$

beta function

$$\langle T^\mu_{\ \mu}\rangle = \frac{\beta(e)}{2e} F_{\mu\nu}F^{\mu\nu}$$

Currents

$$j^\mu_V = \bar{\psi}\gamma^\mu\psi,$$

Vector

$$j^\mu_A = \bar{\psi}\gamma^5\gamma^\mu\psi$$

Axial

$$j^\mu_D = T^{\mu\nu}x_\nu$$

Dilatation

Full list: AVV, ATT, TVV, TAA, AAA, TTT (not counting torsion!)

Energy-Momentum tensor

$$T^{\mu\nu} = -F^{\mu\alpha}F^\nu_{\ \alpha} + \frac{1}{4}\eta^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta}$$

$$+\frac{i}{2}\bar{\psi}(\gamma^\mu D^\nu + \gamma^\nu D^\mu)\psi - \eta^{\mu\nu}\bar{\psi}i\not{D}\psi$$

Conformal anomaly and the beta function

Massless Dirac fermions $$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} i \not{D} \psi$$

are (classically) invariant under the global (scale) transformations:

$$x \to \lambda^{-1} x, \qquad A_\mu \to \lambda A_\mu, \qquad \psi \to \lambda^{3/2} \psi$$

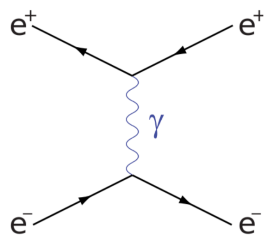
The quantum theory generates an intrinsic scale due to a renormalization (in this particular case) of the electric charge:

$$\beta(e) = \frac{de(\mu)}{d \ln \mu}$$

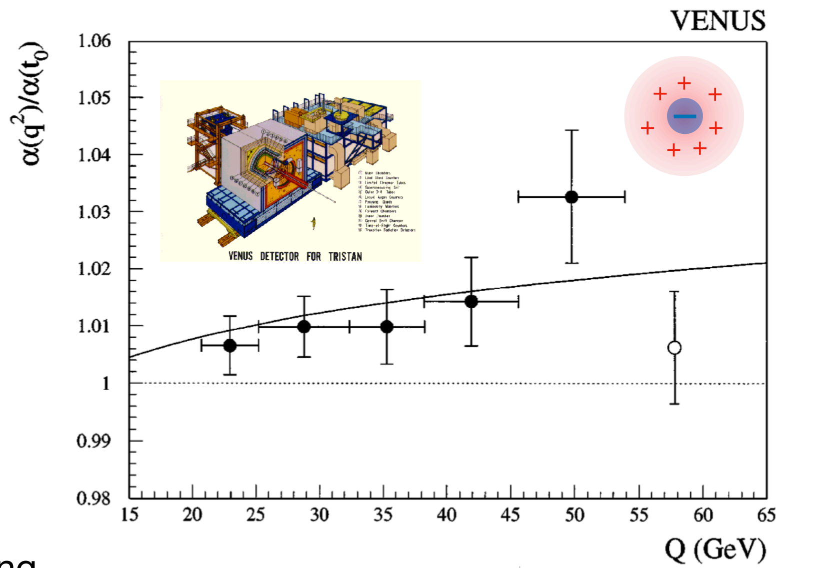
renormalization (energy) scale

In QED (for one Dirac fermion)

$$\beta_{\text{QED}}^{1-\text{loop}} = \frac{e^3}{12\pi^2}$$



Bhabha scattering



S. Odaka et all (VENUS collaboration/KEK), PRL 81, 2428(1998)

→ conformal/scale symmetry is broken at the quantum level

Quantum anomaly → anomalous transport

Axial anomaly $\partial_\mu j^\mu_A = \frac{e^2}{16\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$

Transport $\boldsymbol{j}_A = \frac{\mu_V}{2\pi^2} e\boldsymbol{B}, \qquad \boldsymbol{j}_V = \frac{\mu_A}{2\pi^2} e\boldsymbol{B}$

Chiral separation and chiral magnetic effects

topological = exact in one loop = interesting!

Mixed axial-gravitational anomaly $\partial_\mu j^\mu_A = -\frac{1}{384\pi^2} R_{\mu\nu\alpha\beta}\tilde{R}^{\mu\nu\alpha\beta}$

Transport $\boldsymbol{j}_V = \frac{\mu_V \mu_A}{\pi^2} \boldsymbol{\Omega}, \qquad \boldsymbol{j}_A = \left(\frac{T^2}{6} + \frac{\mu_V^2 + \mu_A^2}{2\pi^2} \right) \boldsymbol{\Omega}$

Thermal contribution to chiral vortical effects

topological = exact in one loop = interesting!

Conformal anomaly $\partial_\mu j^\mu_D = T^\alpha_\alpha \qquad \langle T^\mu_\mu \rangle = \frac{\beta(e)}{2e} F_{\mu\nu} F^{\mu\nu}$

Transport?

not topological … not one-loop exact … not interesting?

Conformal anomaly →
Scale Electric Effect (SEE) and Scale Magnetic Effect (SME)

Oversimplified picture

Gravitational background: Weyl-transformed flat space

flat (Minkowski) metric

$$g_{\mu\nu}(x) = e^{2\tau(x)}\eta_{\mu\nu}$$

scale factor (arbitrary function of coordinates)

The conformal anomaly leads to scale electromagnetic effects:

$$\boxed{\langle T^{\mu}_{\ \mu} \rangle = \frac{\beta(e)}{2e} F_{\mu\nu} F^{\mu\nu}}$$

$$j^{\mu} \equiv \langle j^{\mu}_V \rangle = -\frac{2\beta(e)}{e} F^{\mu\nu} \partial_\nu \tau$$

"Generation of an electric current in a background of electromagnetic and gravitational fields"

The matter is absent. This is a quantum effect in the vacuum.

Fine print: 1) natural in a linear-response theory
2) very unnatural (would-be-wrong) in general relativity
→ to be refined (and made less simple) later

M.Ch., PRL 117, 141601 (2016)

Scale electric effect (SEE)

Time-dependent background: $\tau = \tau(t)$

Metric: $g_{\mu\nu}(x) = e^{2\tau(x)}\eta_{\mu\nu}$

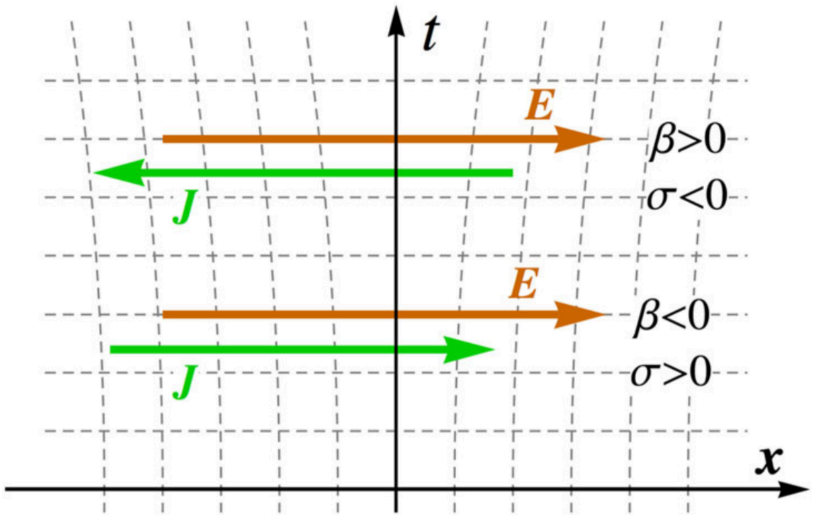
Scale Electric Effect:

$$\langle \boldsymbol{j}(t,\boldsymbol{x})\rangle_{\text{scale}} = \sigma(t)\boldsymbol{E}(t,\boldsymbol{x}) \quad \text{for } \boldsymbol{\nabla}\tau = 0$$

Negative conductivity in an expanding space-time!

Conformal conductivity:

$$\sigma(t,\boldsymbol{x}) = -\frac{2\beta(e)}{e}\frac{\partial\tau(t,\boldsymbol{x})}{\partial t}$$



Independently obtained in the de Sitter spacetime (a version of the Schwinger effect, both for fermions and bosons)

- → T. Hayashinaka, T. Fujita, and J. Yokoyama, Fermionic Schwinger effect and induced current in de Sitter space, J. Cosmol. Astropart. Phys. 07 (2016) 010; T. Hayashinaka and J. Yokoyama, Point splitting renormalization of Schwinger induced current in de Sitter spacetime, J. Cosmol. Astropart. Phys. 07 (2016) 012.
- → T. Kobayashi and N. Afshordi, Schwinger effect in 4D de Sitter space and constraints on magnetogenesis in the early Universe, J. High Energy Phys. 10 (2014) 166.
- → **Photon production in time-dependent cosmological backgrounds: A.D.Dolgov, PRD 48, 2499 (1993).**

Scale magnetic effect (SME)

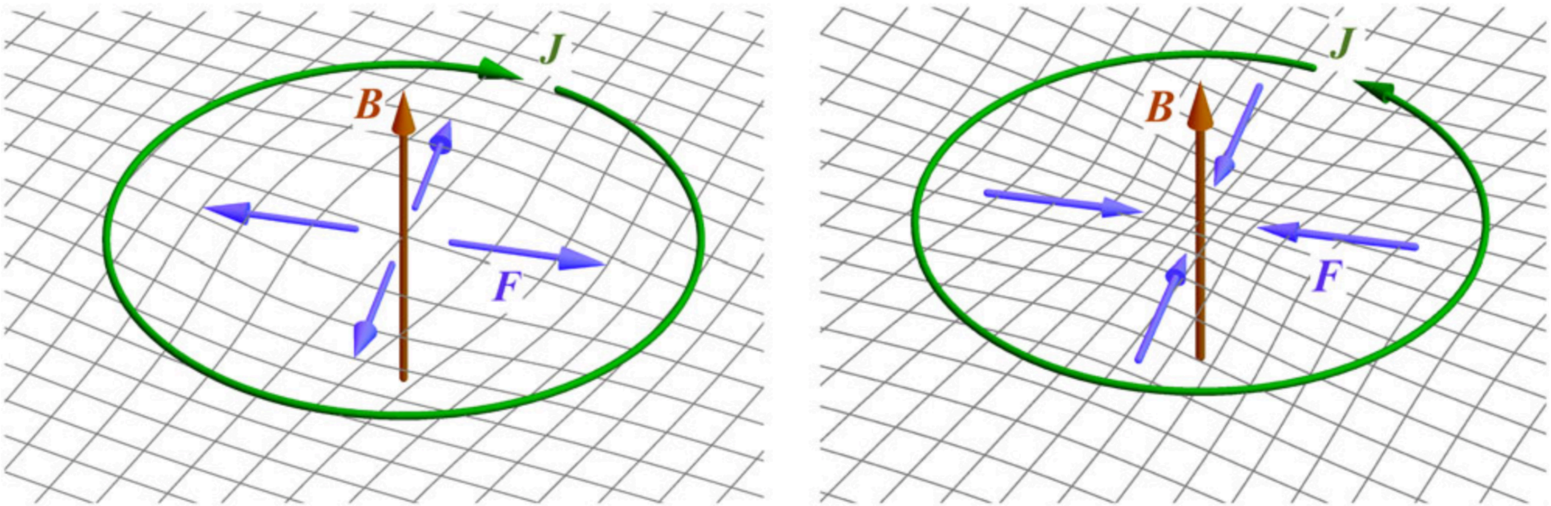
Space-dependent background: $\tau = \tau(\boldsymbol{x})$

Scale Magnetic Effect:

$$\langle \boldsymbol{j}(t, \boldsymbol{x}) \rangle_{\text{scale}} = \boldsymbol{F}(\boldsymbol{x}) \times \boldsymbol{B}(t, \boldsymbol{x}) \quad \text{for } \partial_t \tau = 0$$

Gravitational deformation vector:

$$\boldsymbol{F}(t, \boldsymbol{x}) = \frac{2\beta(e)}{e} \boldsymbol{\nabla} \tau(t, \boldsymbol{x})$$



Distantly similar to Hall effects (no electric field, though).

Formal derivation of scale electromagnetic effects

Conformal anomaly in curved space-time in electromagnetic field background

$$\langle T^{\mu}_{\ \mu} \rangle \equiv -\frac{2}{\sqrt{-g}} g_{\mu\nu} \frac{\delta S_{\rm anom}}{\delta g_{\mu\nu}}$$

anomalous action which generates anomaly

$$= -\frac{1}{4}\left[\underbrace{bC^2 + b'\left(E - \frac{2}{3}\Box R\right)}_{\text{gravitational (curved space-time) background}} + \underbrace{cF_{\mu\nu}F^{\mu\nu}}_{\text{electromagnetic background}} \right]$$

M. Duff, Nucl.Phys.B 125, 334 (1977)

$$C^2 = C_{\mu\nu\alpha\beta} C^{\mu\nu\alpha\beta}$$
$$\equiv R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 2R_{\mu\nu}R^{\mu\nu} + \frac{R^2}{3}$$

Weyl tensor squared

$$E = {}^*R_{\mu\nu\alpha\beta}{}^*R^{\mu\nu\alpha\beta}$$
$$\equiv R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} - 4R_{\mu\nu}R^{\mu\nu} + R^2$$

$${}^*R_{\mu\nu\alpha\beta} = \frac{1}{2}\epsilon_{\mu\nu\mu'\nu'} R^{\mu'\nu'}{}_{\alpha\beta}$$

Euler (topological) density

Couplings

$$b = \frac{1}{320\pi^2}, \qquad b' = -\frac{11}{5670\pi^2}, \qquad c = -\frac{e^2}{24\pi^2}.$$

comes from beta function

Formal derivation of scale electromagnetic effects

Conformal anomaly in curved space-time in electromagnetic field background

$$\langle T^{\mu}_{\ \mu} \rangle \equiv -\frac{2}{\sqrt{-g}} g_{\mu\nu} \frac{\delta S_{\text{anom}}}{\delta g_{\mu\nu}}$$

what is the anomalous action?

$$= -\frac{1}{4} \left[bC^2 + b' \left(E - \frac{2}{3} \Box R \right) + c F_{\mu\nu} F^{\mu\nu} \right]$$

M. Duff, Nucl.Phys.B 125, 334 (1977)

Anomalous action:

$$S_{\text{anom}}[g, A] = \frac{1}{8} \int d^4x \sqrt{-g(x)} \int d^4y \sqrt{-g(y)}$$

$$\cdot H(x) \Delta_4^{-1}(x, y) \left[2bC^2(y) + b' H(y) + 2c F_{\mu\nu}(y) F^{\mu\nu}(y) \right]$$

R. J. Riegert, Phys. Lett. B 134, 56 (1984)

Nonlocality: Green's function for 4th order conformal differential operator

$$\Delta_4 = \nabla_\mu \left(\nabla^\mu \nabla^\nu + 2R^{\mu\nu} - \frac{2}{3} R g^{\mu\nu} \right) \nabla_\nu$$

$$H = E - \frac{2}{3} \Box R$$

S. Paneitz, MIT preprint (1983)

Formal derivation of scale electromagnetic effects

Anomalous electric current from anomalous conformal action

$$J^\mu(x) = -\frac{1}{\sqrt{-g(x)}}\frac{\delta S_{\text{anom}}}{\delta A_\mu(x)}$$ variation with respect to the gauge field

$$= -\frac{1}{\sqrt{-g(x)}}\frac{\partial}{\partial x^\nu}\Big[\sqrt{-g(x)}\,F^{\mu\nu}(x)$$ electromagnetic field strength

$$\cdot \int d^4y \sqrt{-g(y)}\,\Delta_4^{-1}(x,y)\Big(E(y) - \frac{2}{3}\Box R(y)\Big)\Big]$$ space-time curvature enters here

Green's function for 4th order Paneitz conformal differential operator

Weak curvature, linear order:

$$J^\mu(x) = +\frac{e^2}{6\pi^2}F^{\mu\nu}(x)\partial_\nu\varphi(x)$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Minkowski   small perturbation

Effective scale factor:

$$\varphi(x) = \frac{1}{6}\int d^4y\,\Box^{-1}_{x,y}\left[\partial_\alpha\partial_\beta h^{\alpha\beta}(y) - \eta_{\alpha\beta}\Box h^{\alpha\beta}(y)\right]$$

for $g_{\mu\nu}(x) = e^{2\tau(x)}\eta_{\mu\nu}$

$$J^\mu = \frac{2\beta(e)}{e}F^{\mu\nu}\partial_\nu\tau$$

A. Cortijo, M. Vozmediano, M.Ch., PRL 120, 206601 (2018)

Tolman-Ehrenfest law

Temperature gradient vs. gravitational field?

In a static background gravitational field, the temperature of a system in a thermal equilibrium is not constant (Tolman-Ehrenfest):

$$T(\boldsymbol{x})\sqrt{g_{00}(\boldsymbol{x})} = T_0$$

A temperature gradient that drives system out of equilibrium may be mimicked by a gravitational potential (Luttinger):

$$\frac{1}{T}\boldsymbol{\nabla} T = -\frac{1}{c^2}\boldsymbol{\nabla}\Phi \qquad g_{00} = 1 + \frac{2\Phi}{c^2}$$

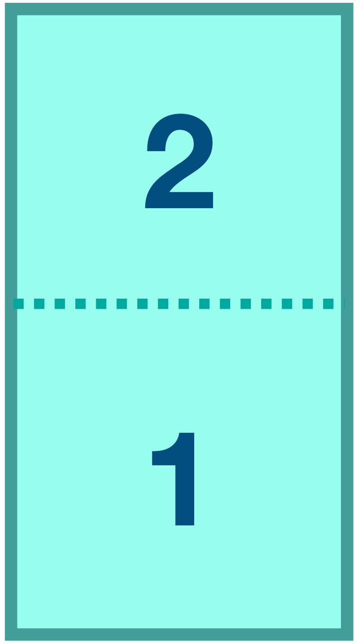
R. C. Tolman, “On the Weight of Heat and Thermal Equilibrium in General Relativity,” Phys. Rev. 35, 904 (1930);
R. Tolman and P. Ehrenfest, “Temperature Equilibrium in a Static Gravitational Field,” Phys. Rev. 36, 1791 (1930).
J.M. Luttinger, Theory of Thermal Transport Coefficients, Phys.Rev. 135 (1964) A1505

Thermal equilibrium

– Consider a closed system divided arbitrarily into two subsystems

– Thermal equilibrium happens when the total entropy reaches its maximum

$$S = S_1 + S_2 \qquad dS_1 + dS_2 = 0$$



– Assume that we have no gravitational field

– If the quantity of heat leaves the first subsystem, it always enters the second subsystem:

$$dE_1 = -dE \rightarrow dE_2 = dE \rightarrow dS_1/dE_1 = dS_2/dE_2 \rightarrow T_1 = T_2$$

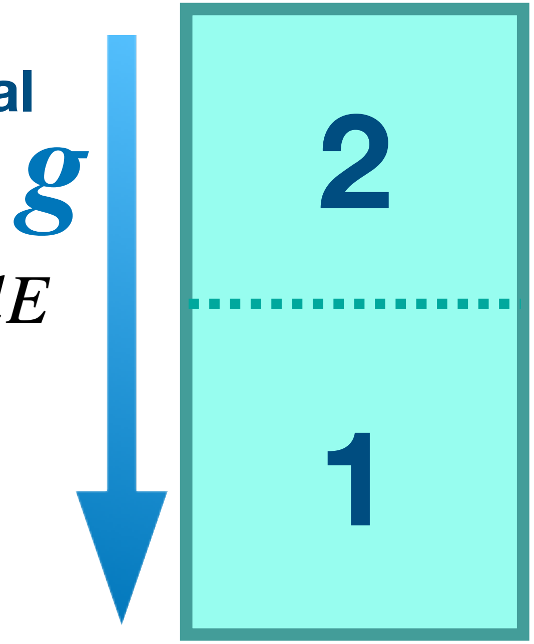
definition of temperature $T = dS/dE$

In the absence of gravitational field, the temperature is constant

How to understand the Tolman-Ehrenfest law?

– In a static gravitational field Φ, the heat quantity dE possesses an inertial mass $$dm = dE/c^2$$

– the equivalence between inertial and gravitational masses: a quantity of heat has a weight



– When heat leaves the first subsystem, $dE_1 = -dE$ it enters the second subsystem, and performs work agains the gravity (heat = mass):

$$dE_2 = dE + (\Phi_2 - \Phi_1)dm = dE_2(1 + \Delta\Phi/c^2)$$

$$\Delta\Phi = \Phi_2 - \Phi_1$$

change of the gravitational potential

$$T_1 = T_2$$

– Entropy maximum

$$dS_1 + dS_2 = 0$$

– Local temperature

$$g_{00} = 1 + \frac{2\Phi}{c^2}$$

$$T_2 = T_1(1 + \Delta\Phi/c^2) \longrightarrow T(x) = T_0/\sqrt{g_{00}(x)}$$

Tolman-Ehrenfest law

C. Rovelli and M. Smerlak, “Thermal time and Tolman-Ehrenfest effect: temperature as the speed of time” Class. Quant. Grav. 28, 4 (2010)

The SME in Dirac and Weyl semimetals

PHYSICAL REVIEW LETTERS **120,** 206601 (2018)

Generation of a Nernst Current from the Conformal Anomaly in Dirac and Weyl Semimetals

M. N. Chernodub,[1,2] Alberto Cortijo,[3] and María A. H. Vozmediano[3]

1.) A temperature gradient that drives system out of equilibrium may be mimicked by a gravitational potential (Luttinger):

$$\frac{1}{T}\nabla T = -\frac{1}{c^2}\nabla\Phi \qquad g_{00} = 1 + \frac{2\Phi}{c^2}$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

flat metric + small perturbation

2.) In a magnetic-field background in a curved space-time, the conformal anomaly generates an electric current via the scale magnetic effect.

→ Conformal anomaly generates thermoelectric transport!

Anomalous electric current: $J^\mu(x) = +\frac{e^2}{6\pi^2} F^{\mu\nu}(x)\partial_\nu\varphi(x)$

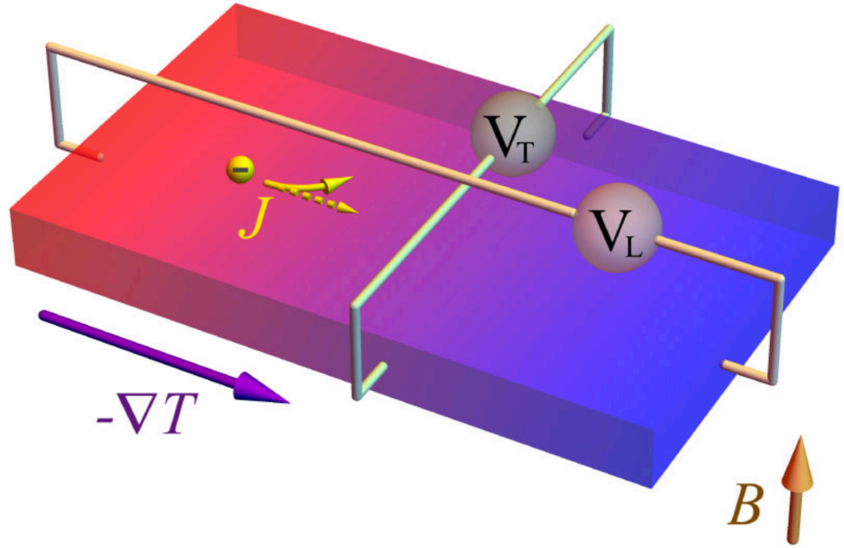
with the effective scale factor: $\varphi(x) = \frac{1}{6}\int d^4y\, \Box^{-1}_{x,y}\left[\partial_\alpha\partial_\beta h^{\alpha\beta}(y) - \eta_{\alpha\beta}\Box h^{\alpha\beta}(y)\right]$

(it will become a local quantity)

Nernst-Ettingshausen Effect in Weyl semimetals

Giant Nernst Effect due to SME

$$\boldsymbol{J} = \frac{e^2 v_F}{18\pi^2 T\hbar}\boldsymbol{B}\times\boldsymbol{\nabla}T$$



The electric current is proportional to the beta function (conformal anomaly!)

Longitudinal anomalous transport in Weyl semimetals:

$$S_{11} \equiv \frac{E_1}{B_3 \nabla_1 T} = \frac{\rho_{12}\mathcal{L}^{12}_{21}}{B_3} \sim \frac{v_F}{9|\boldsymbol{b}|T}$$

$$J_i = \sigma_{ij}E_j + \mathcal{L}^{12}_{ir}(-\nabla_r T) = 0$$

$$E_j = \rho_{ji}\mathcal{L}^{12}_{ir}(-\nabla_r T)$$

$$\mathcal{L}^{12}_{21} = \frac{e^2 v_F B_3}{18\pi^2\hbar T}$$

Estimations for an undoped Weyl semimetal ($v_F \sim 10^5$ m/s, $T \sim 10$ K, $|2\boldsymbol{b}| \sim 0.3$ Å^{-1})

$$S_{11}/T \sim 0.6\ \mu\mathrm{V/T\,K^{-2}}$$

Accessible experimentally!

works via the anomalous Hall current

$$\boldsymbol{J} = \frac{e^2}{2\pi^2\hbar}\boldsymbol{b}\times\boldsymbol{E}.$$

A. Cortijo, M. A. H. Vozmediano, M.Ch., PRL 120, 206601 (2018)

Remarks on the Scale Magnetic Effect (SME)

- Appears due to conformal anomaly $\langle T^{\mu}_{\ \mu} \rangle = \frac{\beta(e)}{2e} F_{\mu\nu} F^{\mu\nu}$
- Bulk phenomenon, works at zero chemical potential
- Leads to the Nernst effect in Dirac/Weyl semimetals

$$\boldsymbol{J} = \frac{e^2 v_F}{18\pi^2 T \hbar} \boldsymbol{B} \times \boldsymbol{\nabla} T$$

- Is not related to axial or axial-gravitational anomalies
- Strength is given by the beta function $\beta(e) = \frac{de(\mu)}{d \ln \mu}$
- Universal: works both in fermionic and bosonic systems

Purely gravitational part of conformal anomaly

Conformal anomaly in curved space-time in electromagnetic field background

anomalous action

electromagnetic part

gravitational part

$$\langle T^{\mu}_{\ \mu} \rangle \equiv -\frac{2}{\sqrt{-g}} g_{\mu\nu} \frac{\delta S_{\rm anom}}{\delta g_{\mu\nu}}$$

$$= -\frac{1}{4}\left[\boxed{bC^2 + b'\left(E - \frac{2}{3}\Box R\right)} + \boxed{cF_{\mu\nu}F^{\mu\nu}}\right]$$

pure gravity

couplings

$$b = \frac{1}{320\pi^2}$$

$$b' = -\frac{11}{5670\pi^2}$$

$$c = -\frac{e^2}{24\pi^2}$$

comes from beta function

$$\beta(e) = \frac{de(\mu)}{d\ln\mu}$$

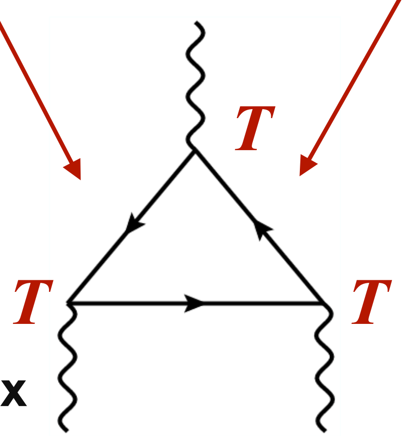
Anomalous action:

$$S_{\rm anom}[g, A] = \frac{1}{8}\int d^4x\sqrt{-g(x)}\int d^4y\sqrt{-g(y)}$$

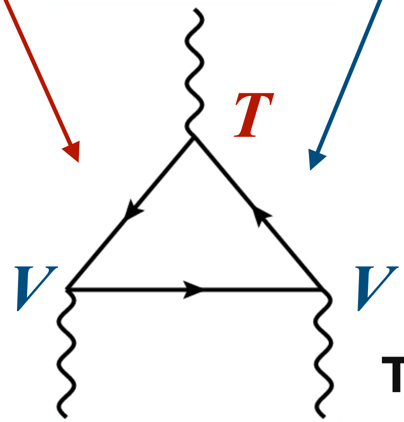
$$\cdot \boxed{H(x)\Delta_4^{-1}(x,y)\left[2bC^2(y) + b'H(y)\right.} + \boxed{\left.2cF_{\mu\nu}(y)F^{\mu\nu}(y)\right]}$$

gravity

electromagnetism



TTT vertex



TVV vertex

Anomalous TTT vertex

A temperature gradient that drives a system out of equilibrium may be mimicked by a gravitational potential (Luttinger):

$$\frac{1}{T}\boldsymbol{\nabla}T = -\frac{1}{c^2}\boldsymbol{\nabla}\Phi \qquad g_{00} = 1 + \frac{2\Phi}{c^2} \qquad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

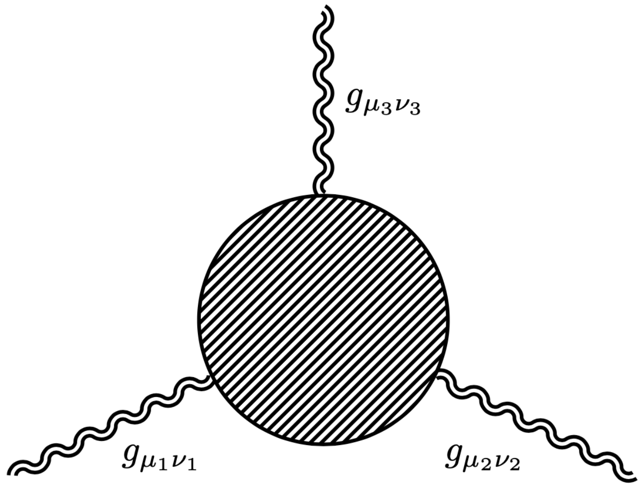
Expectation value of the energy-momentum tensor:

$$\langle T^{\mu_1\nu_1}(x_1)\rangle = \frac{1}{8}\int dx_2\, dx_3\, \underline{\langle T^{\mu_1\nu_1}(x_1)T^{\mu_2\nu_2}(x_2)T^{\mu_3\nu_3}(x_3)\rangle}\; h_{\mu_2\nu_2}(x_2)\, h_{\mu_3\nu_3}(x_3)$$

Inspired by Tolman–Ehrenfest effect:

$$T(x) = T_0\sqrt{g_{00}(x)}$$

$$\Phi(\boldsymbol{x}) \equiv \frac{h_{00}(\boldsymbol{x})}{2} = -\frac{1}{2}\left(\frac{T^2(\boldsymbol{x})}{T_0^2} - 1\right)$$



TTT vertex

Contribution of the anomalous TTT correlator to the energy-momentum tensor.

C. Corianò, M. M. Maglio, M.Ch., Phys.Lett.B 802 (2020) 135236

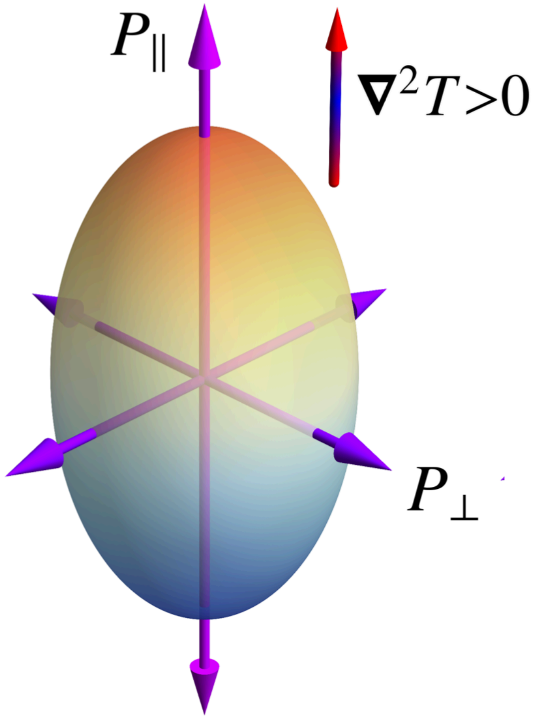
Anomalous TTT vertex

The expectation value of the energy-momentum tensor

$$\langle T^{00}\rangle_{TTT} = \frac{4b}{9}\left[3\left(\partial_3^2\,\Phi\right)^2 + 4\left(\partial_3\Phi\right)\left(\partial_3^3\Phi\right) + 2\Phi\partial_3^4\,\Phi\right]$$

$$\langle T^{11}\rangle_{TTT} = \langle T^{22}\rangle_{TTT} = \frac{4b}{9}\left[2\left(\partial_3\Phi\right)\left(\partial_3^3\Phi\right) + \Phi\partial_3^4\Phi\right].$$

other components are zero



Pressure asymmetry:

$$P_\parallel = \langle T^{33}\rangle$$

$$\delta P = P_\parallel - P_\perp$$

$$P_\perp = \frac{1}{2}\left(\langle T^{11}\rangle + \langle T^{22}\rangle\right)$$

Leading contribution:

comes from the gravitational part of the conformal anomaly

$$\delta P = \frac{16\boxed{b}}{3}\hbar c\left(\frac{\nabla T}{T}\right)^2\left(\frac{\nabla^2 T}{T}\right) \equiv \frac{\hbar c}{60\pi^2}\left(\frac{\nabla T}{T}\right)^2\left(\frac{\nabla^2 T}{T}\right)$$

Relative pressure asymmetry:

$$\frac{\delta P}{P_{\rm th}} = \frac{3}{7}\left(\frac{\nabla T}{\pi T^2}\right)^2\left(\frac{\nabla^2 T}{\pi^2 T^3}\right)$$

very small effect

$\delta P/P_{\rm th} \sim 10^{-7}$

Dirac/Weyl semimetals & quark–gluon plasma

Conformal anomaly and transport effects at the edge

What is about the boundaries? Take massless QED (or any theory …)

beta function

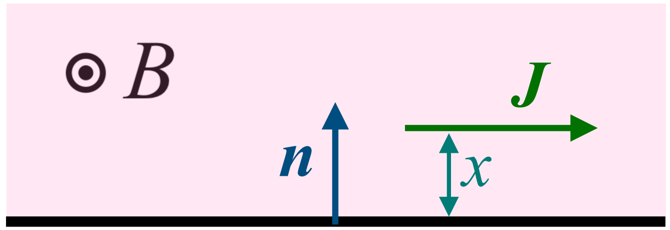
a normal vector to the boundary

electric current

$$J^\mu = -\frac{2c\beta_e}{e\hbar}\frac{F^{\mu\nu}n_\nu}{x}$$

spatial distance to the reflective boundary

In the magnetic-field background:



Current generation at the boundary: D.M. McAvity, H. Osborn, Class. Quantum Gravity 8, 603 (1991).
Relation to the scale anomaly: C.-S. Chu and R.-X. Miao, JHEP 07, 005 (2018), PRL 121, 251602 (2018);
Numerical evidence: V.A.Goy, A.V.Molochkov, M.Ch. PLB 789 (2019) 556

Scale Magnetic Effect at the Edge (SMEE):

Electric current along the edge due to tangential magnetic field

$$\boldsymbol{j}(\boldsymbol{x}) = -f_{\boldsymbol{n}}(\boldsymbol{x})\boldsymbol{n}\times\boldsymbol{B} \qquad f_{\boldsymbol{n}}(\boldsymbol{x}) = \frac{2\beta(e)}{e}\frac{1}{|\boldsymbol{n}\cdot\boldsymbol{x}|}$$

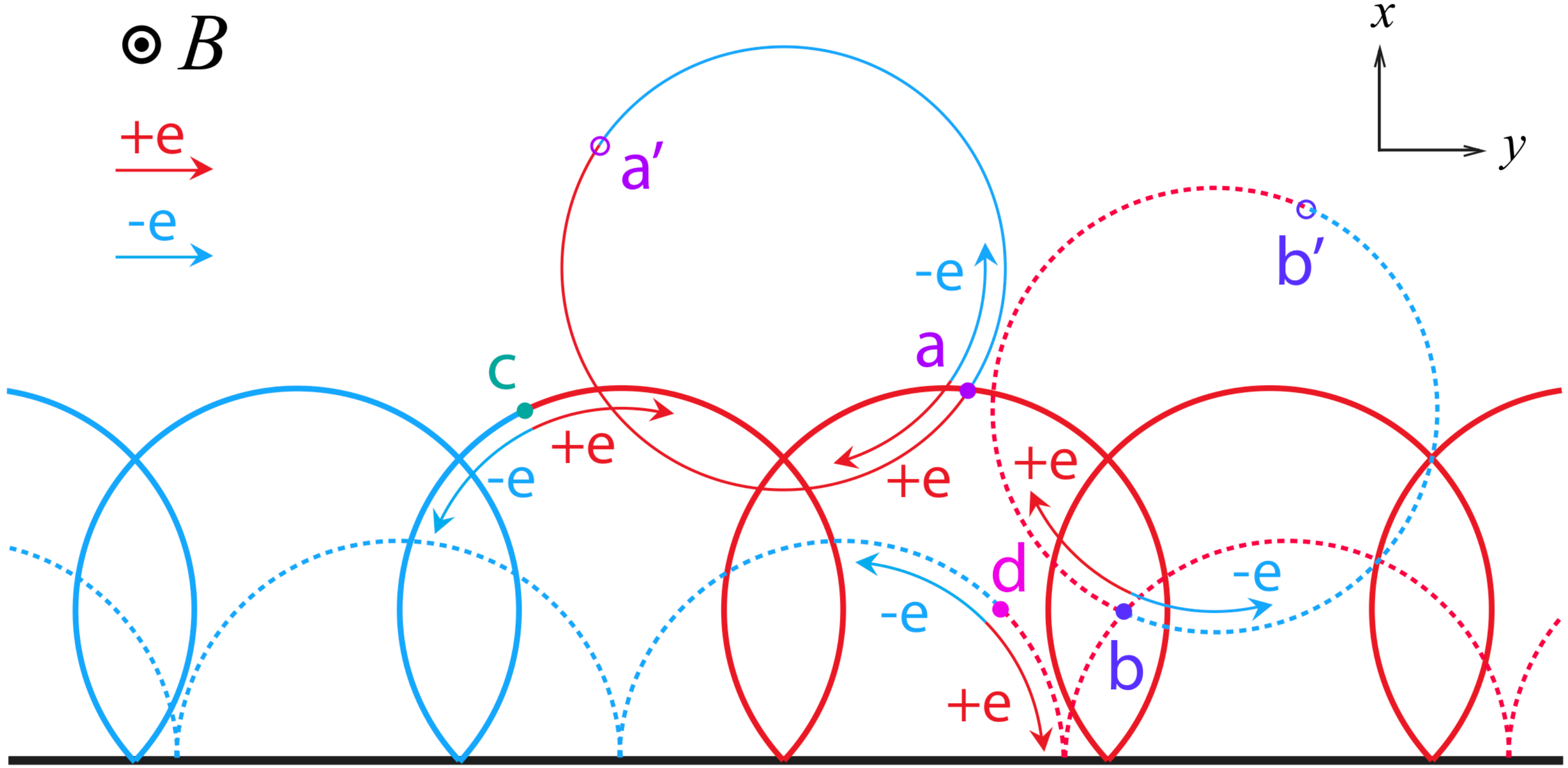
— Effect due to conformal anomaly
— No topology (Berry, Chern, etc)
— No matter at all (= quantum vacuum)

diverges at the boundary!

Scale Magnetic Effect at the Edge (SMEE)

A physical picture

Ingredients: vacuum, edge and magnetic field



Skipping orbits (like in the Hall effect, but now in the vacuum)

Absent: No Fermi surface, no temperature.

SMEE: numerical check

Generates the current at the boundary?

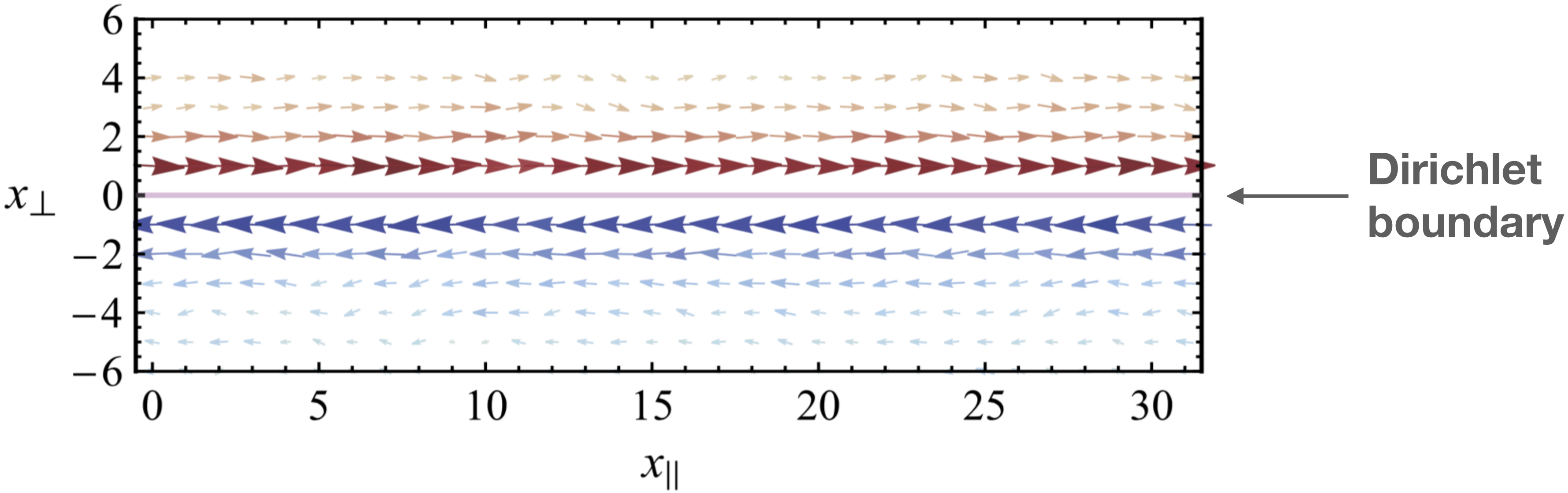
Scalar electrodynamics at a conformal point:

$$\mathcal{L}_{\mathrm{sQED}} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \left[\left(\partial_\mu - ieA_\mu\right)\phi\right]^* \left(\partial^\mu - ieA^\mu\right)\phi$$

Massless one-component electrically-charged scalar field

Numerical Monte-Carlo simulations

We see the generated electric current!

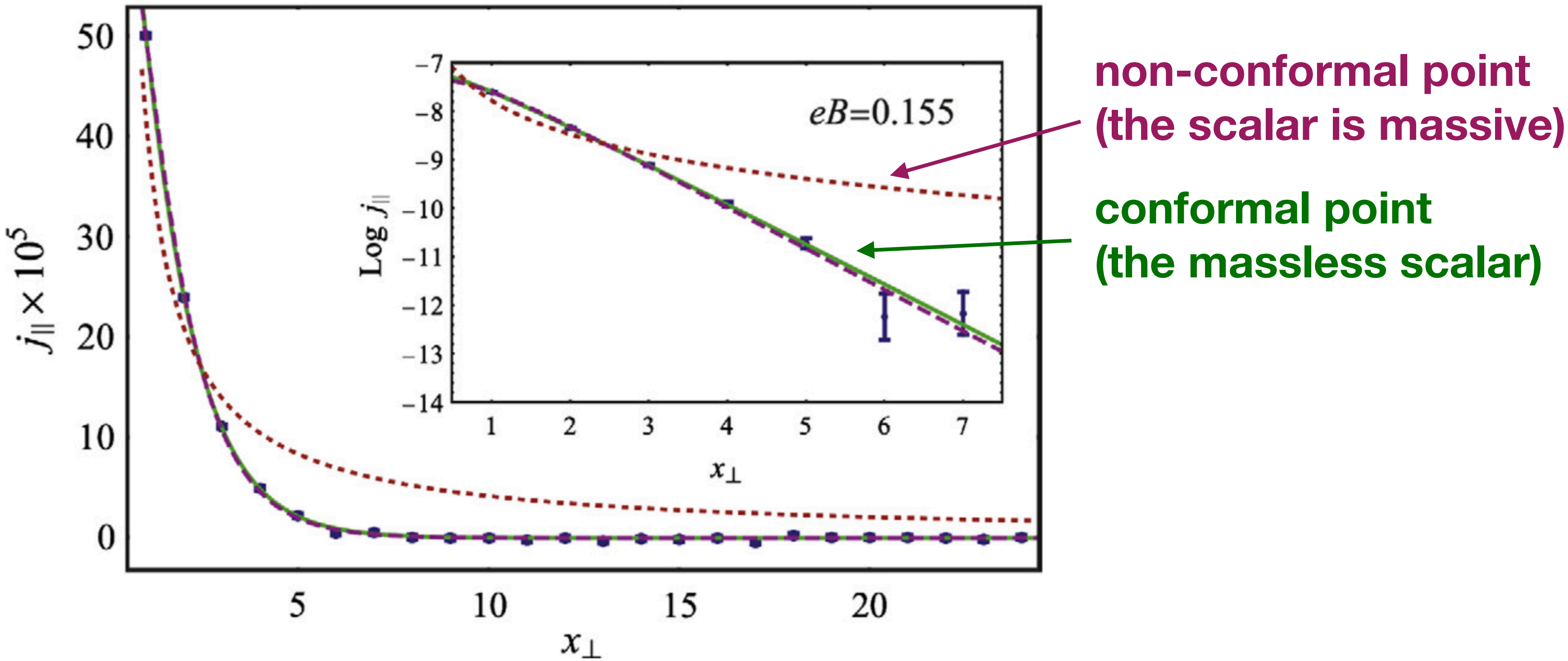


V.A. Goy, A.V. Molochkov, M.Ch., Phys. Lett. B 789, 556 (2019)

SMEE: numerical check

Diverges at the boundary?

We see the *1/x* divergence of the current at the boundary



We see the correct coefficient and recover the beta function!

$$\beta^{\text{1-loop}}_{\text{sQED}} = \frac{e^3}{48\pi^2}$$

(Notice that the beta function of the scalar QED is four times smaller than the beta function in the usual QED)

V.A. Goy, A.V. Molochkov, M.Ch., Phys. Lett. B 789, 556 (2019)

Scale magnetic effect at the edge and superconductivity

Scalar electrodynamics at a conformal point (massless scalar):

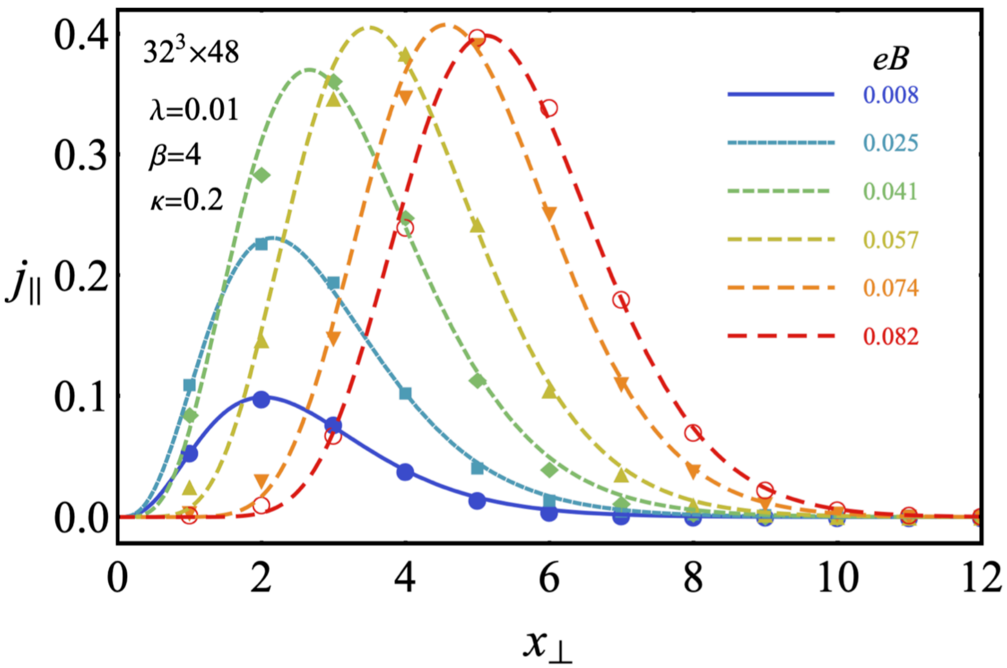
$$\mathcal{L}_{\rm sQED} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \left[\left(\partial_\mu - ieA_\mu \right) \phi \right]^* \left(\partial^\mu - ieA^\mu \right) \phi$$

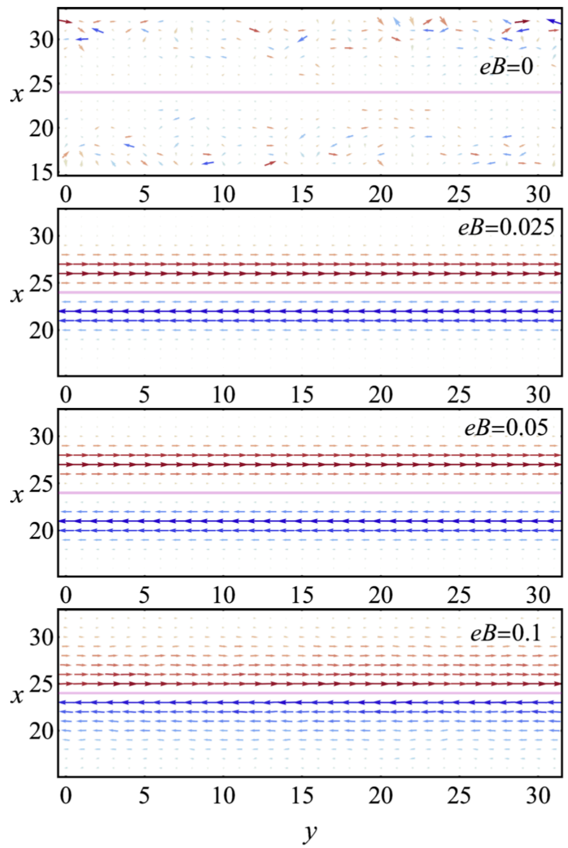
Charged fields may condense!

potential

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + (D_\mu \varphi)^* D^\mu \varphi - V(\varphi)$$

What happens to the SEEE current in the superconducting phase?





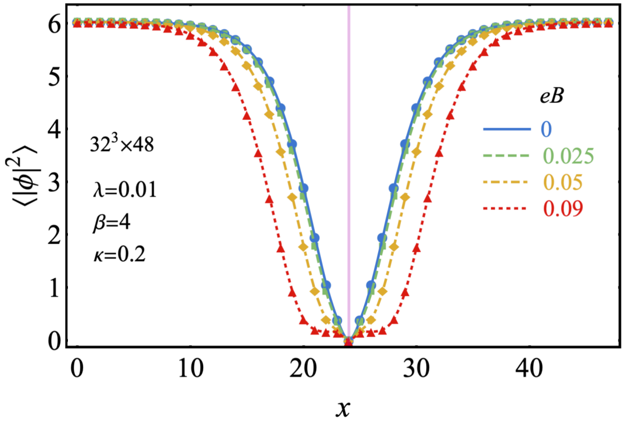
V.A. Goy, A.V. Molochkov, M.Ch., in preparation

Scale magnetic effect at the edge and superconductivity

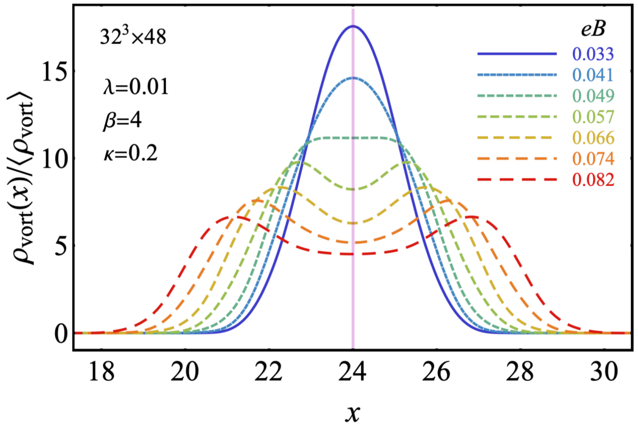
What happens to the SEEE boundary current in superconducting phase?

It becomes the Meissner current!

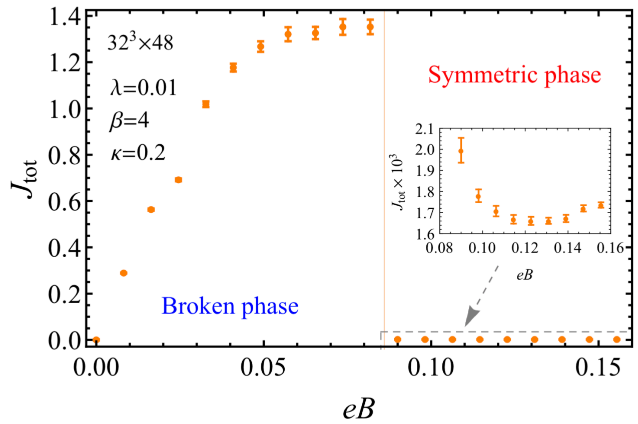
Superconducting condensate

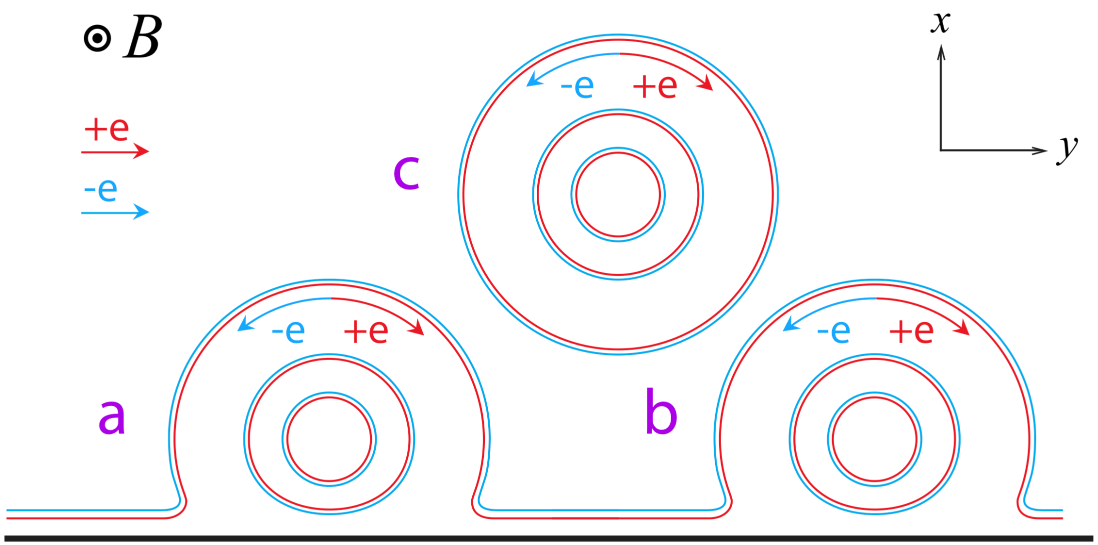


Vortex density



Total (integrated) electric current





Meissner: total current is large, but it vanishes at the boundary

SMEE: total current is small, but it diverges at the boundary

Scale electric effect at the edge: conformal screening

Screening of electrostatic field in metals:

$$E(x) \sim E(0) e^{-x/\lambda}$$

Fermi momentum

$$p_F = (2\pi^2 n)^{1/3} \hbar$$

Screening lengths:

Density of carriers n

$$\lambda_{\mathrm{D}} = \sqrt{\frac{\varepsilon_0 k_B T}{n e^2}}, \qquad \lambda_{\mathrm{FT}} = \sqrt{\frac{\varepsilon_0 \pi^2 \hbar^3}{m e^2 p_F}}$$

Debye **Fermi-Thomas**

What if the medium is totally conformal and possess no dimensionful parameters?

For example, take a Dirac semimetal at particle-hole symmetric point.

– We have the mobile carriers (massless fermionic quasiparticles)

– Classically, there is no dimensionful scale.

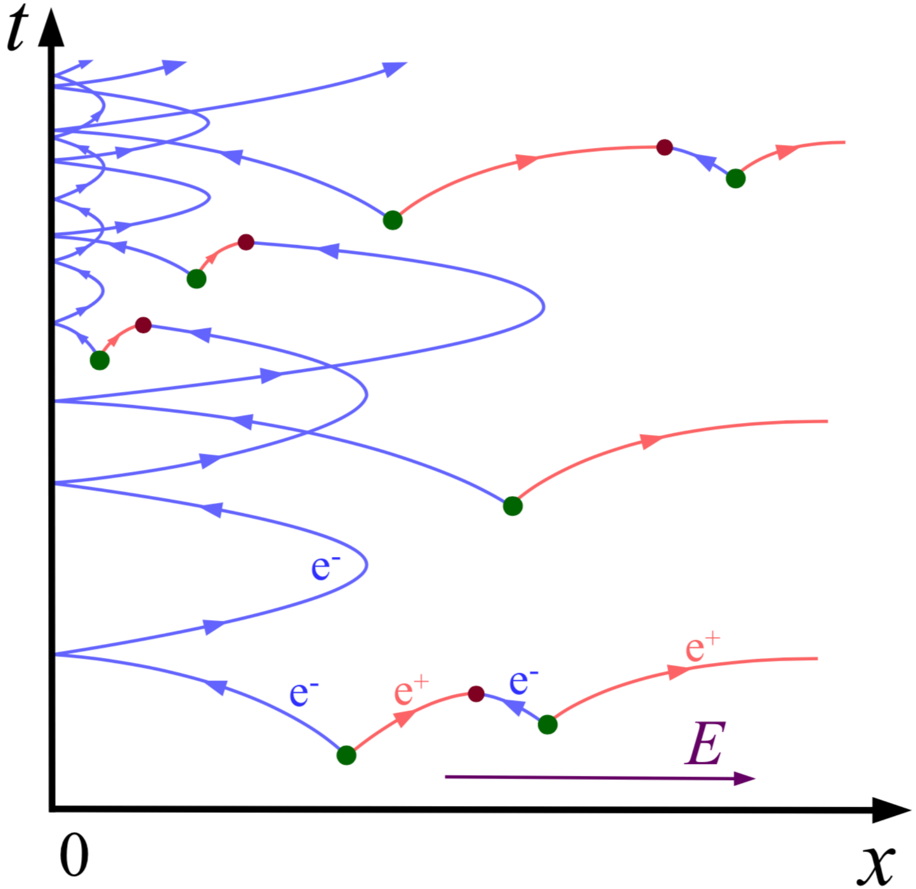
No quantity to construct the screening length from!

– No classical scale → no screening?

Scale electric effect at the edge

$$J^\mu = -\frac{2c\beta_e}{e\hbar}\frac{F^{\mu\nu}n_\nu}{x} \longrightarrow \rho = -\frac{2\beta_e}{e\hbar}\frac{\boldsymbol{nE}}{x}$$

the density of the electric charge accumulated at the boundary



Physics: the screening is due to the Schwinger effect (skipping orbits in time)

Works efficiently due to the absence of a mass gap

Generated by the conformal anomaly!
(proportional to the beta function)

Mechanism in semimetals: creation of electron-hole pairs in the presence of a uniform electric field (the Zener effect)

Scale electric effect at the edge of a semimetal

Consider QED:

$$\mathcal{L} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \sum_{a=1}^{N_f} \bar{\psi}_a i \gamma^\mu D_\mu \psi_a$$

Charge density due to conformal anomaly: $\rho = -\frac{2\beta_e}{e\hbar} \frac{\boldsymbol{n}\boldsymbol{E}}{x}$

Solve the Maxwell equation $\partial_x E_x(x) = \frac{1}{\varepsilon_0} \rho(x)$

At the boundary the conformal screening is polynomial:

$E_x(x) = \frac{C}{x^\nu}$	$\rho(x) = -\frac{C \varepsilon_0 \nu}{x^{1+\nu}}$	$\phi(x) = \phi_0 - \frac{C x^{1-\nu}}{1-\nu}$
Electric field	**Charge density**	**Electrostatic potential**

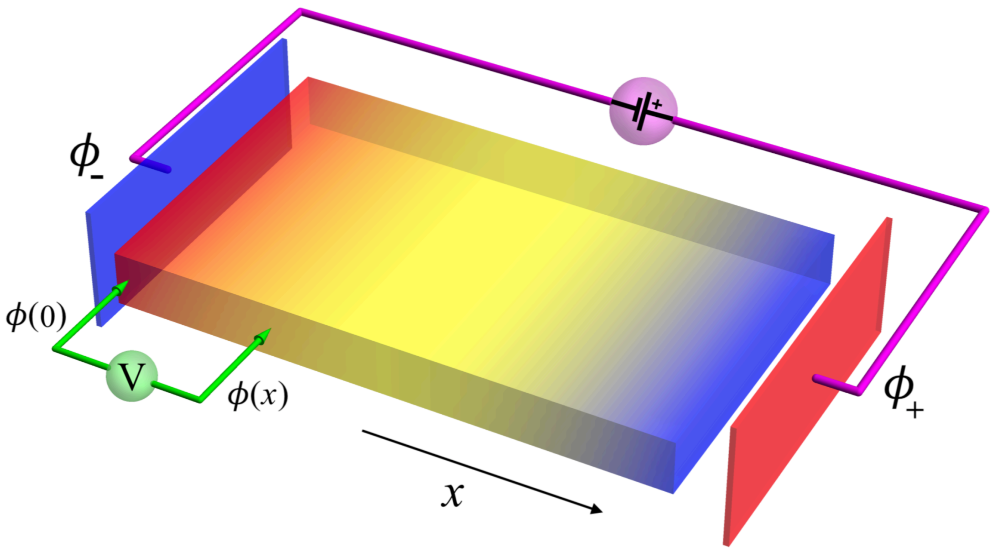
Conformal screening exponent: $\nu = \frac{2\beta_e}{e c \hbar \varepsilon_0}$

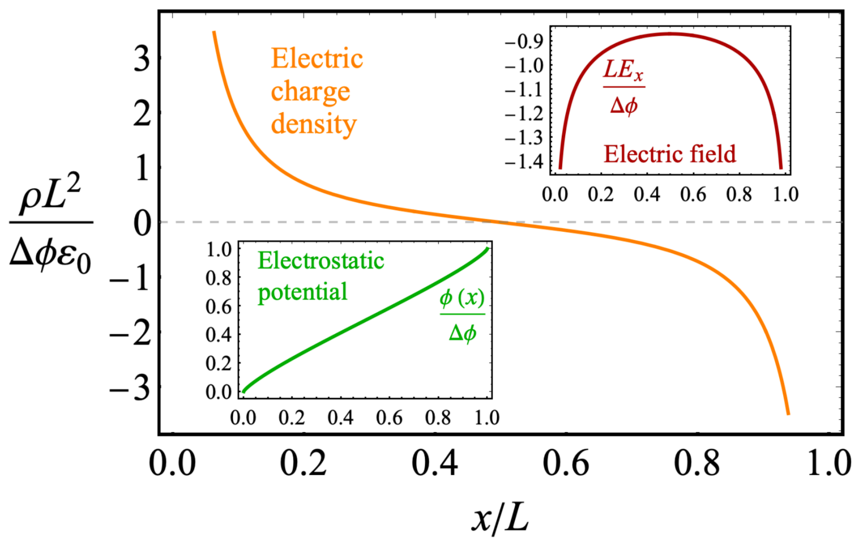
Scale electric effect at the edge

Conformal exponent in a Dirac semimetal: $\nu = \dfrac{e^2}{6\pi^2 \hbar v_F \varepsilon \varepsilon_0} = \dfrac{2\beta_e}{e c \hbar \varepsilon_0}$

Particle density in a finite sample with two boundaries:

$$\rho(x) = \frac{\Delta\phi}{L^2}\varepsilon_0 \nu h(\nu) \left(1 - \frac{2x}{L}\right) \left[\frac{x}{L}\left(1 - \frac{x}{L}\right)\right]^{-1-\nu}$$





Direct measurement of the beta function. Indirect evidence of the Schwinger effect.

M. A. H. Vozmediano, M.Ch., Phys. Rev. Research 1, 032002(R) (2019)

Accessible experimentally

- direct measurement of the beta function associated with the renormalization of the electric charge

(never done in solid state)

- evidence of the elusive Schwinger effect

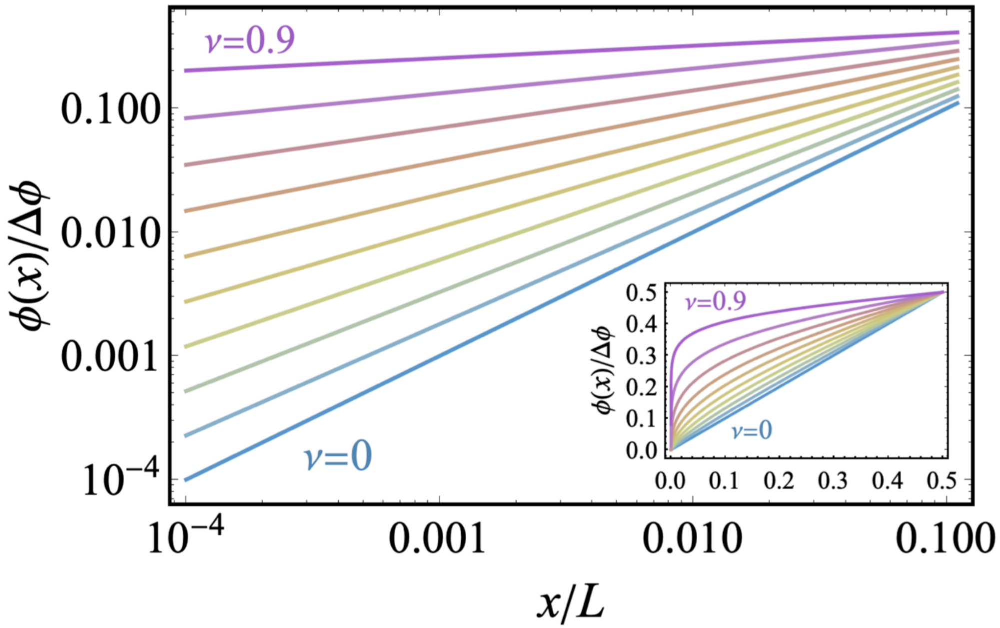
(particle-antiparticle production by electric field)

Conformal exponent in a Dirac semimetal: $\nu = \dfrac{e^2}{6\pi^2 \hbar v_F \varepsilon \varepsilon_0}$

In typical Dirac/Weyl materials $v_F \sim 10^{-3}c$ and $\varepsilon \sim 10$

→ large, experimentally accessible conformal exponent: $\nu \sim 10^{-1}$

Electrostatic screening potential vs. distance from the boundary of a Dirac material (at the Lifshitz point)



Summary

Conformal anomaly leads to a number of new transport effects:
— in the bulk (unbounded systems)
— at reflective boundaries (edges) of bounded systems

Currents are proportional to the beta function.
Accessible experimentally in Dirac and Weyl semimetals.

Scale electric effect:
- — negative conductivity in expanding systems
- — polynomial screening of electrostatic fields
- — particle creation via the Schwinger effect

Scale magnetic effect:
- —thermoelectric transport, Nernst effects
- —edge currents in the absence of matter