Bottomonium suppression in the QGP

From EFTs to non-unitary quantum evolution

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N. Brambilla, M.-A. Escobedo, A. Islam, MS, A. Tiwari, A. Vairo, P. Vander Griend, 2205.10289
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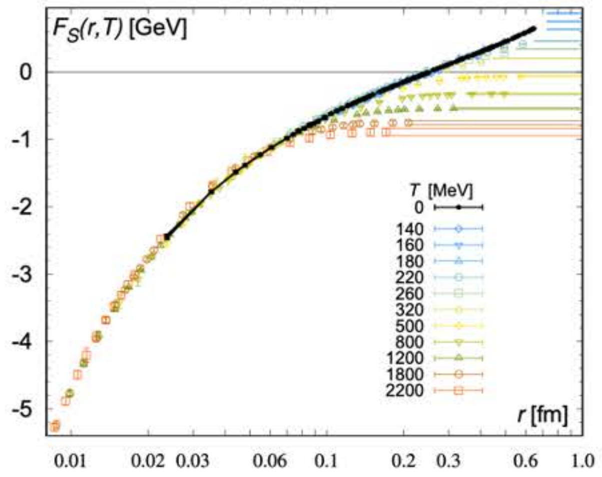
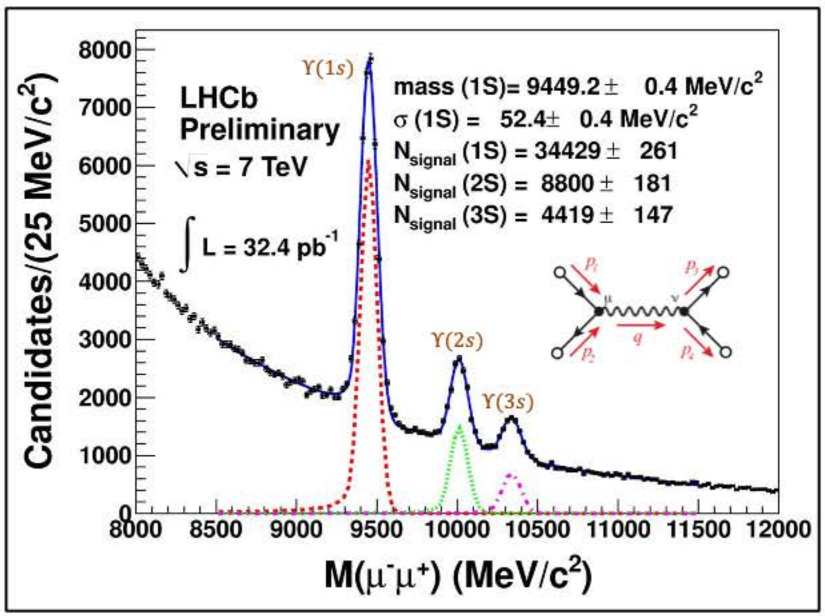
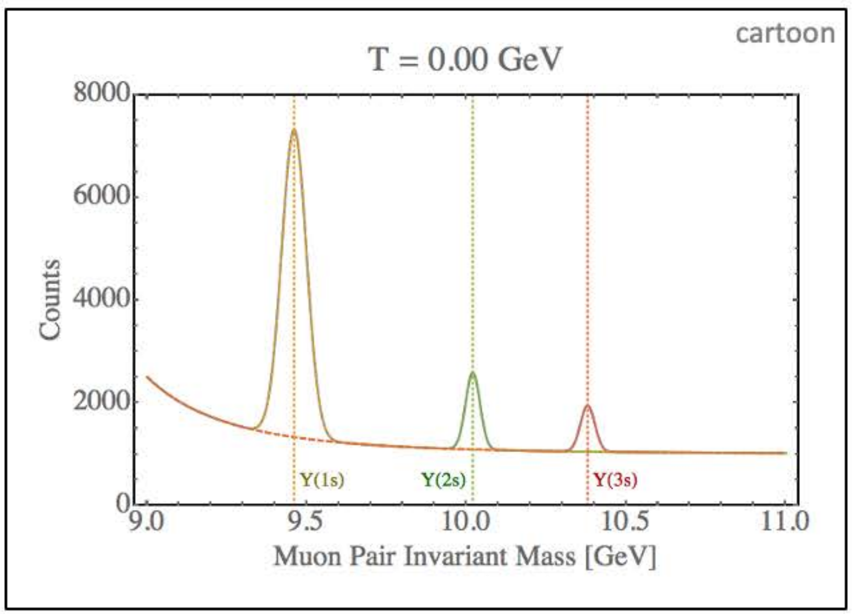
T. Matsui, and H. Satz, Phys. Lett. B178, 416 (1986)
F. Karsch, M. T. Mehr, and H. Satz, Z. Phys. C37, 617 (1988)

- Also, high energy plasma particles which slam into the bound states cause them to have shorter lifetimes → **larger spectral widths**

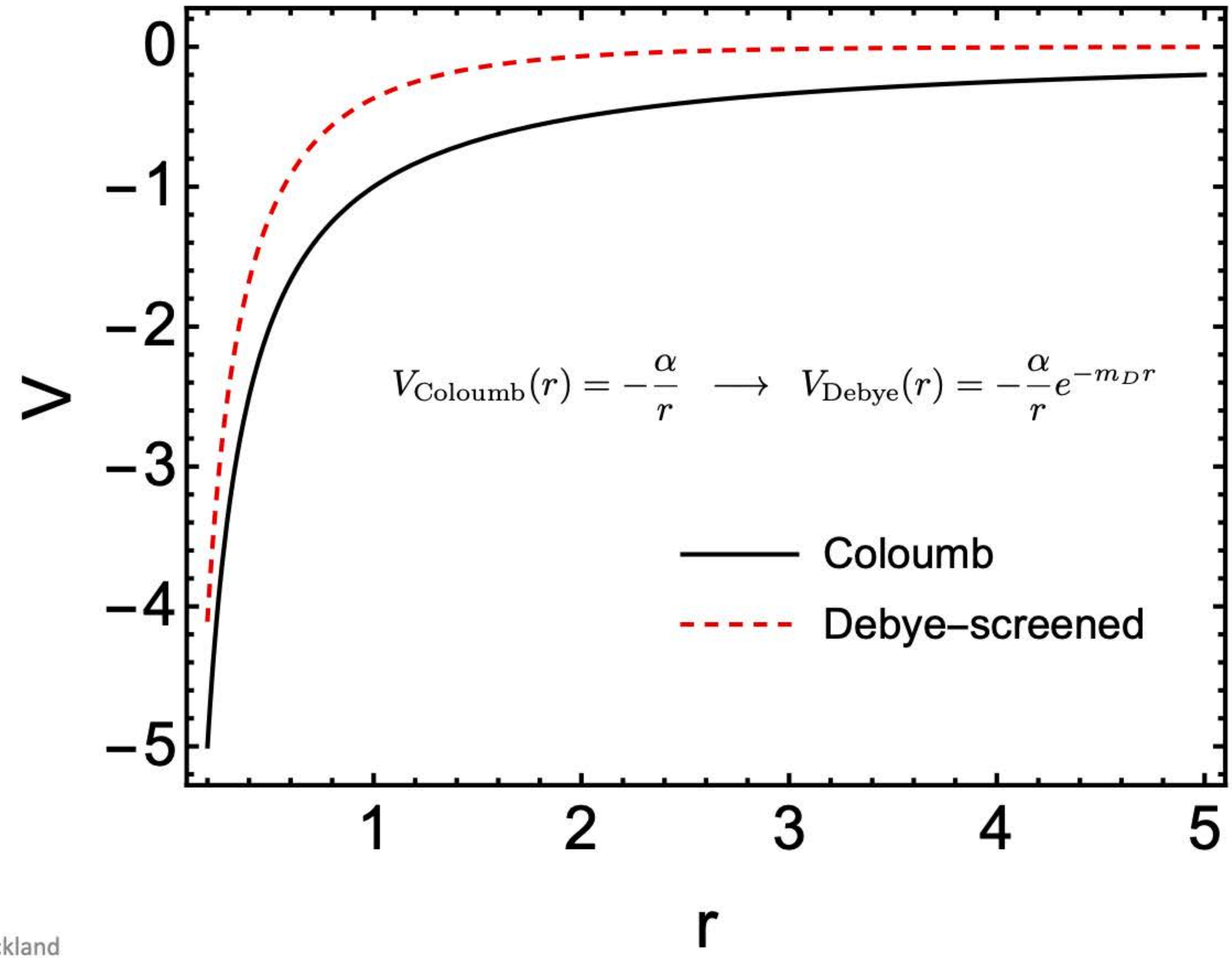
TUMQCD Collaboration, 1804.10600







Debye-screening in a plasma



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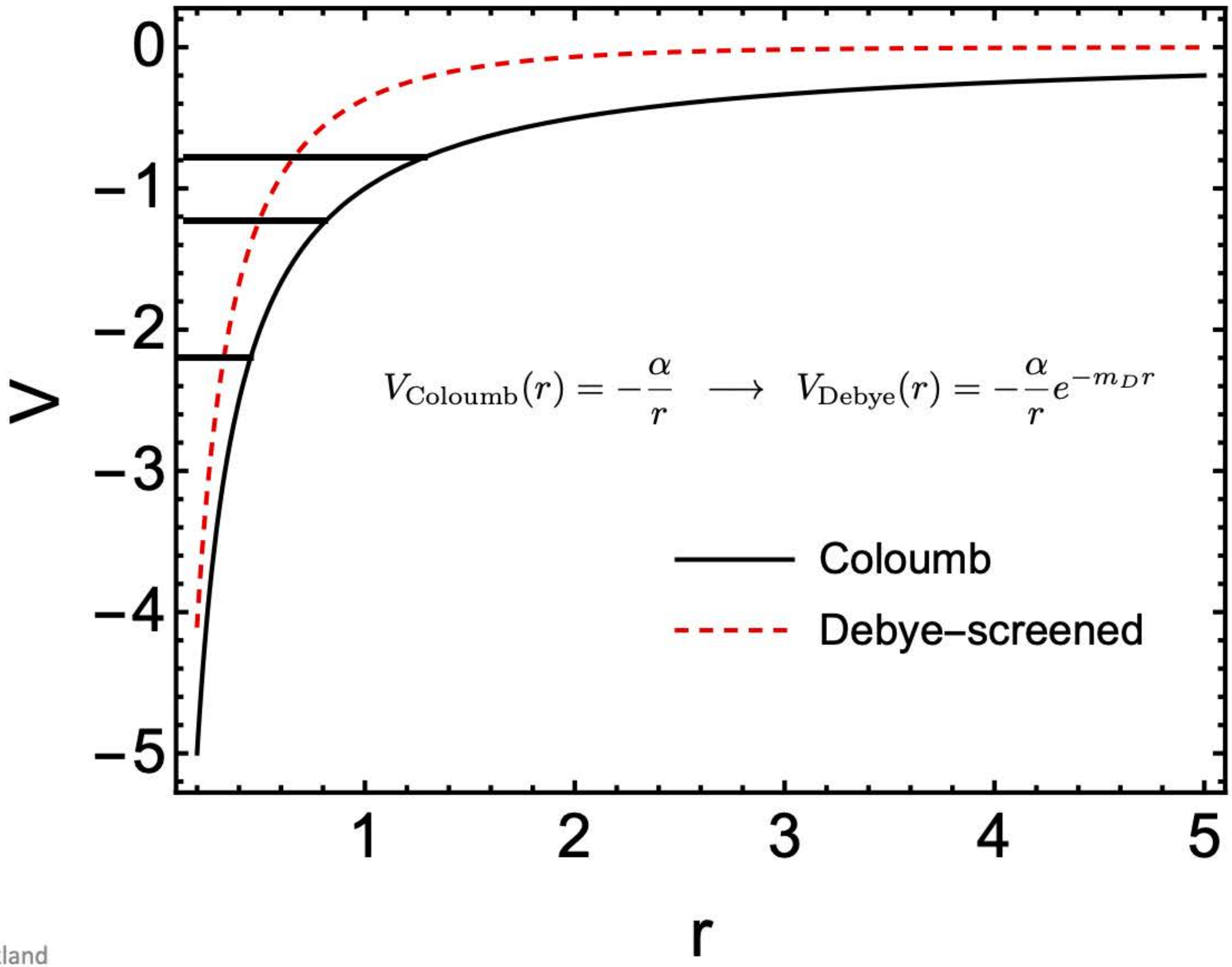

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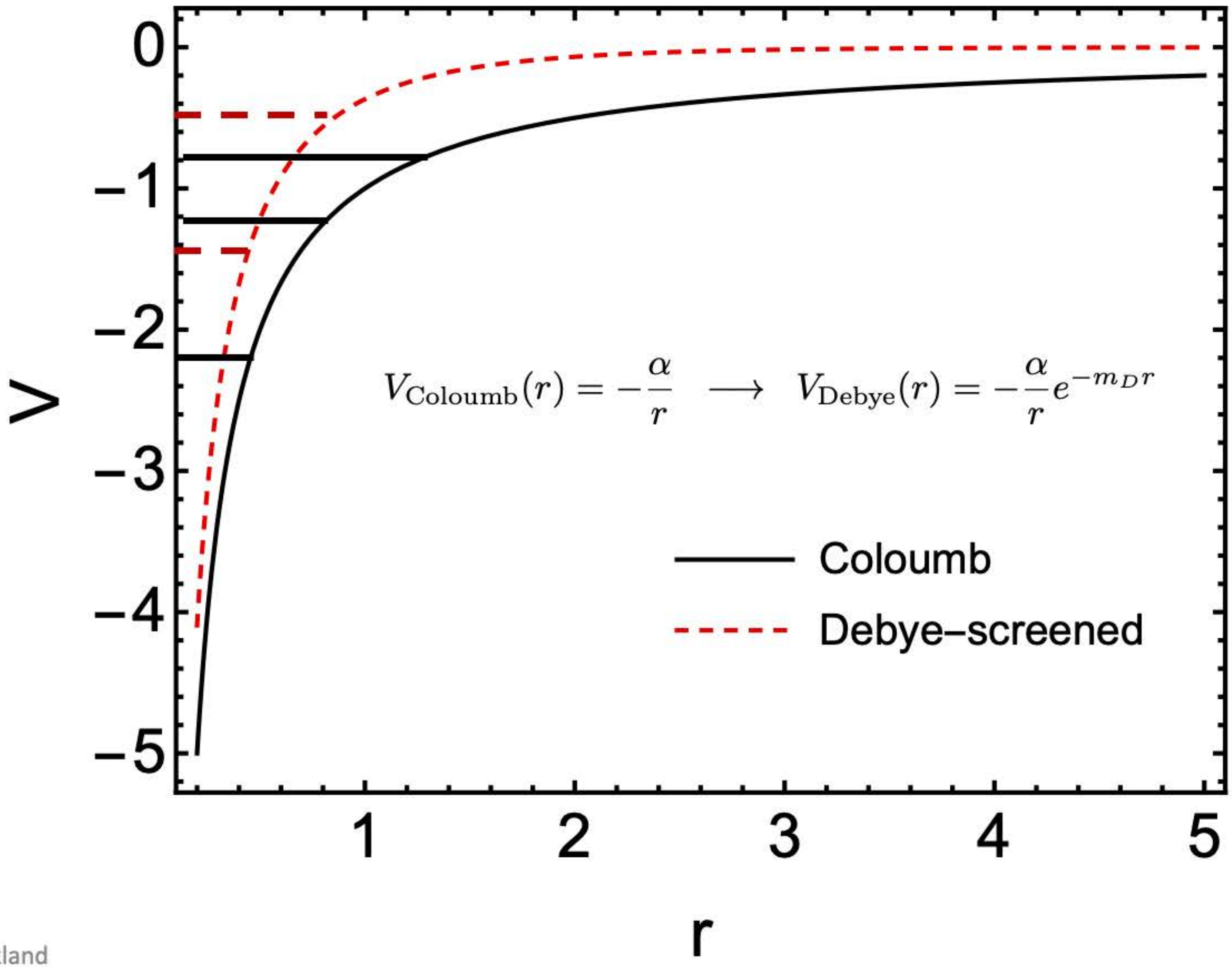

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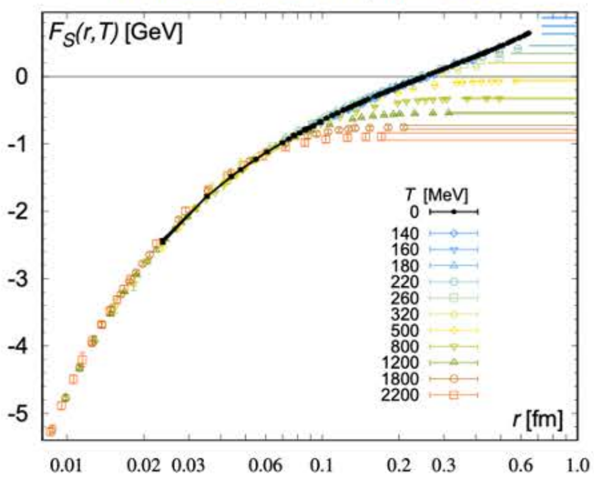

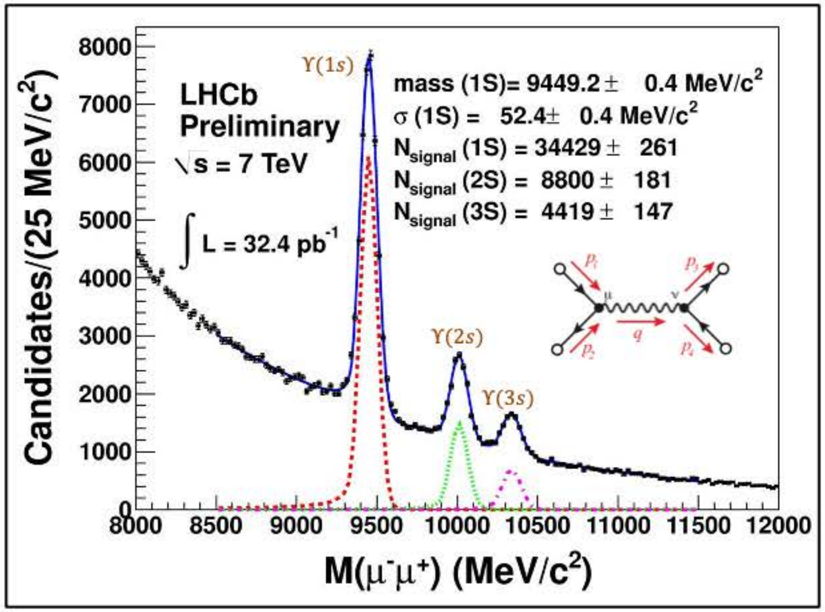

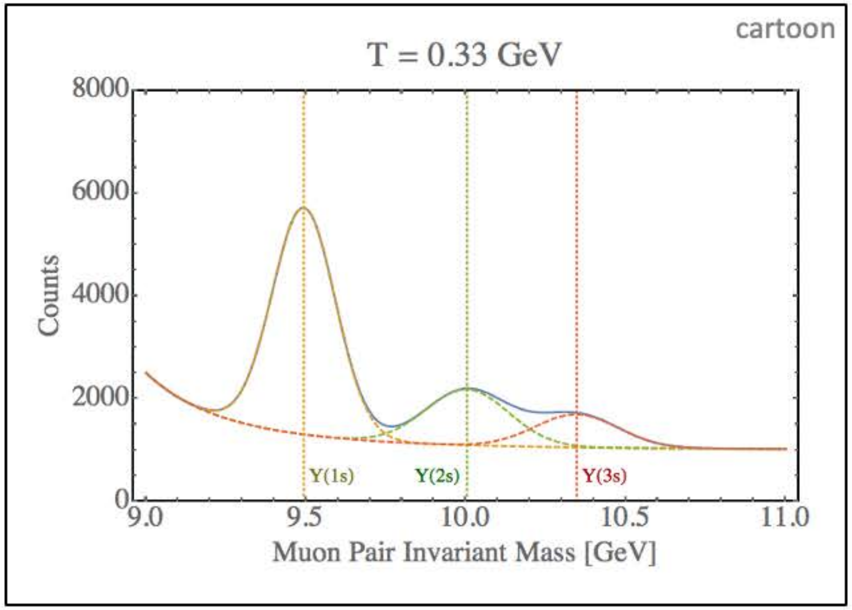

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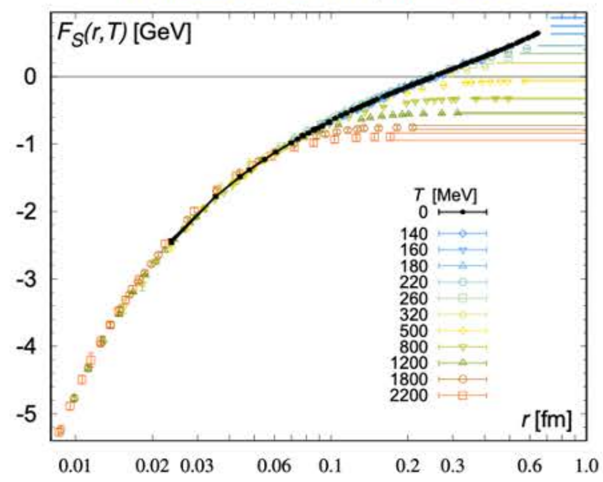

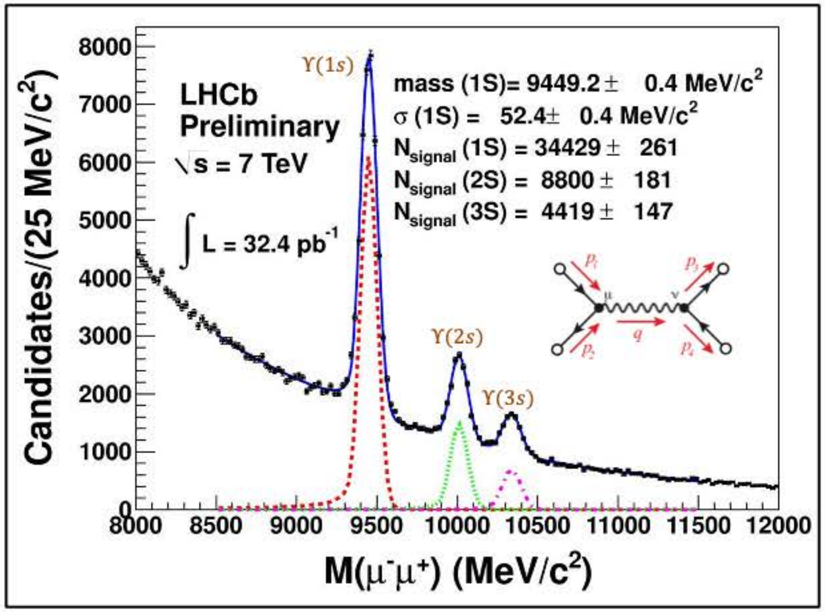

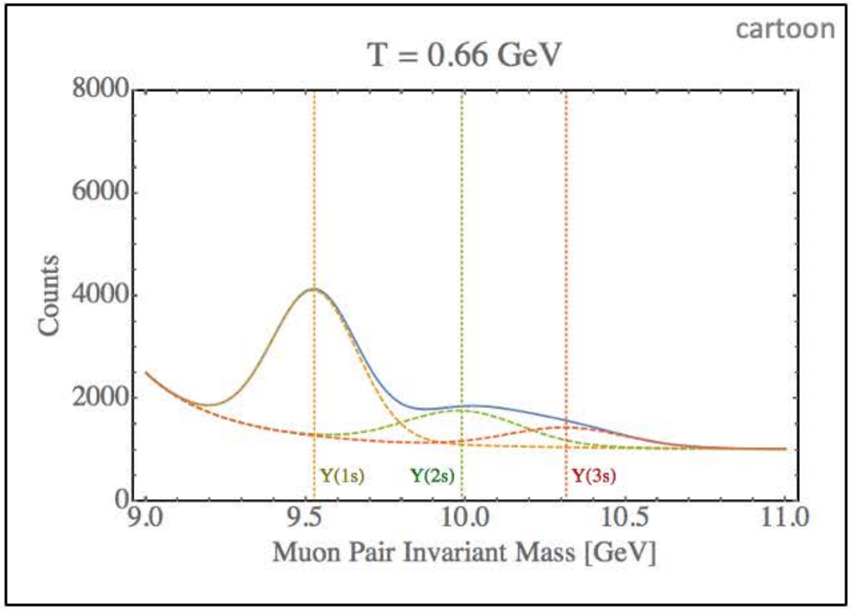

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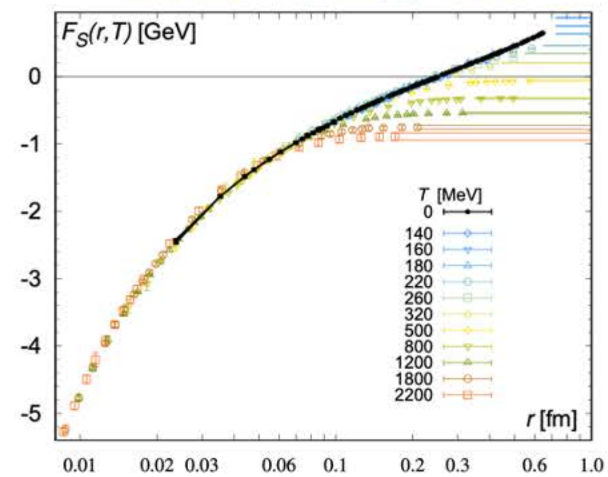
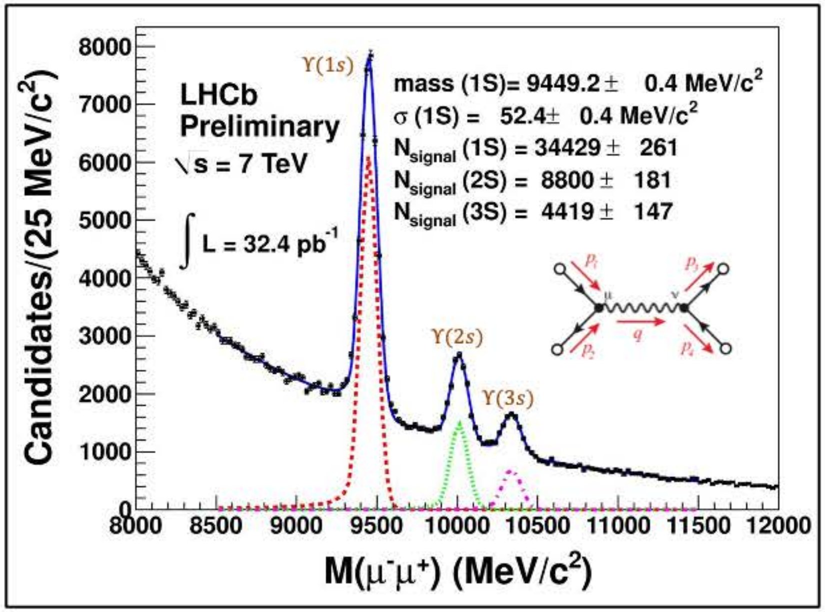
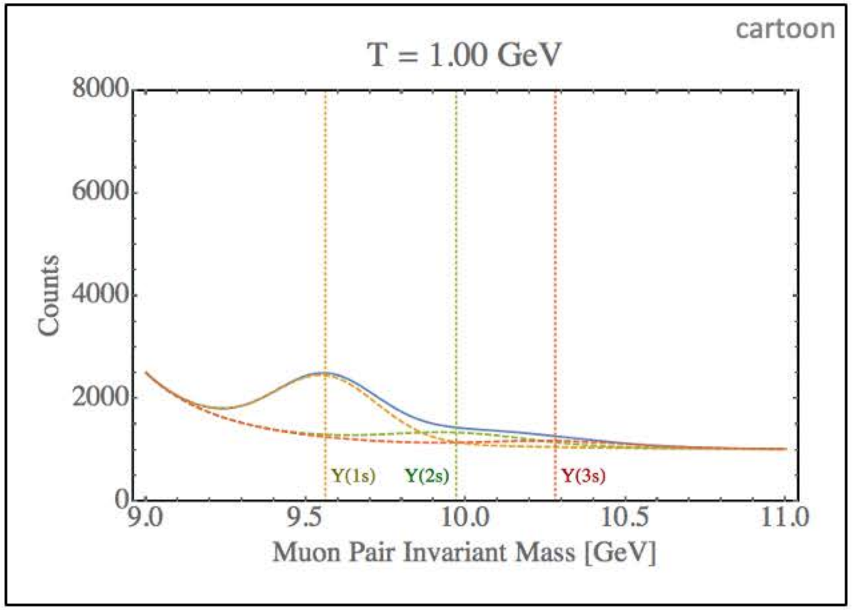


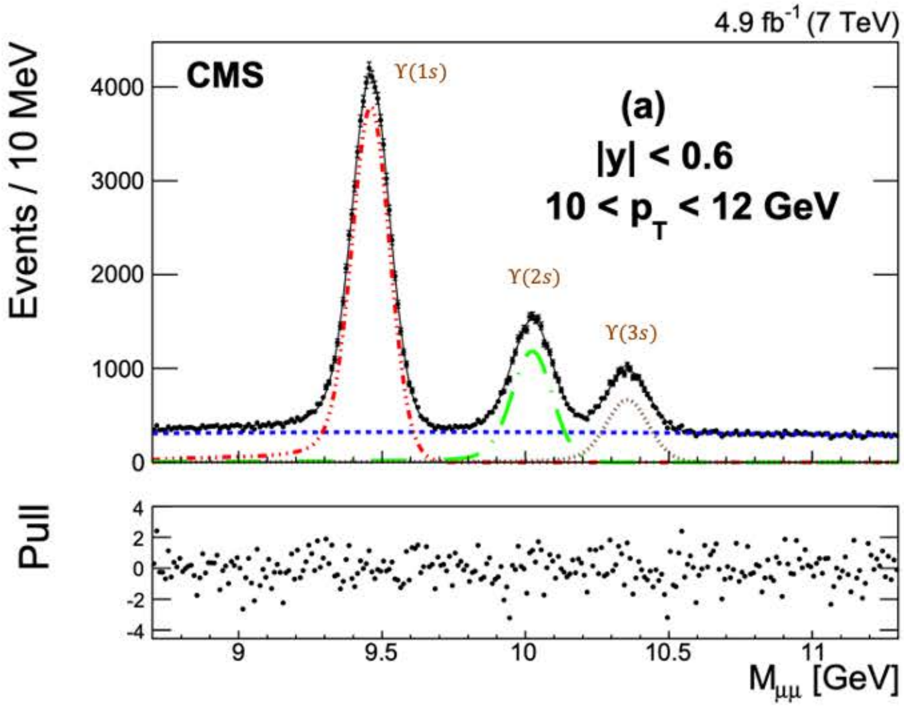




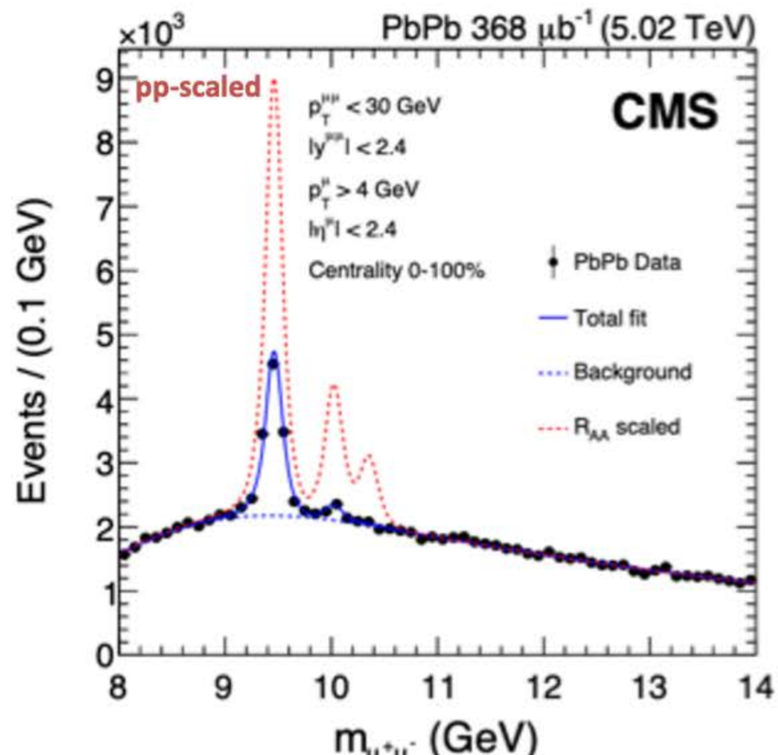
Experimental data – 5.02 TeV Dimuon Spectra

The **CMS, ALICE,** and **ATLAS** experiments have measured bottomonium production in both pp and Pb-Pb collisions at 5.02 TeV. Here I show CMS results.

pp

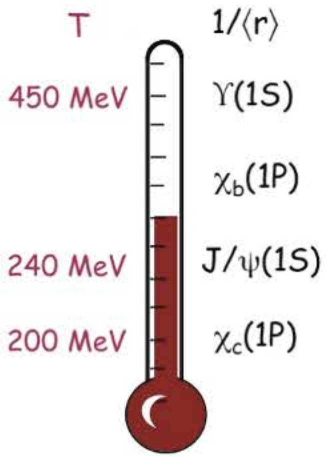


PbPb



Why focus on bottomonia?

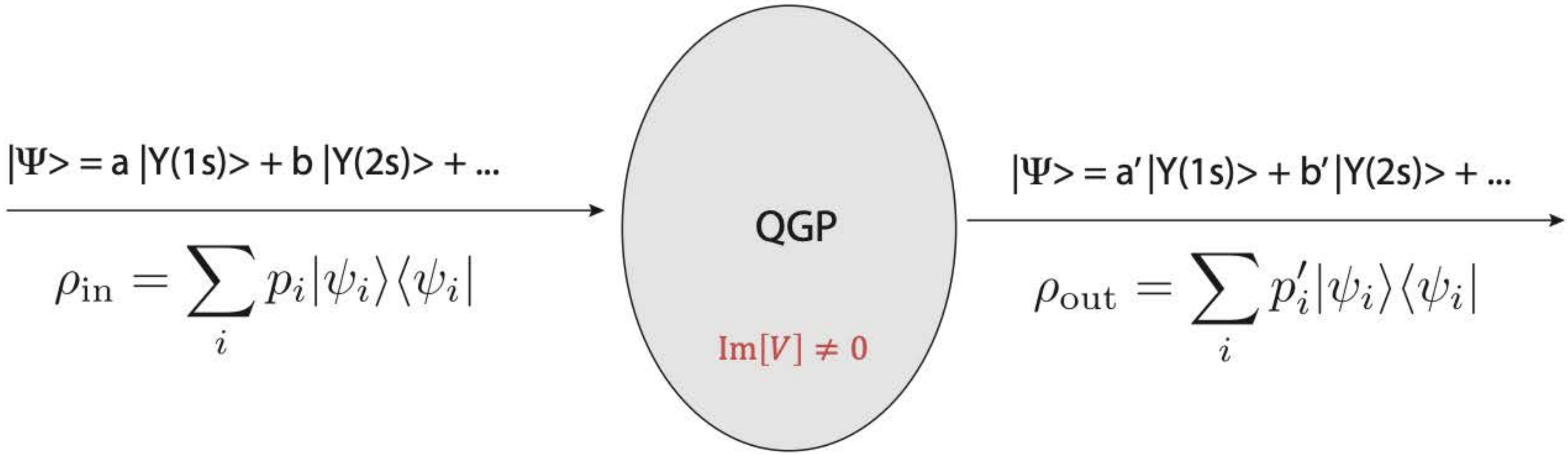
- Can trust heavy quark effective theory more.
- Cold nuclear matter (CNM) effects in AA decrease with increasing quark mass.
- The masses of bottomonia (m ~ 10 GeV) are much higher than the temperature generated in HICs (T < 1 GeV) → bottomonia production dominated by initial hard scatterings.



A. Mocsy, P. Petreczky, and MS, 1302.2180

- Since bottom quarks and anti-quarks are relatively rare in RHIC and LHC HICs, the probability for regeneration of bottomonia through statistical recombination is much smaller than for charm quarks. [see e.g. E. Emerick, X. Zhao, and R. Rapp, arXiv:1111.6537 and others]

Conceptual problem



- Bottomonium states are produced locally (hard processes) at early times in hard collisions ($t < 1$ fm/c).
- They then propagate through the plasma and interact with the medium.
- Bound states can break up and potentially re-form due to in-medium transitions induced by in-medium gluon absorption and emission.

Heuristic understanding – Noisy QM

- Heavy quark bound states have an in-medium potential with both real and imaginary parts. This results in a large in-medium width.
- How can we understand the emergence of the imaginary part in a simple manner in an intuitive manner?
- **Consider a non-relativistic bound state subject to a noisy potential**

$$H(\mathbf{r},t) = -\frac{\nabla_{\mathbf{r}}^{2}}{M} + V(\mathbf{r}) + \Theta(\mathbf{r},t)$$

$$\Theta(\mathbf{r},t) = \theta\left(\mathbf{R}+\frac{\mathbf{r}}{2},t\right) - \theta\left(\mathbf{R}-\frac{\mathbf{r}}{2},t\right)$$

Noise due to environment (assumed here to be color neutral).

- Noise has zero mean, is uncorrelated in time, and has a spatial correlation function $D(\boldsymbol{r})$

$$\langle\theta(\mathbf{x},t)\rangle = 0 \qquad \langle\theta(\mathbf{x},t)\theta(\mathbf{x}',t')\rangle = D(\mathbf{x}-\mathbf{x}')\delta(t-t')$$

Note: The treatment presented here does not include possibility of color-charged noise, more on this coming …

Heuristic understanding – Noisy QM

- Expanding the time evolution operator up to $O(\Delta t^{3/2})$

$$e^{-i\Delta t\, H(\mathbf{r},t)} \simeq 1 - i\Delta t\, H(\mathbf{r},t) - \frac{1}{2}\left\{\Delta t\, H(\mathbf{r},t)\right\}^2 + \ldots$$

$$\approx 1 - i\Delta t\left[H(\mathbf{r},t) - \frac{i}{2}\Delta t\left\{\theta(\mathbf{x},t)^2 + \theta(\mathbf{x}',t)^2 - 2\,\theta(\mathbf{x},t)\theta(\mathbf{x}',t)\right\}\right]$$

- Now construct an effective Hamiltonian that is averaged over the noise

$$\langle \mathrm{H}_{\rm eff}(\mathbf{r},t)\rangle \simeq H(\mathbf{r},t) - \frac{i}{2}\Delta t\left\{\langle\theta(\mathbf{x},t)^2\rangle + \langle\theta(\mathbf{x}',t)^2\rangle - 2\,\langle\theta(\mathbf{x},t)\theta(\mathbf{x}',t)\rangle\right\}$$

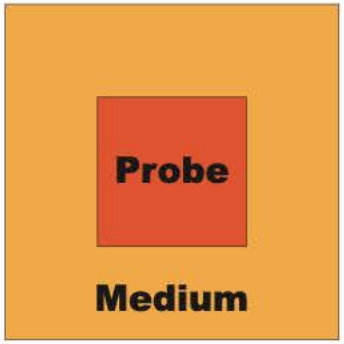
Imaginary part of the potential

$$\longrightarrow \quad \langle \mathrm{H}_{\rm eff}(\mathbf{r},t)\rangle = -\frac{\nabla_{\mathbf{r}}^2}{M} + V(\mathbf{r}) - i\left\{D(\mathbf{0}) - D(\mathbf{r})\right\}$$

$$\Im[V(r)] = D(\mathbf{r}) - D(\mathbf{0})$$

Imaginary part emerges through interference of wave function with itself when summing over environmental noise.

Open quantum system (OQS) approach



Probe = heavy-quarkonium state

Medium = light quarks and gluons that comprise the QGP

- Can treat heavy quarkonium states propagating through QGP using an open quantum system approach

$$H_{\rm tot} = H_{\rm probe} \otimes I_{\rm medium} + I_{\rm probe} \otimes H_{\rm medium} + H_{\rm int}$$

- Total density matrix

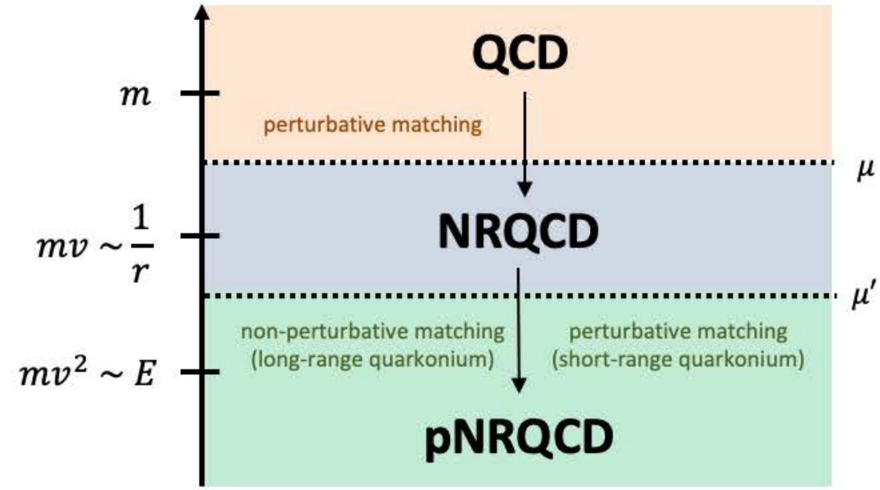
$$\rho_{\rm tot} = \sum_j p_j |\psi_j\rangle\langle\psi_j| \longrightarrow \frac{d}{dt}\rho_{\rm tot} = -i[H_{\rm tot}, \rho_{\rm tot}]$$

- Reduced density matrix (medium DOF integrated out)

$$\rho_{\rm probe} = {\rm Tr}_{\rm medium}[\rho_{\rm tot}] \longrightarrow$$ **"Master equation"**

OQS + pNRQCD → Lindblad equation

- What are the relevant scales?
  - Temperature T
  - Bound state mass $m \gg T$
  - Bound state size $r \sim 1/mv \sim a_0$ (Bohr radius)
  - Debye mass m_D
  - Binding energy $E \sim mv^2$



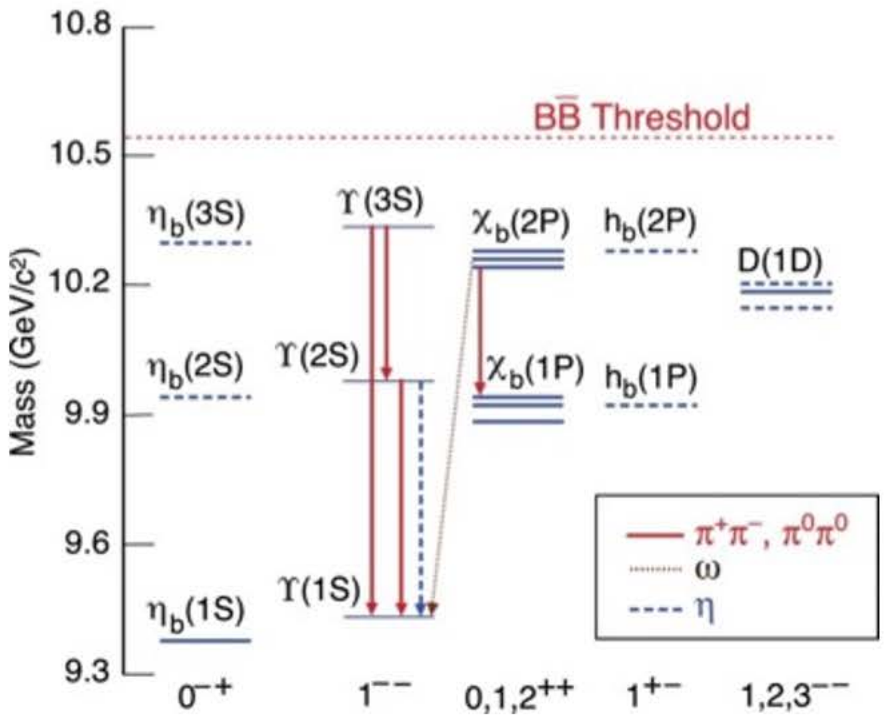
- Separation of time scales
  - Medium relaxation time scale $\langle \hat{O}_{\rm M}(t)\hat{O}_{\rm M}(0)\rangle \sim e^{-t/t_{\rm M}} \rightarrow \frac{1}{T}$
  - Intrinsic probe time scale $t_P \sim \frac{1}{\omega_i - \omega_j} \rightarrow \frac{1}{E}$
  - Probe relaxation time scale $\langle p(t)\rangle \sim e^{-t/t_{\rm rel}} \rightarrow \frac{1}{\text{self-energy}} \sim \frac{1}{\alpha_s a_0^2 \Lambda^3}$ $\quad \Lambda = T, E$

$$\xrightarrow[t_{\rm rel}\,,\, t_{\rm P} \gg t_{\rm M}]{1/r \gg T \sim m_D \gg E} \quad \boxed{\frac{d\rho_{\rm probe}}{dt} = -i[H_{\rm probe}, \rho_{\rm probe}] + \sum_n \left(C_n\, \rho_{\rm probe}\, C_n^\dagger - \frac{1}{2}\{C_n^\dagger C_n, \rho_{\rm probe}\} \right)}$$

N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, 1612.07248, 1711.04515

Bottomonium scales

- The mass scale is perturbative: $m_b \sim 5$ GeV
- The system is non-relativistic ($v \ll 1$), with $v_b \sim 0.1$.
- $\Delta_n E \sim mv^2$ and $\Delta_{\rm fs} E \sim mv^4$



Results of a non-relativistic potential model

State	Name	Exp. [92]	Model	Rel. Err.
1^1S_0	$\eta_b(1S)$	9.398 GeV	9.398 GeV	0.001%
1^3S_1	$\Upsilon(1S)$	9.461 GeV	9.461 GeV	0.004%
1^3P_0	$\chi_{b0}(1P)$	9.859 GeV	9.869 GeV	0.21%
1^3P_1	$\chi_{b1}(1P)$	9.893 GeV		
1^3P_2	$\chi_{b2}(1P)$	9.912 GeV		
1^1P_1	$h_b(1P)$	9.899 GeV		
2^1S_0	$\eta_b(2S)$	9.999 GeV	9.977 GeV	0.22%
2^3S_1	$\Upsilon(2S)$	10.002 GeV	9.999 GeV	0.03%
2^3P_0	$\chi_{b0}(2P)$	10.232 GeV	10.246 GeV	0.05%
2^3P_1	$\chi_{b1}(2P)$	10.255 GeV		
2^3P_2	$\chi_{b2}(2P)$	10.269 GeV		
2^1P_1	$h_b(2P)$	-		
3^1S_0	$\eta_b(3S)$	-	10.344 GeV	-
3^3S_1	$\Upsilon(3S)$	10.355 GeV	10.358 GeV	0.03%

J. Alford and MS, 1309.3003

Non-Relativistic QCD (NRQCD)

Caswell and Lepage (1986), Bodwin, Braaten and Lepage (1994)

$$\mathcal{L}_{NRQCD} = \mathcal{L}_g + \mathcal{L}_q + \mathcal{L}_\psi + \mathcal{L}_\chi + \mathcal{L}_{\psi\chi}$$

$$\mathcal{L}_g = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} + \frac{d_2}{m_Q^2} F^a_{\mu\nu} D^2 F^{\mu\nu a} + d_g^3 \frac{1}{m_Q^2} g f_{abc} F^a_{\mu\nu} F^{\mu b}_\alpha F^{\nu\alpha c}$$

$$\mathcal{L}_\psi = \psi^\dagger \left(iD_0 + c_2 \frac{\mathrm{D}^2}{2m_Q} + c_4 \frac{\mathrm{D}^4}{8m_Q^3} + c_F g \frac{\sigma \mathrm{B}}{2m_Q} + c_D g \frac{\mathrm{DE}-\mathrm{ED}}{8m_Q^2} \right.$$
$$\left. + i c_S g \frac{\sigma(\mathrm{D}\times\mathrm{E}-\mathrm{E}\times\mathrm{D})}{8m_Q^2} \right) \psi$$

$$\mathcal{L}_\chi = c.c \text{ of } \mathcal{L}_\psi$$

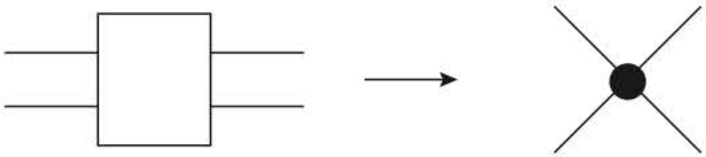
$$\mathcal{L}_{\psi\chi} = \frac{f_1(^1S_0)}{m_Q^2} \psi^\dagger \chi \chi^\dagger \psi + \frac{f_1(^3S_1)}{m_Q^2} \psi^\dagger \sigma \chi \chi^\dagger \sigma \psi + \frac{f_8(^1S_0)}{m_Q^2} \psi^\dagger T^a \chi \chi^\dagger T^a \psi$$
$$+ \frac{f_8(^3S_1)}{m_Q^2} \psi^\dagger T^a \sigma \chi \chi^\dagger T^a \sigma \psi$$

- **Integrating out the scale m can be done perturbatively** and is not affected by the presence of the medium since $m \gg \Lambda_{QCD}, T$.
- **Hard gluons**, with energy and momentum of order m.
- **Soft gluons**, with energy and momentum of order mv.
- **Potential gluons**, with energy of order mv^2 and momentum of order mv.
- **Ultrasoft gluons**, with both energy and momentum of order mv^2

NRQCD → Potential NRQCD (pNRQCD)

Pineda and Soto, '97; Brambilla, Pineda, Soto, and Vairo '99, '00, '03

Degrees of freedom at scale $\frac{1}{r} = mv$ are integrated out



$\times \quad V(r)$

Power counting

$$r \sim \frac{1}{mv} \qquad t, R \sim \frac{1}{mv^2}, \frac{1}{\Lambda_{\rm QCD}}$$

Gauge fields are multiple expanded

$$A(r,R,t) = A(R,t) + \mathbf{r}\cdot\nabla A(R,t) + \ldots$$

Non-analytic behavior in r → matching coefficients V

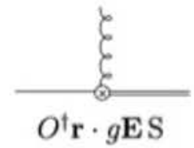
- Resulting degrees of freedom are singlet and octet states (see Lagrangian on next slide).
- Allows to obtain manifestly gauge-invariant results.
- Easier connection lattice QCD.
- If $1/r \gg T$ we can use this as a starting point.
- In other cases, the matching between NRQCD and pNRQCD will be modified.

NRQCD → Potential NRQCD (pNRQCD)

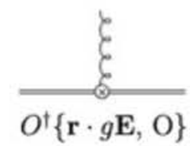
Pineda and Soto, '97; Brambilla, Pineda, Soto, and Vairo '99, '00, '03

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu,a} + \mathrm{Tr}\left\{ S^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_s \right) S + O^\dagger \left(i\partial_0 - \frac{\mathbf{p}^2}{m} - V_o \right) O \right\}$$

$$+V_A \mathrm{Tr}\left\{ O^\dagger\, \mathbf{r}\cdot g\mathbf{E}\, S + S^\dagger\, \mathbf{r}\cdot g\mathbf{E}\, O \right\} \rightarrow$$



$$+\frac{V_B}{2}\mathrm{Tr}\left\{ O^\dagger\, \mathbf{r}\cdot g\mathbf{E}\, O + O^\dagger O\, \mathbf{r}\cdot g\mathbf{E} \right\} \rightarrow$$

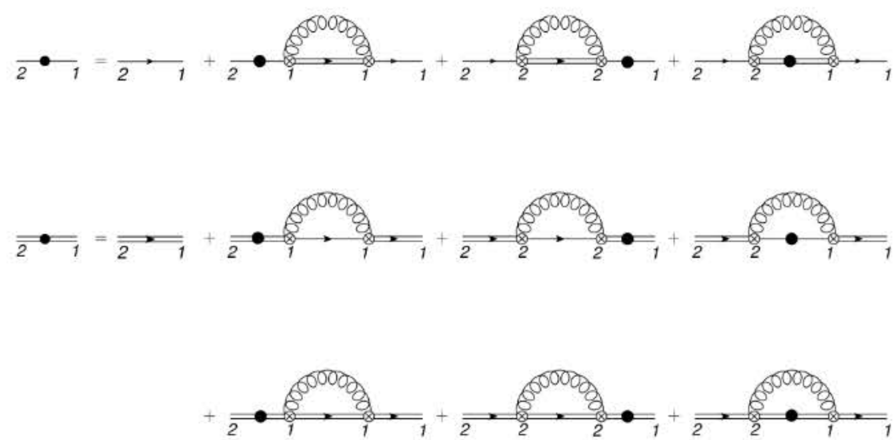


Singlet and octet potentials

$$V_s(r) = -C_F \frac{\alpha_s}{r}$$

$$V_o(r) = \frac{\alpha_s}{2N_c r}$$

- Based on this Lagrangian, we can perform first-principles calculations.
- Right figure shows diagrams contributing to singlet and octet self-energies.
- These enter into the calculation of Lindblad/collapse/jump operators.



OQS + pNRQCD – Lindblad reorganization

$$\frac{d\rho_{\rm probe}}{dt} = -i[H_{\rm probe}, \rho_{\rm probe}] + \sum_n \left(C_n \, \rho_{\rm probe} \, C_n^\dagger - \frac{1}{2}\{C_n^\dagger C_n, \rho_{\rm probe}\} \right)$$

- $H_{\rm probe}$ is Hermitian (includes singlet and octet states)
- C_n are the **collapse (or jump) operators** (connect different internal states)
- Partial and **total decay width operators** are

$$\Gamma_n = C_n^\dagger C_n \qquad \Gamma = \textstyle\sum_n \Gamma_n$$

- Can reorganize Lindblad equation by defining

$$H_{\rm eff} = H_{\rm probe} - \frac{i}{2}\Gamma$$

← **Non-Hermitian effective Hamiltonian**

$$\longrightarrow \quad \frac{d\rho_{\rm probe}}{dt} = -iH_{\rm eff}\rho_{\rm probe} + i\rho_{\rm probe}H_{\rm eff}^\dagger + \sum_n C_n \, \rho_{\rm probe} \, C_n^\dagger$$

LO OQS + pNRQCD Hamiltonian and collapse operators

N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, 1612.07248, 1711.04515

$$\frac{d\rho_{\rm probe}}{dt} = -iH_{\rm eff}\rho_{\rm probe} + i\rho_{\rm probe}H_{\rm eff}^\dagger + \sum_n C_n\,\rho_{\rm probe}\,C_n^\dagger$$

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$

$$H_{\rm probe} = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{r^2}{2}\gamma \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix}$$

mass shift

$$C_i^0 = \sqrt{\frac{\kappa}{N_c^2-1}}\, r^i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2-1} & 0 \end{pmatrix},$$

$$C_i^1 = \sqrt{\frac{(N_c^2-4)\kappa}{2(N_c^2-1)}}\, r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

$$\Gamma = \kappa r^i \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix} r^i$$

Total width → Im[V]

$$H_{\rm eff} = H_{\rm probe} - \frac{i}{2}\Gamma$$

$$\gamma \equiv \frac{g^2}{6\,N_c}\,{\rm Im}\int_{-\infty}^{+\infty} ds\,\langle T\,E^{a,i}(s,\mathbf{0})E^{a,i}(0,\mathbf{0})\rangle$$

$$\kappa \equiv \frac{g^2}{6\,N_c}\,{\rm Re}\int_{-\infty}^{+\infty} ds\,\langle T\,E^{a,i}(s,\mathbf{0})E^{a,i}(0,\mathbf{0})\rangle$$

Six collapse operators cover

- singlet → octet,
- octet → singlet
- octet → octet

LO OQS + pNRQCD Hamiltonian and collapse operators

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$$\frac{d\rho_{\rm probe}}{dt} = -iH_{\rm eff}\rho_{\rm probe} + i\rho_{\rm probe}H_{\rm eff}^\dagger + \sum_n C_n\,\rho_{\rm probe}\,C_n^\dagger$$

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$

$$C_i^0 = \sqrt{\frac{\kappa}{N_c^2-1}}\,r^i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2-1} & 0 \end{pmatrix},$$

$$C_i^1 = \sqrt{\frac{(N_c^2-4)\kappa}{2(N_c^2-1)}}\,r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

Six collapse operators cover
- singlet → octet,
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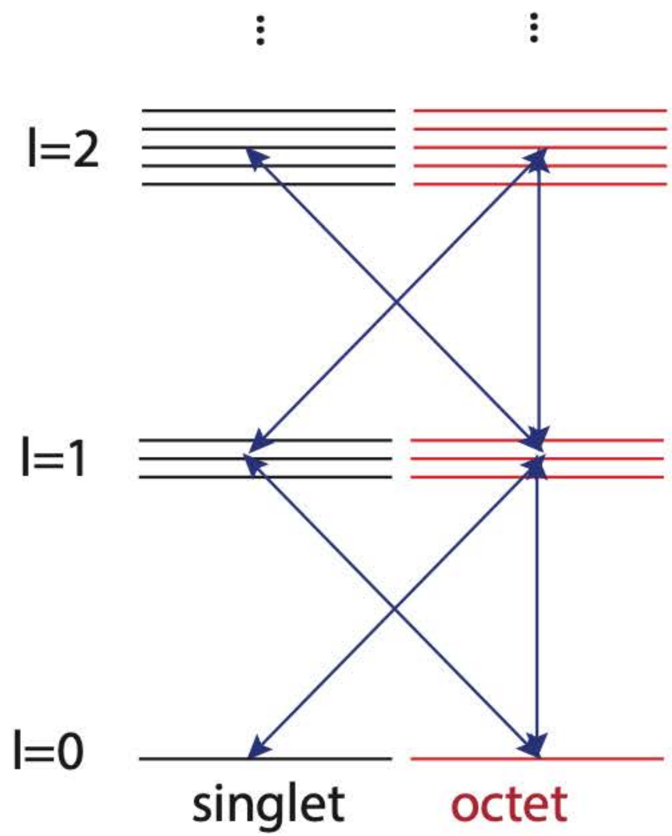
- octet → octet



Going beyond leading order in E/T

N. Brambilla, M.-A. Escobedo, A. Islam, M.S., A. Tiwari, A. Vairo, and P. Vander Griend, 2205.10289

- Results on previous slides were obtained by truncating at leading order (LO) in E/T (binding energy over temperature). This was extended to NLO in the reference above.

- By decomposing states into radial and angular components, for an isotropic system we can rewrite the operators as acting only on the 1d effective wavefunction
  u(r,t) = r R(r,t).

- At NLO, there are six jump operators (shown on the right).

- Based on these, one can write down the singlet and octet non-Hermitian effective Hamiltonian (see reference above for details).

$$C^{\uparrow}_{s\to o} = r - \frac{N_c\alpha_s}{8T} + \frac{1}{2MT}\left(\frac{\partial}{\partial r} - \frac{l+1}{r}\right),$$

$$C^{\downarrow}_{s\to o} = r - \frac{N_c\alpha_s}{8T} + \frac{1}{2MT}\left(\frac{\partial}{\partial r} + \frac{l}{r}\right),$$

$$C^{\uparrow}_{o\to s} = r + \frac{N_c\alpha_s}{8T} + \frac{1}{2MT}\left(\frac{\partial}{\partial r} - \frac{l+1}{r}\right),$$

$$C^{\downarrow}_{o\to s} = r + \frac{N_c\alpha_s}{8T} + \frac{1}{2MT}\left(\frac{\partial}{\partial r} + \frac{l}{r}\right),$$

$$C^{\uparrow}_{o\to o} = r + \frac{1}{2MT}\left(\frac{\partial}{\partial r} - \frac{l+1}{r}\right),$$

$$C^{\downarrow}_{o\to o} = r + \frac{1}{2MT}\left(\frac{\partial}{\partial r} + \frac{l}{r}\right).$$

Full NLO effective Hamiltonian

The effective Hamiltonian for singlet and octet evolution is defined by $H^{\rm eff}_{s,o} = h_{s,o} + {\rm Im}(\Sigma_{s,o}) - i\Gamma_{s,o}/2$ with $\Gamma_s = \sum_{i\in\{\uparrow,\downarrow\}}\Gamma^i_{s\to o}$ and $\Gamma_o = \sum_{i\in\{\uparrow,\downarrow\}}(\Gamma^i_{o\to s} + \Gamma^i_{o\to o})$. When expressed as operators acting on the reduced wave function, the singlet effective Hamiltonian $\overline{H}^{\rm eff}_s$ is given by

$$\mathrm{Re}[\overline{H}^{\rm eff}_s] = \frac{\overline{\mathcal{D}}^2}{M} - \frac{C_f\,\alpha_{\rm s}}{r} + \frac{\hat{\gamma}T^3}{2}r^2 + \frac{\hat{\kappa}T^2}{4M}\{r, p_r\}\,, \tag{3.12}$$

$$\mathrm{Im}[\overline{H}^{\rm eff}_s] = -\frac{\hat{\kappa}T^3}{2}\left[\left(r - \frac{N_c\alpha_{\rm s}}{8T}\right)^2 - \frac{3}{2MT} + \frac{\overline{\mathcal{D}}^2}{(2MT)^2} + \frac{1}{2MT}\left(\frac{N_c\alpha_{\rm s}}{4T}\right)\frac{1}{r}\right], \tag{3.13}$$

where $p_r = -i\partial_r$. Similarly, the octet effective Hamiltonian $\overline{H}^{\rm eff}_o$ is given by

$$\mathrm{Re}[\overline{H}^{\rm eff}_o] = \frac{\overline{\mathcal{D}}^2}{M} + \frac{1}{2N_c}\frac{\alpha_{\rm s}}{r} + \frac{N_c^2-2}{2(N_c^2-1)}\left[\frac{\hat{\gamma}T^3}{2}r^2 + \frac{\hat{\kappa}T^2}{4M}\{r, p_r\}\right], \tag{3.14}$$

$$\begin{aligned}\mathrm{Im}[\overline{H}^{\rm eff}_o] = &-\frac{\hat{\kappa}T^3}{2(N_c^2-1)}\left[\left(r + \frac{N_c\alpha_{\rm s}}{8T}\right)^2 - \frac{3}{2MT} + \frac{\overline{\mathcal{D}}^2}{(2MT)^2} - \frac{1}{2MT}\left(\frac{N_c\alpha_{\rm s}}{4T}\right)\frac{1}{r}\right] \\ &-\frac{\hat{\kappa}T^3}{4(N_c^2-1)}\left[r^2 - \frac{3}{2MT} + \frac{\overline{\mathcal{D}}^2}{(2MT)^2}\right],\end{aligned} \tag{3.15}$$

where $\hat{\kappa} = \kappa/T^3$ and $\hat{\gamma} = \gamma/T^3$.

$$\overline{\mathcal{D}}^2 = -\frac{\partial^2}{\partial r^2} + \frac{l(l+1)}{r^2}$$

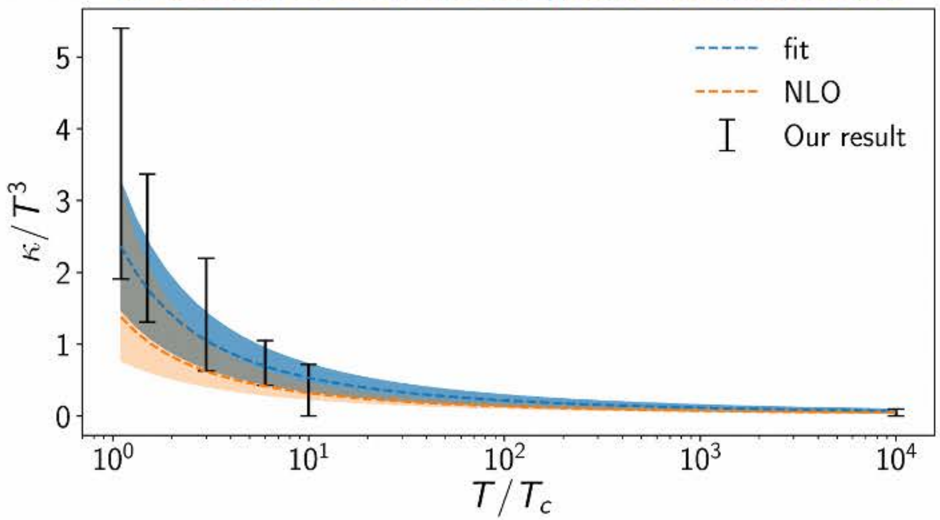
N. Brambilla, M.-A. Escobedo, A. Islam, M.S., A. Tiwari, A. Vairo, and P. Vander Griend, 2205.10289

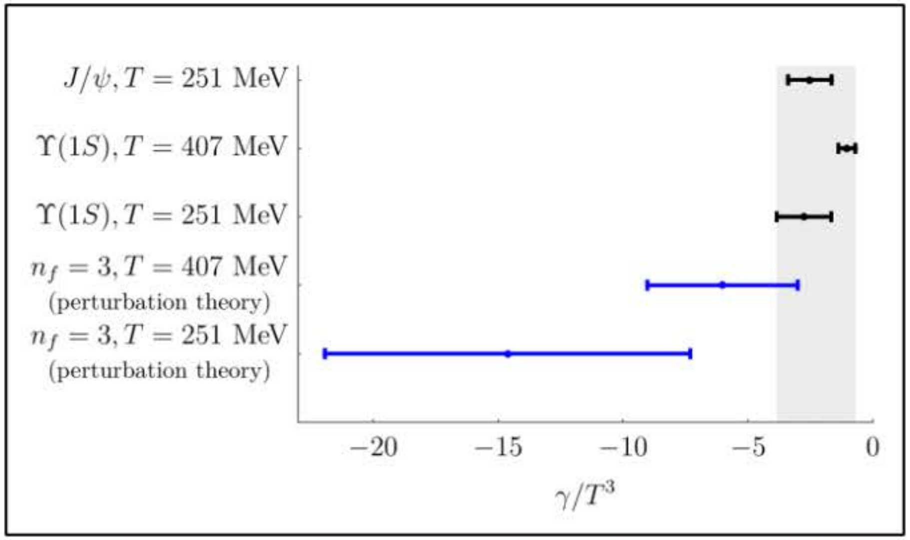
Values of $\hat{\kappa}$ and $\hat{\gamma}$ used

- We used NLO fits to recent lattice measurements of the heavy quark transport coefficient $\hat{\kappa} \equiv \kappa/T^3$. Note that this is related to the heavy quark diffusion constant D.
  - N. Brambilla, V. Leino, P. Petreczky, and A. Vairo, 2007.10078

- The value of $\hat{\gamma} \equiv \gamma/T^3$ is less constrained, we vary it in the range -3.5 < $\hat{\gamma}$ < 0.
  - N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, 1612.07248.
  - N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, 1711.04515.
  - N. Brambilla, M. A. Escobedo, A. Vairo and P. Vander Griend, 1903.08063.

N. Brambilla, V. Leino, P. Petreczky, and A. Vairo, 2007.10078





N. Brambilla, M. A. Escobedo, A. Vairo and P. Vander Griend, 1903.08063.

A parallelizable approach: Quantum trajectories

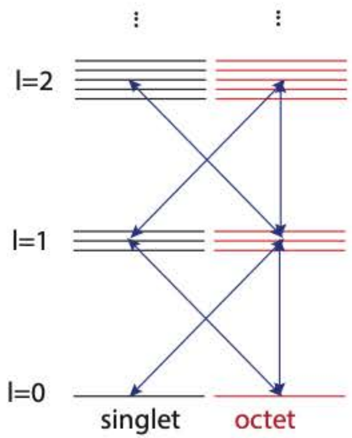
N. Brambilla, M.-A. Escobedo, M.S., A. Vairo, P. Vander Griend, and J.H. Weber, 2012.01240

$$\frac{d\rho_{\rm probe}}{dt} = -iH_{\rm eff}\rho_{\rm probe} + i\rho_{\rm probe}H_{\rm eff}^\dagger + \sum_n C_n\,\rho_{\rm probe}\,C_n^\dagger$$

Non-unitary "no jump" evolution

Can treat this "quantum jump" term stochastically

- Can be reduced to the solution of a large set of "quantum trajectories" in which we solve a 1D Schrödinger equation with a **non-Hermitian Hamiltonian $H_{\rm eff}$**, subject to **stochastic quantum jumps**.
- The evolution with the non-Hermitian $H_{\rm eff}$ preserves the color and angular momentum state of the system (but not norm).
- Collapse/jump operators encode transitions between different color/angular momentum states (subject to selection rules).



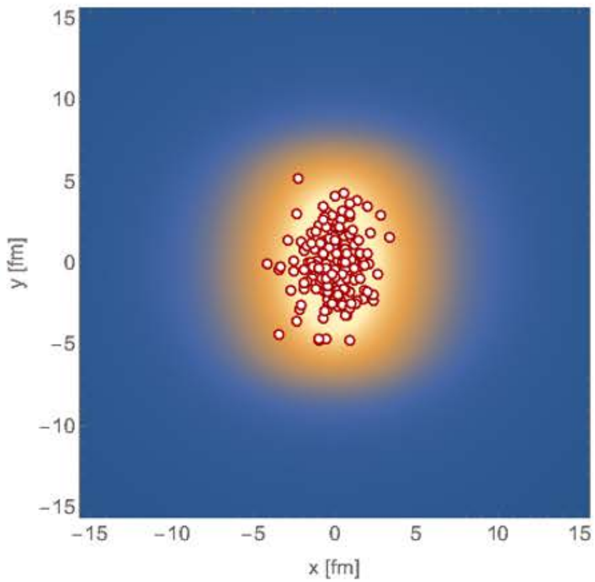
- For each **physical trajectory** (path through the QGP) we average over a large set of <u>**independent**</u> **quantum trajectories** → **Embarrassingly parallel**
- **Added benefit: Can describe all angular momentum states (no cutoff) .**

How can one numerically solve these equations?

$$\frac{d\rho_{\text{probe}}}{dt} = -iH_{\text{eff}}\rho_{\text{probe}} + i\rho_{\text{probe}}H_{\text{eff}}^{\dagger} + \sum_n C_n\,\rho_{\text{probe}}\,C_n^{\dagger}$$

- Each block of the density matrix in color space can be decomposed into orbital angular momentum blockwise.
- Upon truncating in angular momentum ($l \leq l_{max}$) one can reduce both the singlet and octet blocks of the reduced density matrix to size $(l_{max} + 1)^2$.
- One can then discretize the radial wavefunction (N = # of lattice points) and evolve the reduced density matrix using standard differential equation and matrix solvers gives $\sim N^2(l_{max} + 1)^2$ matrix size.
- **Need to describe bound and unbound states with highly localized initial wave function, so the box must be large and have small lattice spacing → large N and large l_{max}.**
- As N and l_{max} become large, **the computation becomes very challenging**.
- **Need a better/faster method which we can easily parallelize.**

Computing survival probabilities with QTraj



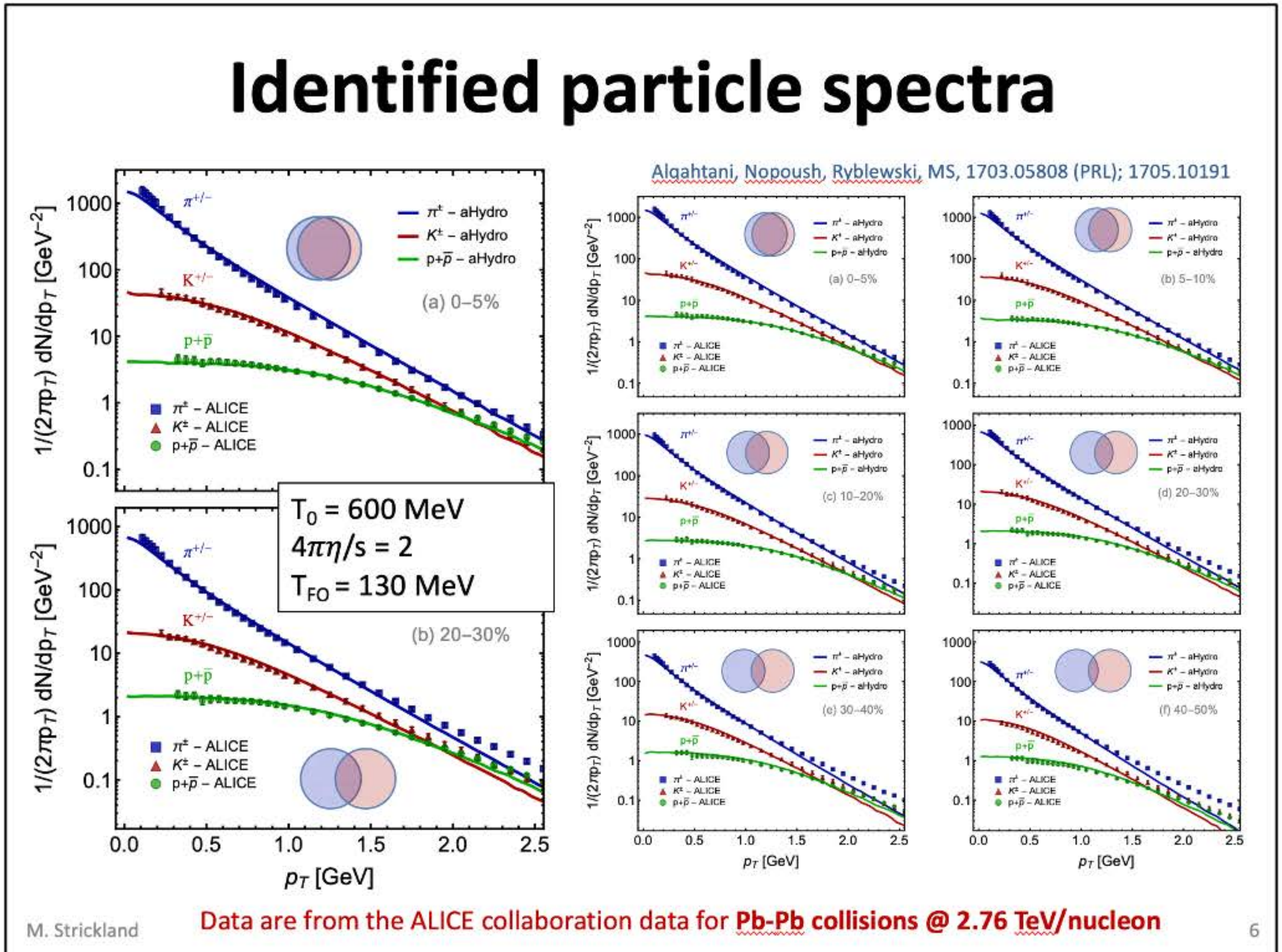
Survival probability

$$SP(n,l) = \frac{|\langle n,l|\psi(t_f)\rangle|^2}{|\langle n,l|\psi(t_0)\rangle|^2}$$

- Used $N = 4096$ points
- $L = 108\ a_0$
- $\Delta t = 2 \times 10^{-4}$ fm

- We sampled bottomonium production points and initial COM transverse momentum using **Monte-Carlo sampling** (physical trajectories).
- Temperature evolution provided **by 3+1D anisotropic hydrodynamics** (good description of identified soft hadron spectra and anisotropic flow, see backup slides for evidence).
- Along each physical trajectory, we solved the **real-time 3D Schrödinger equation with a complex potential and stochastically sampled jumps** → Lindblad equation.
- We then solved for the **survival probability** of S- and P-wave states (see box to the left).

3+1D hydrodynamical background



For 5.02 TeV, T_0 = 630 MeV @ t_0 = 0.25 fm/c

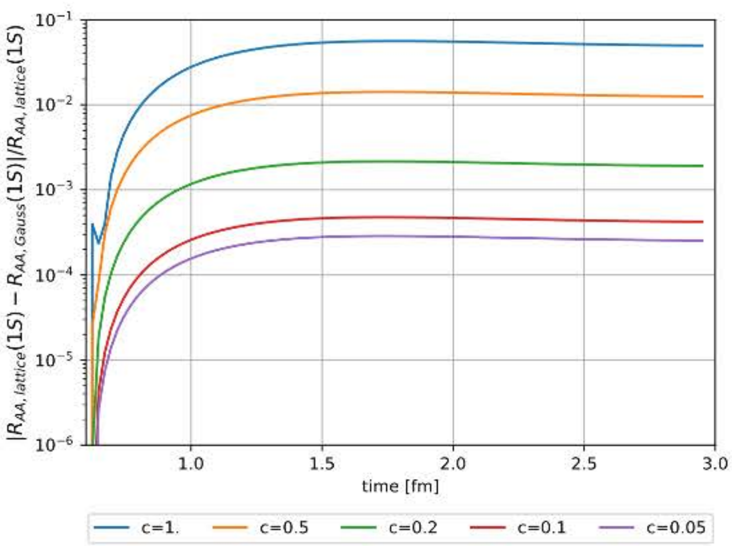
- We use a 3+1D dissipative code for the hydro background (**quasiparticle anisotropic hydrodynamics**)
- Has been tuned to RHIC and LHC heavy ion collisions
- Reproduces spectra, multiplicities, charged and identified elliptic flow of light hadrons, HBT radii, etc.

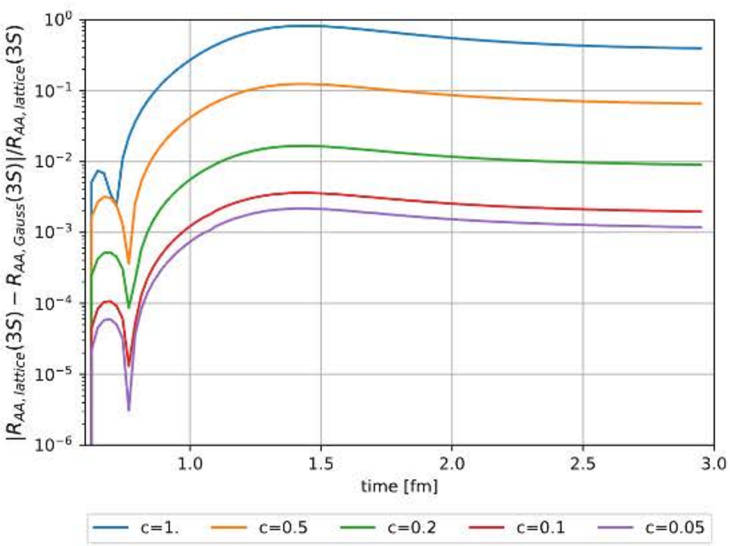
Initial bottomonium wavefunction

- We took the initial wavefunction to be given by a smeared delta function (local production due to large mass, $\Delta \sim 1/M$) of the form

$$u_\ell(r, \tau = 0) \propto r^{\ell+1} \exp(-r^2/\Delta^2)$$

- For a given l, the **initial state is a quantum linear superposition** of the eigenstates of H.
- **Includes both bound and unbound states.**
- We took $\Delta = 0.2\, a_0$ which reproduces results obtained with a true delta to within 1%.

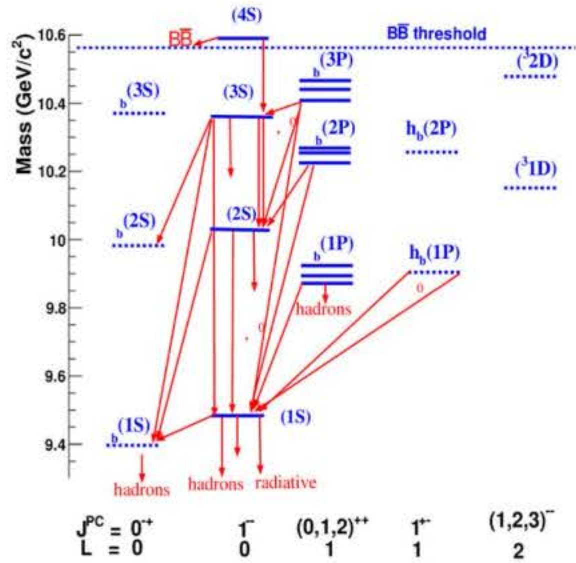




Feed-down implementation

$$\vec{N}_{\rm observed} = F\vec{N}_{\rm direct}$$

$$F = \begin{pmatrix} 1 & 0.2645 & 0.0194 & 0.352 & 0.18 & 0.0657 & 0.0038 & 0.1153 & 0.077 \\ 0 & 1 & 0 & 0 & 0 & 0.106 & 0.0138 & 0.181 & 0.089 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0.0091 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0.0051 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$



- $N_{\rm direct}$ corresponds to $(N_{1S}, N_{2S}, N_{1P} \times 3, N_{3S}, N_{2P} \times 3, N_{2D})^T$ where, e.g., N_{1S} is the final number of Υ(1S) states that can decay in the dilepton channel.

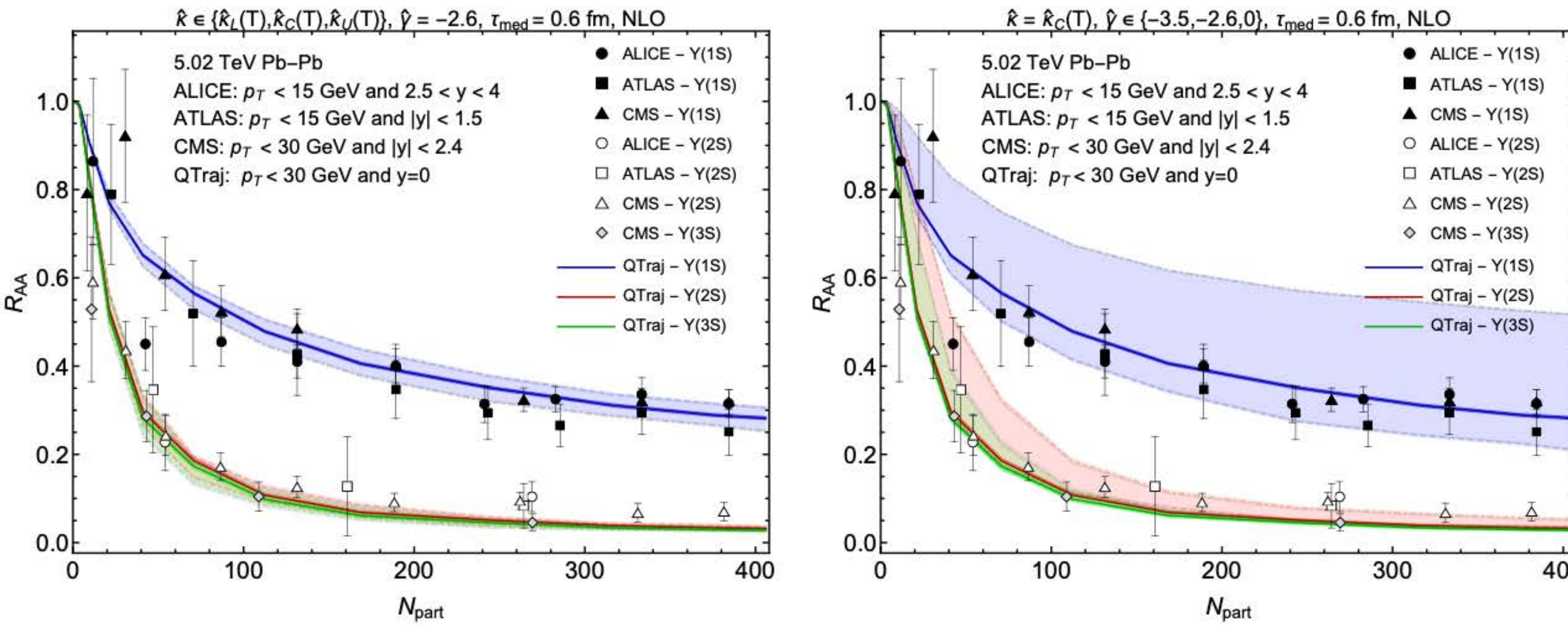
- $N_{\rm direct}$ can be obtained using $\langle N_{\rm bin}(b)\rangle * \sigma_{\rm direct}$ * (Survival probability)

- After feed down, we then normalize by the pp collision result scaled to AA → R_{AA}.

$$R^i_{AA}(c) = \frac{(F \cdot S(c) \cdot \vec{\sigma}_{\rm direct})^i}{\sigma^i_{\rm exp}}$$

NLO OQS + pNRQCD predictions for R_{AA} vs N_{part} at LHC

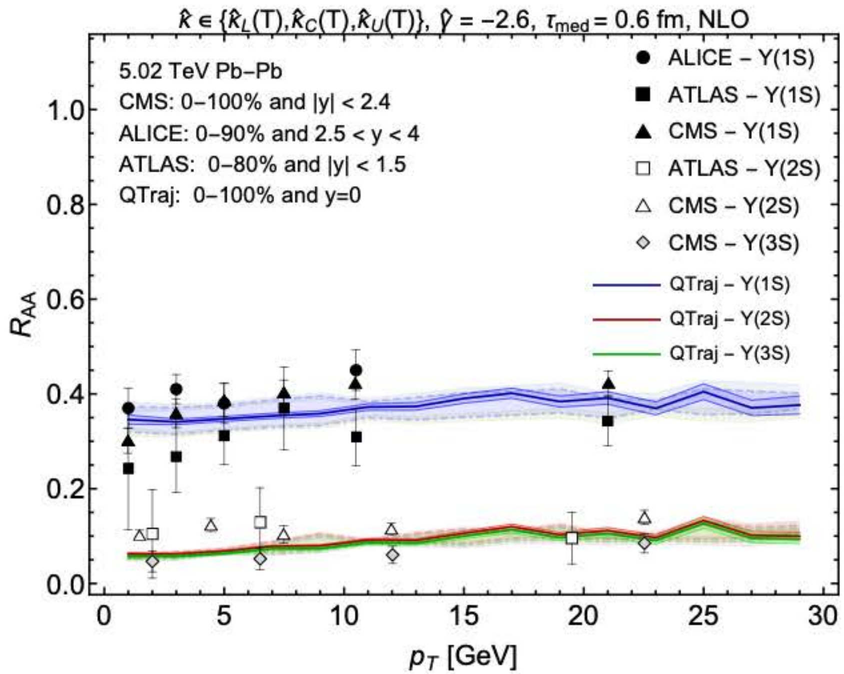
N. Brambilla, M.-A. Escobedo, A. Islam, M.S., A. Tiwari, A. Vairo, P. Vander Griend, 2205.10289

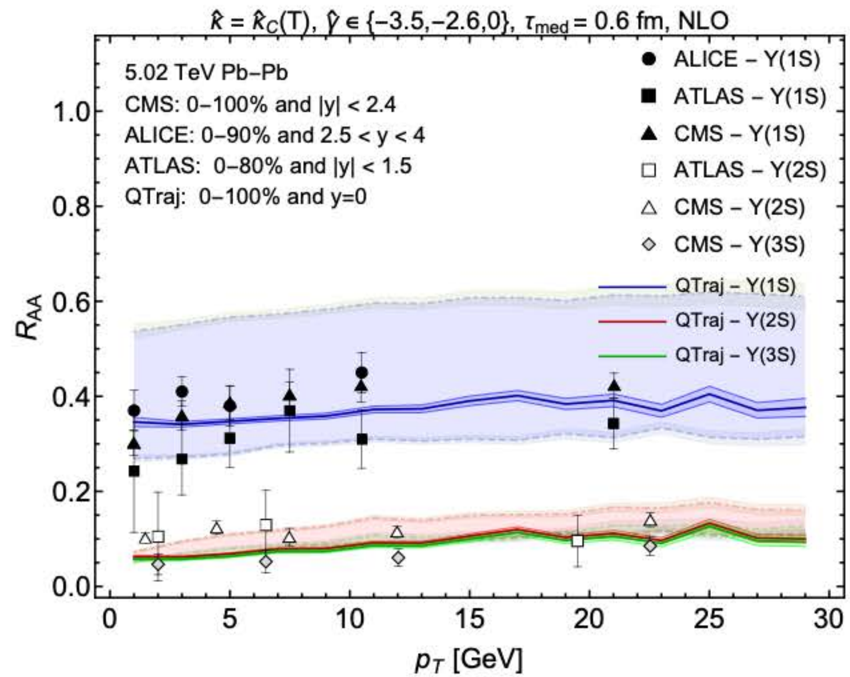


- **Left panel**: Result including feed down, when varying $\hat{\kappa}$ over the theoretical uncertainty.
- **Right panel**: Result including feed down, when varying $\hat{\gamma}$ over the theoretical uncertainty
- **Note:** The NLO results above ignore jumps (H_{eff} only). The effect of jumps is expected to be small based on prior studies, but requires lots of computer time (forthcoming).
- The statistical uncertainty associated with the average over physical trajectories is on the order of the line width.

NLO R_{AA} vs transverse momentum

N. Brambilla, M.-A. Escobedo, A. Islam, M.S., A. Tiwari, A. Vairo, P. Vander Griend, 2205.10289

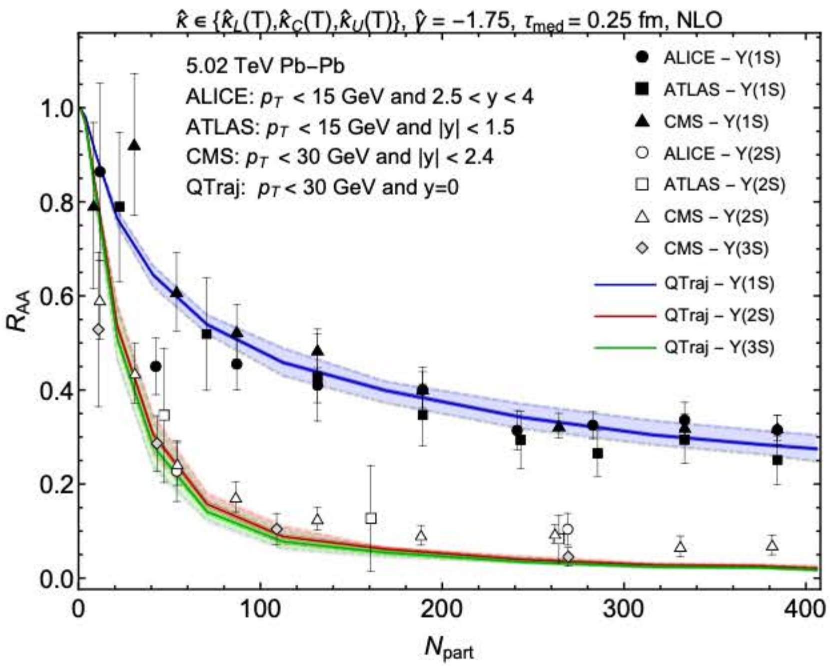


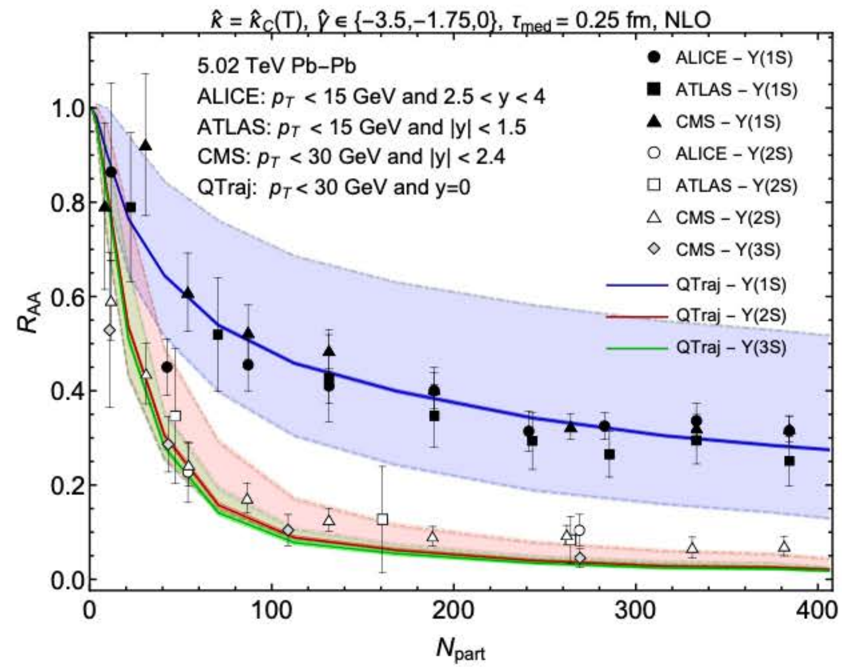


- QTraj predictions consistent with experimental observations.
- Very flat. Small decrease comes from longer time spent in the QGP as the velocity decreases.
- Once again, larger variation from uncertainty in $\hat{\gamma}$.

NLO OQS + pNRQCD predictions for R_{AA} vs N_{part} at LHC

N. Brambilla, M.-A. Escobedo, A. Islam, M.S., A. Tiwari, A. Vairo, P. Vander Griend, 2205.10289

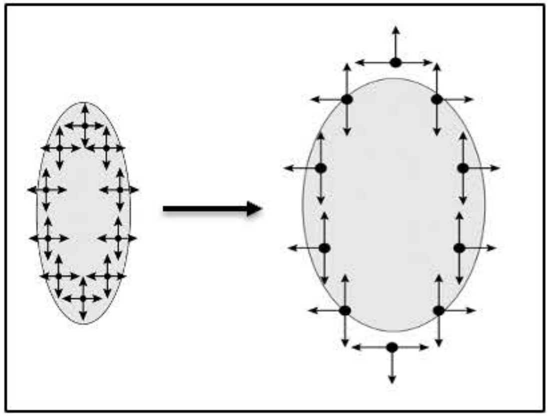




- Note that It is possible to use a shorter medium initialization time and obtain a similar description of the data.
- Above shows results using τ_{med} = 0.25 fm rather than 0.6 fm
- In this case, I changed the central value of gamma plotted, but the range of variation is the same.

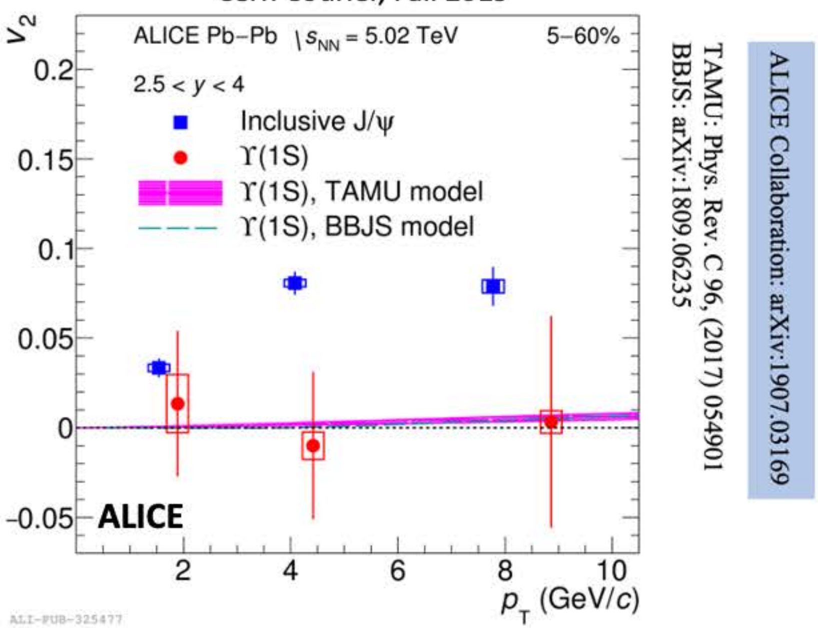
Momentum-space anisotropies

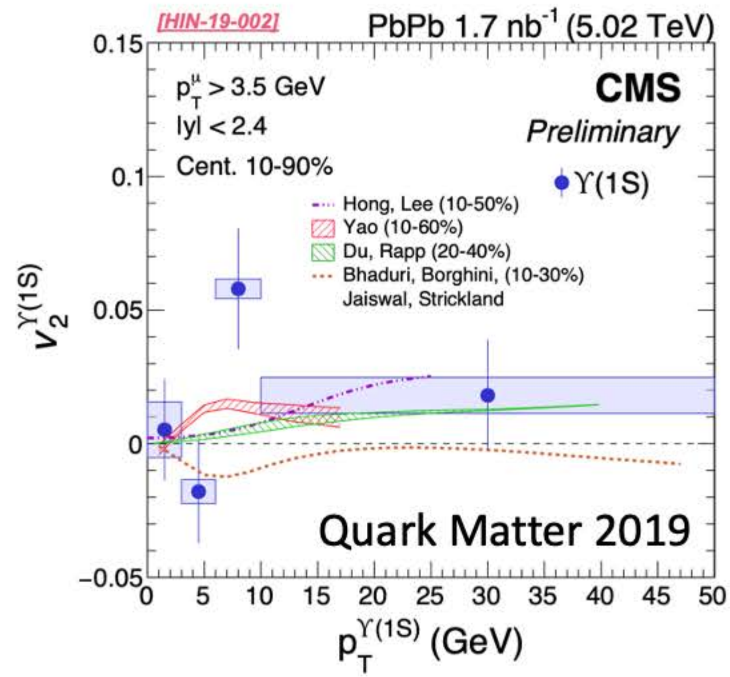
4d flow tomography



- Bottomonium probably doesn't flow in the "collective flow" sense.
- However, there can be momentum-space anisotropies induced by path-length differences in suppression along the short and long sides of the QGP.

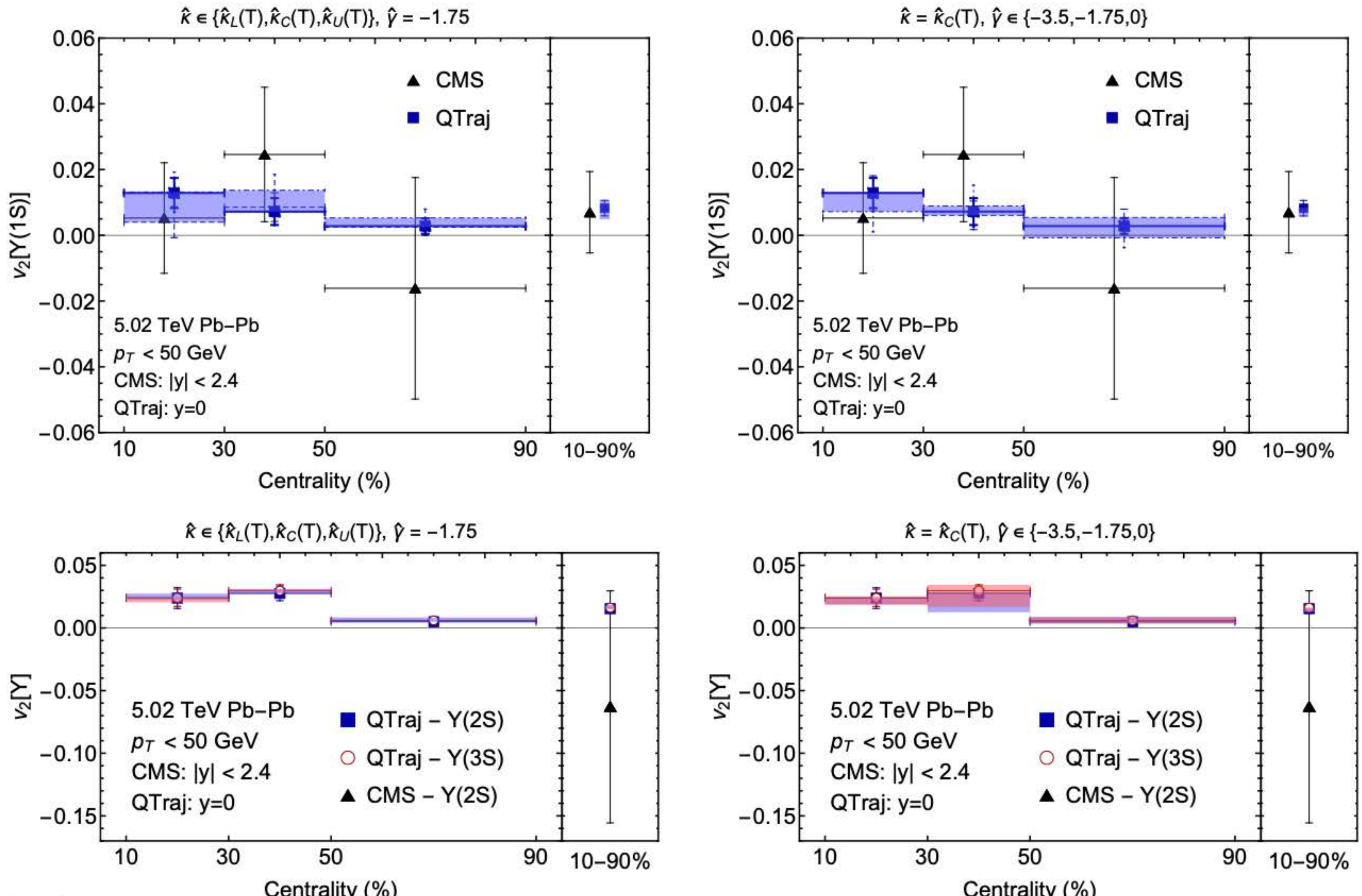
Cern Courier, Fall 2019





LO Momentum-space anisotropies at LHC

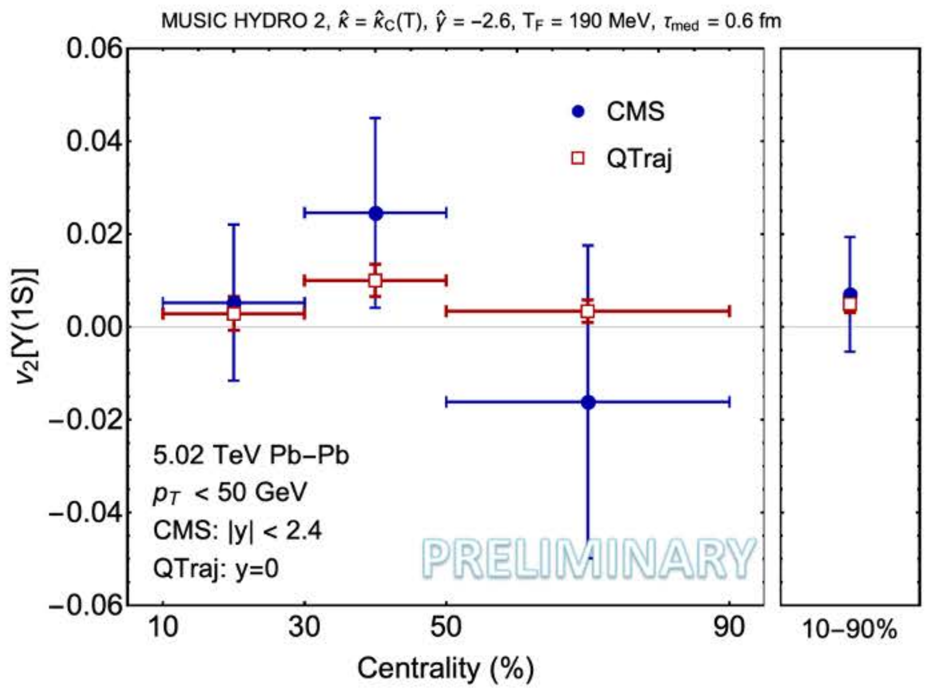
N. Brambilla, M.-A. Escobedo, M.S., A. Vairo, P. Vander Griend, and J.H. Weber, 2107.06222 (Leading-Order)

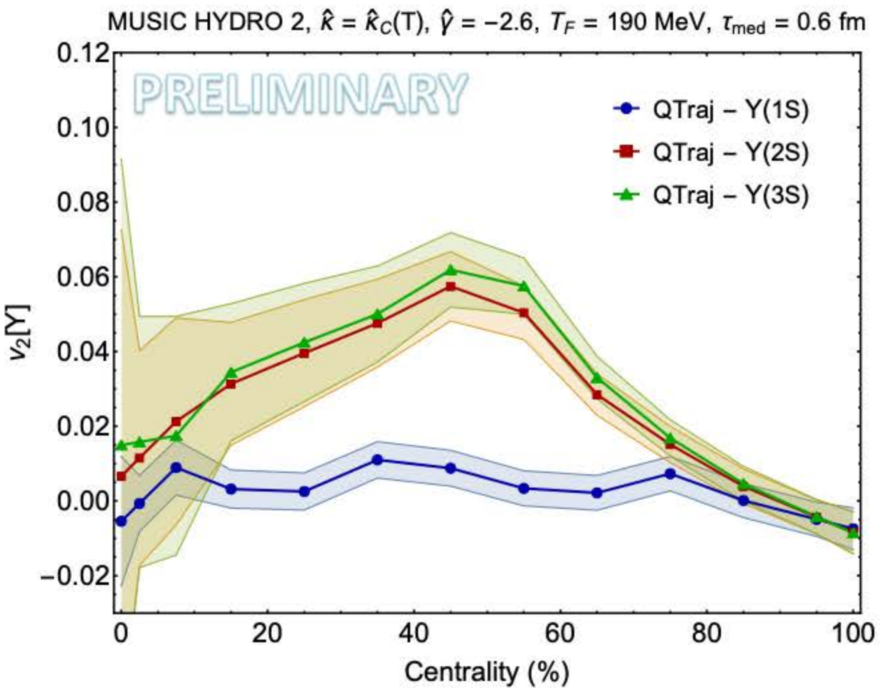


NLO Momentum-space anisotropies at LHC

H. Alalawi, J. Boyd, and M. Strickland, forthcoming

- Forthcoming work on including the effect of fluctuating hydrodynamics.
- Using MUSIC + IPGlasma IC tuned to 5.02 TeV Pb-Pb collisions.
- Agrees well with aHydro results for R_{AA} (not shown).





Conclusions and Outlook

- **First 3D quantum non-abelian treatment of heavy quarkonium within the OQS+pNRQCD framework.**
- Phenomenological treatment has been extended to NLO in E/T.
- Transport coefficients used were **constrained by independent lattice measurements.**
- **OQS + pNRQCD works quite well in describing bottomonium suppression and "flow" vs N_{part} and p_T seen at LHC energies.**
- Demonstrated that Upsilon[nS] R_{AA} (and their double ratios) can provide **experimental constraints on these transport coefficients**.
- The **quantum trajectory algorithm** allows us to include effect of "**quantum jumps"** between color and angular momentum states in a **computationally scalable manner**. Code (Qtraj) has been released as an open-source package.
- Could be useful in other simulations of open quantum systems evolution, such as the evolution of dark matter in the early universe.