Coupled-charge transport: current status of fluid dynamic modeling

CRC-Tr 211 Transport Meeting
Frankfurt

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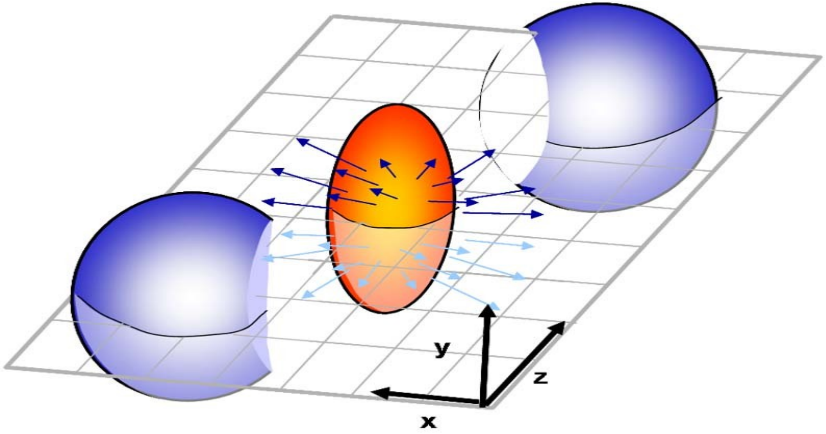
The multi-component nature of nuclear matter

Traditionally:

Viewed as ‘blob’ of one type of matter (single component) with one velocity field

- usually ‘blob’ of energy
  with conserved particle number



https://www.quantumdiaries.org/wp-content/uploads/2011/02/FlowPr.jpg

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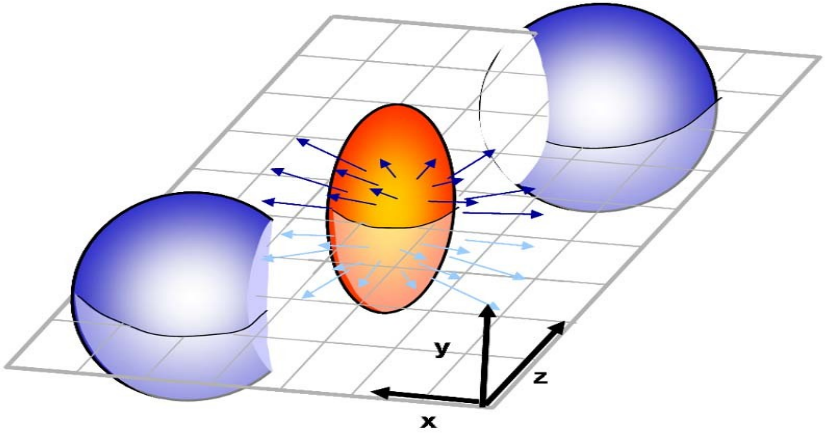

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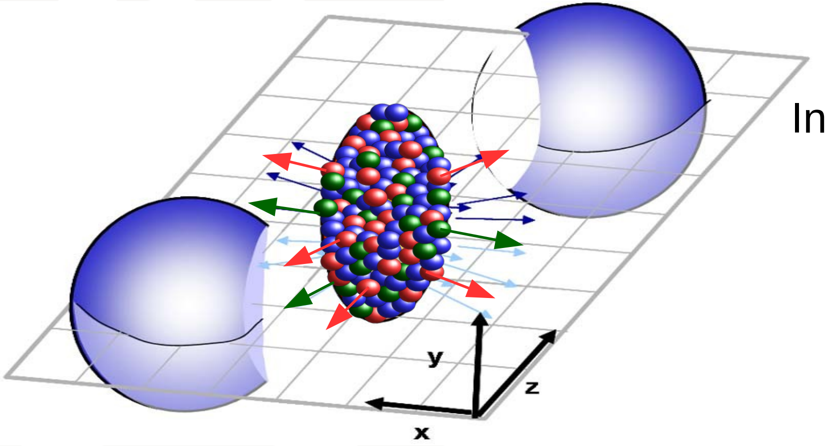


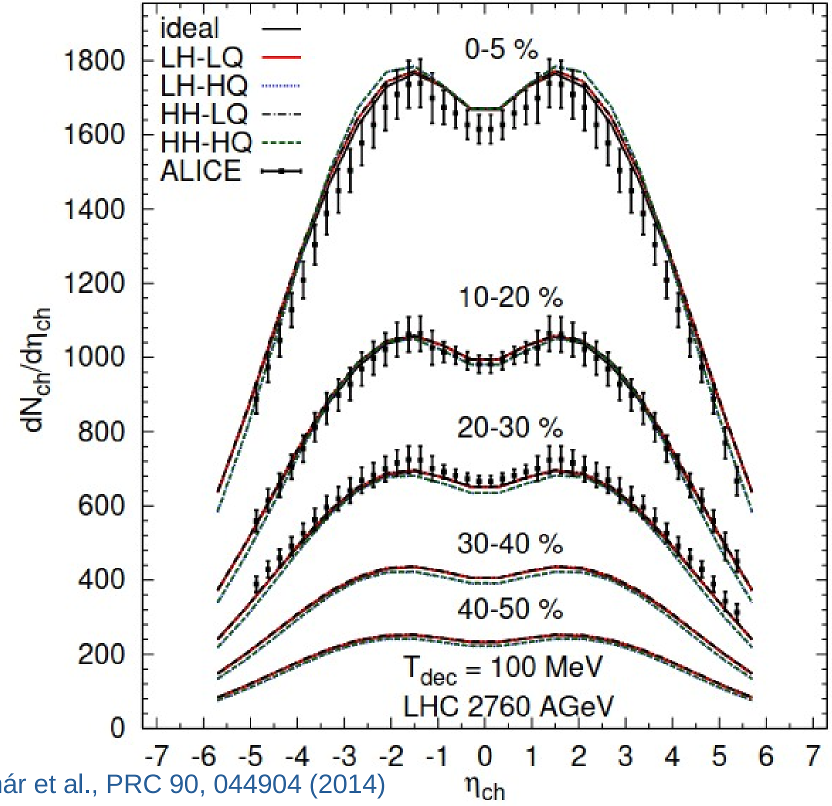
In general:

Consists of multiple components with <u>various properties</u> with multiple velocity fields

- with **multiple conserved quantities**
  (e.g. energy, electric charge, baryon number, strangeness, …)
- mixed chemistry → **<u>coupled charge currents!</u>**

Hydrodynamics applied to heavy ion collisions

Hydro has been very successful in the **effective** description of the evolution of heavy-ion collisions ...



Molnár et al., PRC 90, 044904 (2014)

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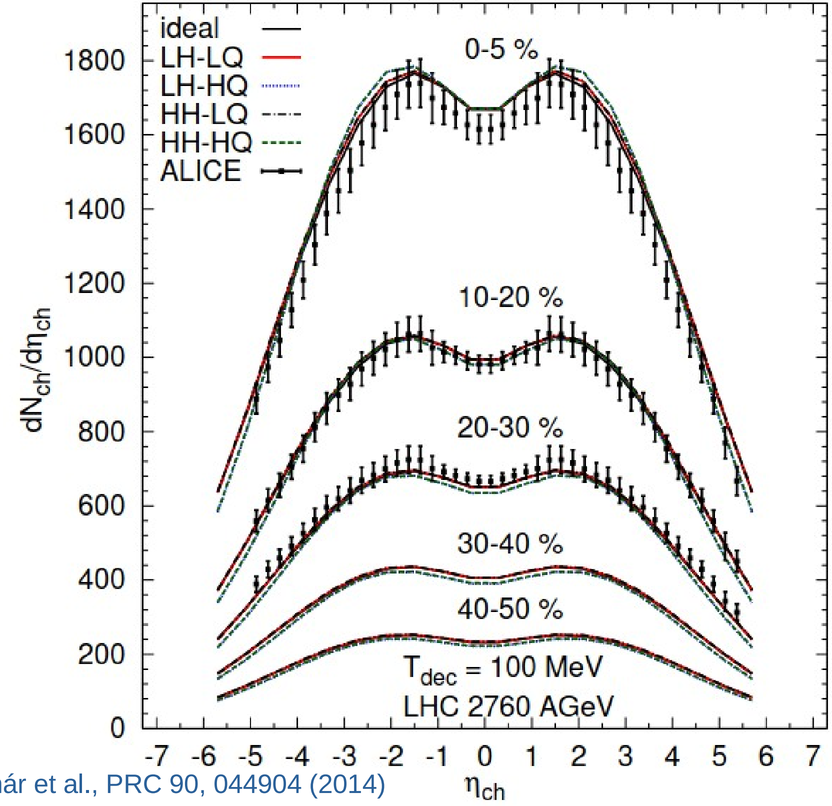

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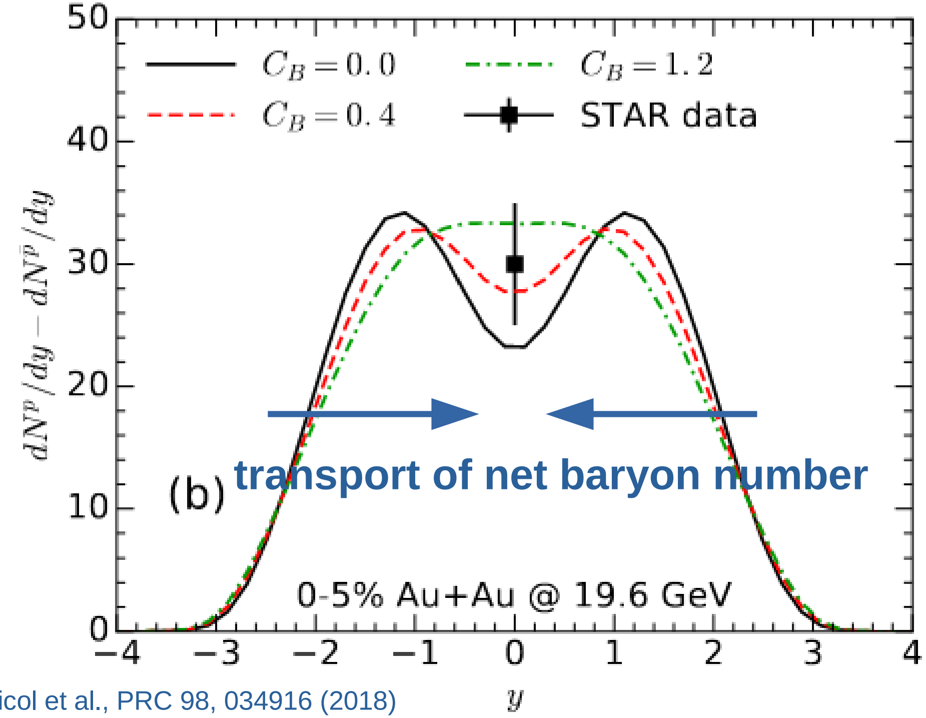
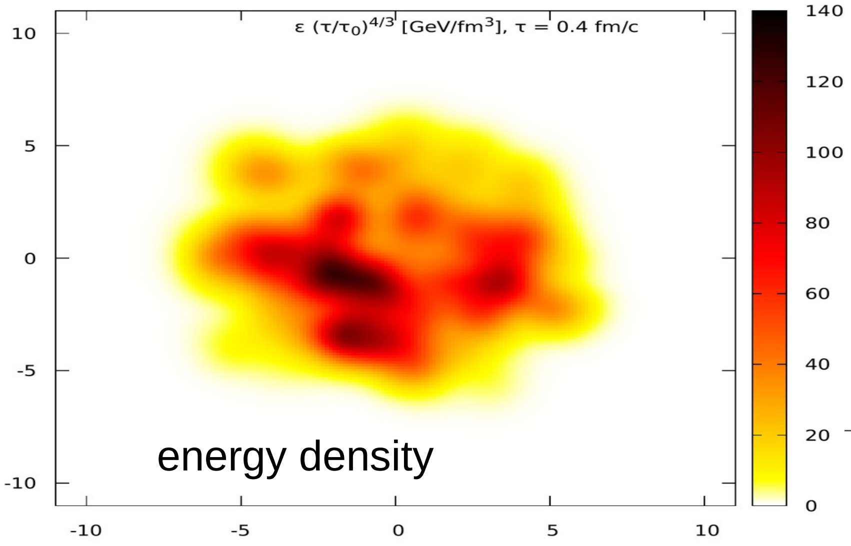
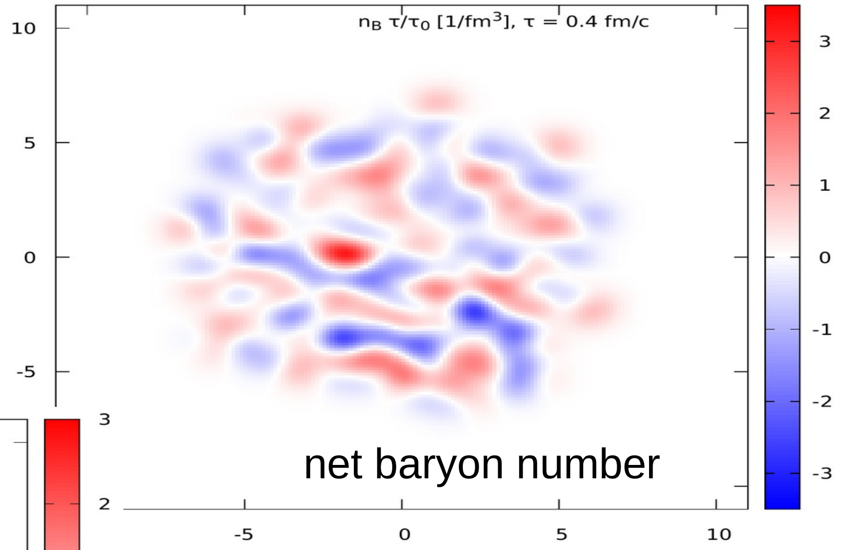
… but the theory has to be extended in order to explicitly account for effects and is dependent on the knowledge of the underlying microscopic properties.

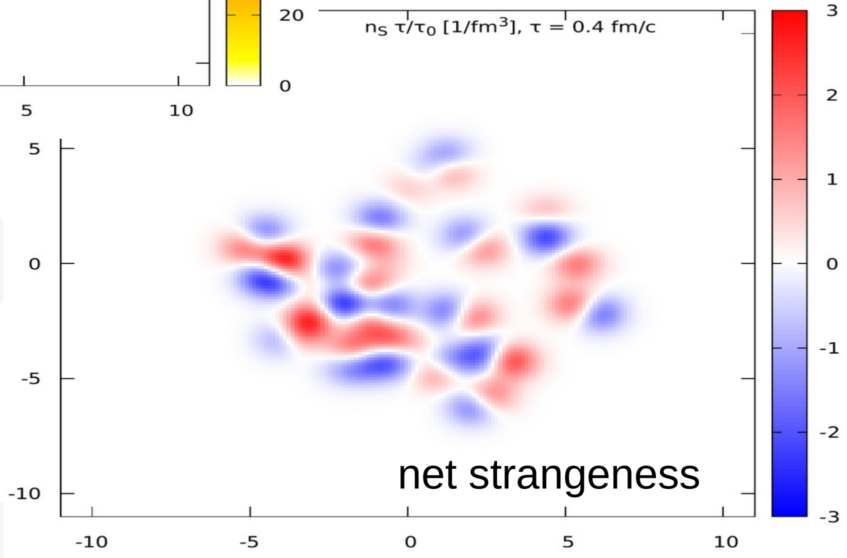


Denicol et al., PRC 98, 034916 (2018)

Initial state with multiple conserved charges







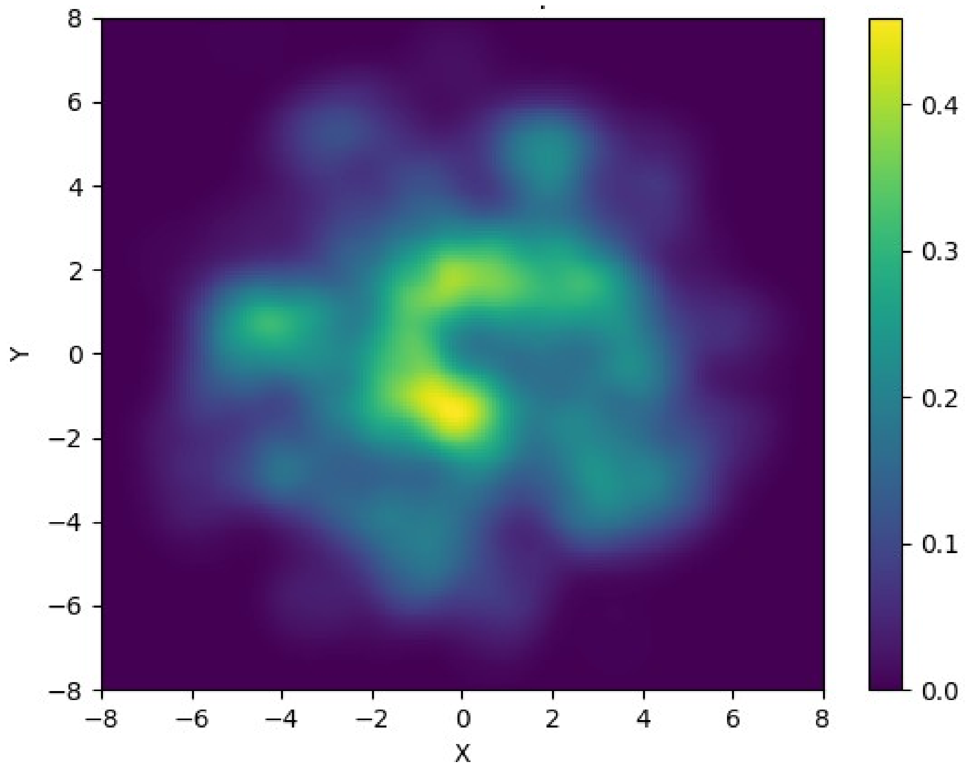
Smoothed initial condition for Pb Pb @ 5 TeV from ICCING

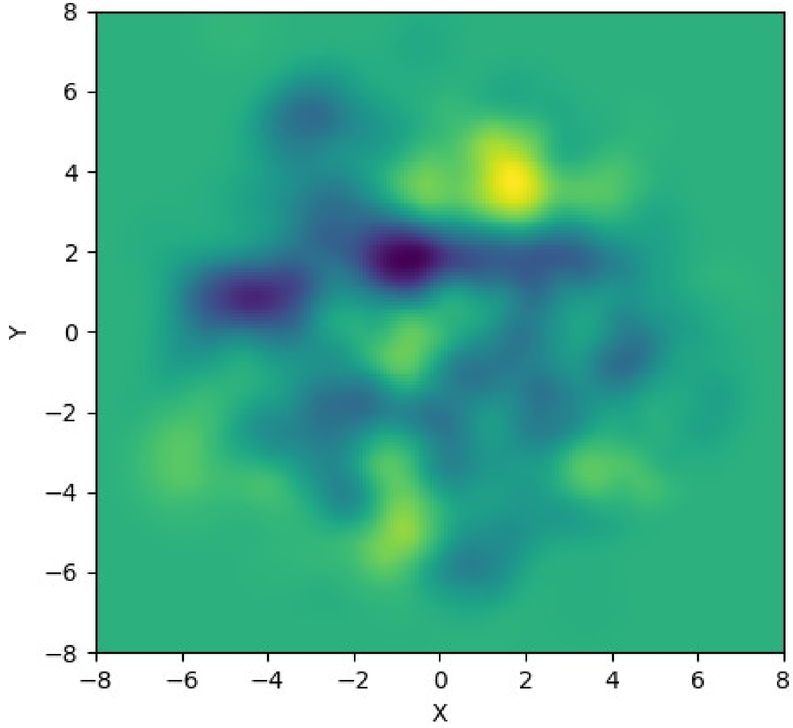
Martinez et al., arXiv:1911.10272, arXiv:1911.12454

Plots not published!

Initial state with multiple conserved charges

Collaborations with T. Dore, O. Garcia-Montero, S. Schlicht (**McDipper / KoMPoST**)
and H. Roch, N, Götz, H. Elfner (**SMASH**)





Fluid dynamics of multi-component systems

Hydrodynamics: macroscopic effective field theory of thermal matter close to local equilibrium

Here: single fluid velocity field u^μ

Conservation of Energy and Momentum: $\partial_\mu T^{\mu\nu} = 0$

Conservation of charge: $\partial_\mu N_q^\mu = 0$

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q-th conserved charge (eg. B,Q,S)

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2nd-order (multi-component) hydro

$\dot{\Pi},\ \dot{V}_q^{\langle\mu\rangle},\ \dot{\pi}^{\langle\mu\nu\rangle}$

$$\tau_\Pi \dot{\Pi} + \Pi = S_\Pi$$
$$\sum_{q'} \tau_{qq'} \dot{V}_{q'}^{\langle\mu\rangle} + V_q^\mu = S_q^\mu$$
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Relaxation equations (Israel-Stewart-type causal theory)

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$$S_q^\mu = \sum_{q'} \kappa_{qq'} \nabla^\mu \alpha_{q'} - \sum_{q'} \tau_{qq'} V_{q',\nu}\omega^{\nu\mu} - \sum_{q'} \delta_{VV}^{(q,q')} V_{q'}^\mu \theta - \sum_{q'} \lambda_{VV}^{(q,q')} V_{q',\nu}\sigma^{\mu\nu}$$
$$- \ell_{V\Pi}^{(q)} \nabla^\mu \Pi + \ell_{V\pi}^{(q)} \Delta^{\mu\nu}\nabla_\lambda \pi_\nu^\lambda + \tau_{V\Pi}^{(q)} \Pi \dot{u}^\mu - \tau_{V\pi}^{(q)} \pi^{\mu\nu}\dot{u}_\nu$$
$$+ \sum_{q'} \lambda_{V\Pi}^{(q,q')} \Pi \nabla^\mu \alpha_{q'} - \sum_{q'} \lambda_{V\pi}^{(q,q')} \pi^{\mu\nu}\nabla_\nu \alpha_{q'}$$

Fotakis, Molnár, Niemi, Greiner, Rischke
PRD 106, 036009 (2022)

Mixed chemistry couples diffusion currents (coupled charge-transport); already present in 1st order term

2nd order terms: couples all currents to each other; depend on all gradients!

Explicit expressions for transport coefficients!

Equations of motion with multiple conserved charges

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Diffusion coefficients: extensively studied!

Fotakis, Molnár, Niemi, Greiner, Rischke PRD 106, 036009 (2022)

Greif, Fotakis et al., PRL 120, 242301 (2018)
Fotakis, Greif et al., PRD 101, 076007 (2020)
Fotakis, Soloveva et al, PRD 104, 034014 (2021)

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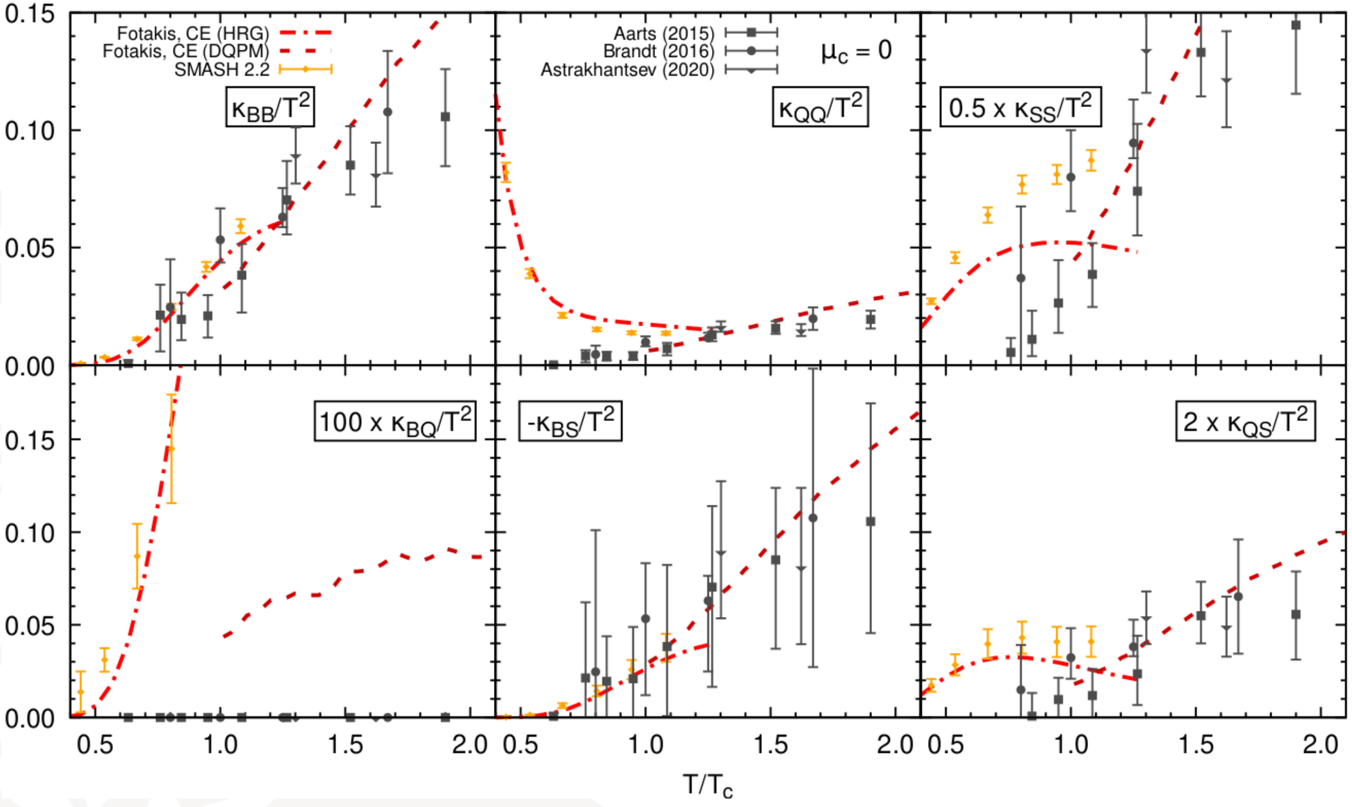
Computation of transport coefficients (Example: diffusion coefficients)

$$\kappa_{qq'} = \sum_{n=0}^{\infty} \sum_{i,j=1}^{N_{\text{species}}} \left(\mathcal{C}^{-1}\right)^{(1)}_{ji,0n} q_j \left(q'_i \mathcal{J}^{(i)}_{n+1,1} - \frac{n_{q'}}{\epsilon + P_0} \mathcal{J}^{(i)}_{n+2,1} \right)$$

Greif, Fotakis et al., PRL 120, 242301 (2018)
Fotakis, Greif et al., PRD 101, 076007 (2020)
Fotakis, Soloveva et al, PRD 104, 034014 (2021)
Hammelmann et al., arXiv:2307.15606 (2023)



Insights in **chemical composition** of nuclear matter?

Upcoming publication!

(Fotakis, Lohr, Greiner)

Upcoming project with T. Dore and S. Schlichting

Single-component vs. Multi-component system

What is with the second-order terms?

$$\mathrm{S}_q^{\mu} = (\ldots) + \ell_{V\pi}^{(q)} \Delta^{\mu\nu} \nabla_{\lambda} \pi_{\nu}^{\lambda} + (\ldots)$$

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Ultrarelativistic, classical system with hard-sphere interactions:

Denicol et al., PRD 85, 114047 (2012)

Used in simulations of heavy-ion collisions!

TABLE I. The coefficients for the particle diffusion for a classical gas with constant cross section in the ultrarelativistic limit, in the 14-moment approximation. The transport coefficient $\tau_{n\pi}$ was incorrectly listed as being zero in Ref. [1]

κ	$\tau_n[\lambda_{\text{mfp}}]$	$\delta_{nn}[\tau_n]$	$\lambda_{nn}[\tau_n]$	$\lambda_{n\pi}[\tau_n]$	$\ell_{n\pi}[\tau_n]$	$\tau_{n\pi}[\tau_n]$
$3/(16\sigma)$	$9/4$	1	$3/5$	$\beta_0/20$	$\beta_0/20$	$\beta_0/80$

$$\tau_n \dot{V}_q^{\langle\mu\rangle} + V_q^\mu = \sum_{q'} \kappa_{qq'} \nabla^\mu \alpha_{q'} - V_{q,\nu}\omega^{\nu\mu} - \tau_n V_{q'}^\mu \theta - \frac{3\tau_n}{5} V_{q,\nu}\sigma^{\mu\nu} + \frac{\tau_n}{20T}\Delta^{\mu\nu}\nabla_\lambda \pi_\nu^\lambda - \frac{\tau_n}{20T}\pi^{\mu\nu}\dot{u}_\nu - \frac{\tau_n}{20T}\pi^{\mu\nu}\nabla_\nu \alpha_q$$

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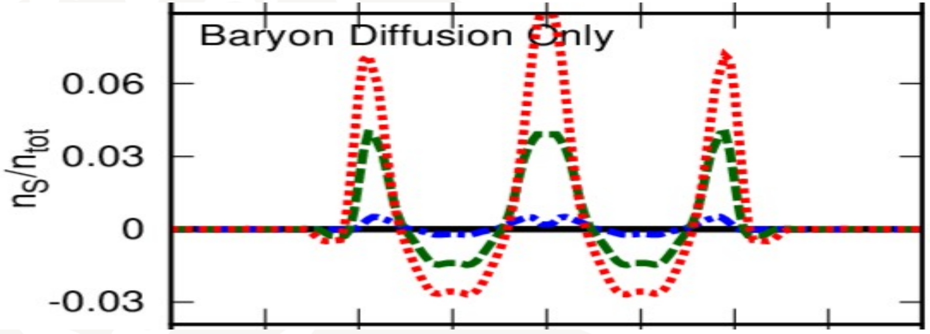

Second-order transport coefficients not consistent with assumed system

→ generation of unphysical charge currents

Consistency is important in charge transport!
Use multi-component expressions.

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Ultrarelativistic, classical system with ha … .14047 (2012)

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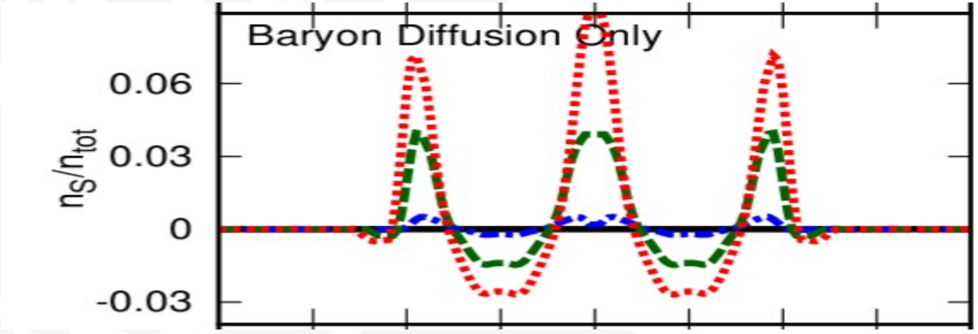

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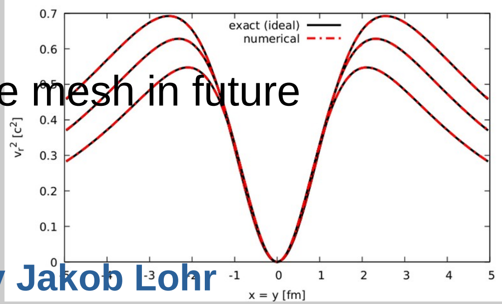
The journey to a software package treating coupled-charged transport

- (3+1)D-hydro simulation
- multiple-conserved charges
- implemented multi-component theory
- user-defined EoS, transport coefficients, theories and and geometries
- maybe adaptive mesh in future



See recent talk by Jakob Lohr

The journey to a software package treating coupled-charged transport

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HYDRA

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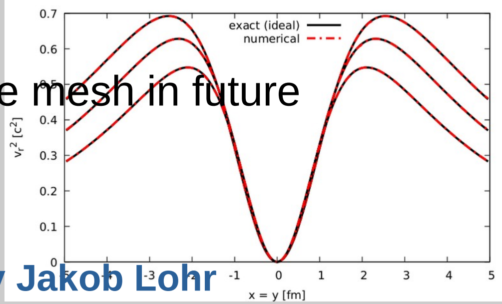

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TraCoLinR

(**TRA**nsport **CO**efficients from **LIN**ear **R**esponse theory)

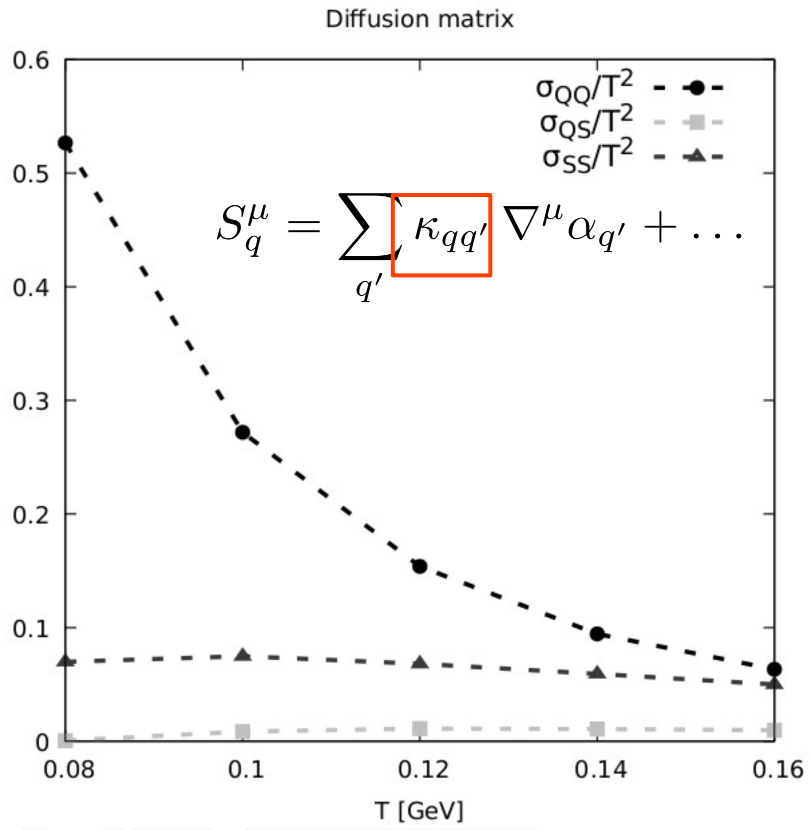
- Calculates <u>all</u> first- and second-order transport coefficients from the linearized Boltzmann equation
- Multi-component systems with isotropic (in-)elastic, s-dependent cross sections
- thermal masses and cross sections possible (e.g. DQPM)
- quantum corrections are planned

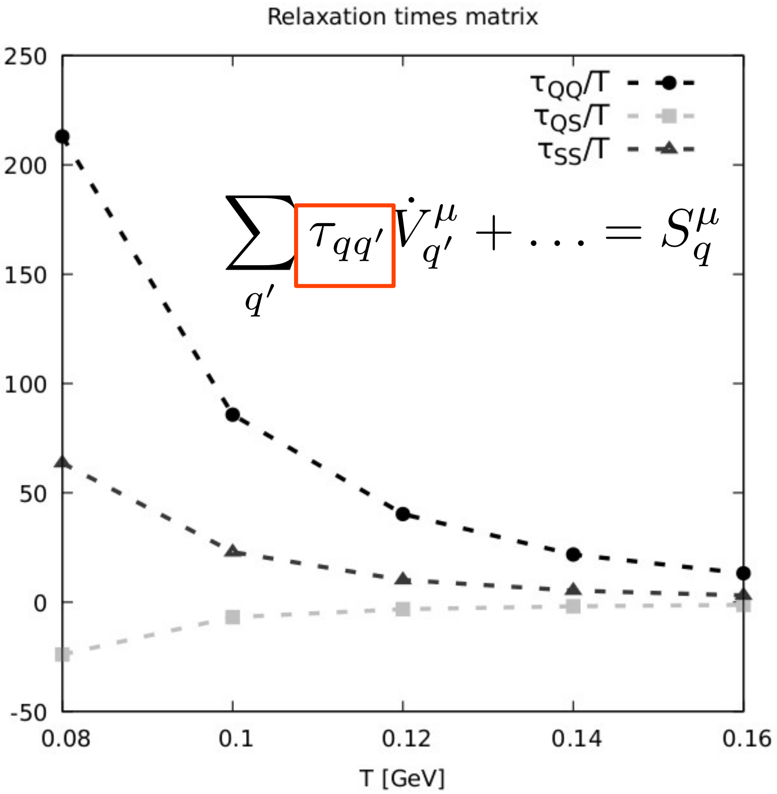
Upcoming publication!

(Fotakis, Lohr, Wagner, Potesnov et al.)

Preliminary results from TraCoLinR

Pion-Kaon gas with resonant cross sections (rho, K*) with hard-sphere “background”





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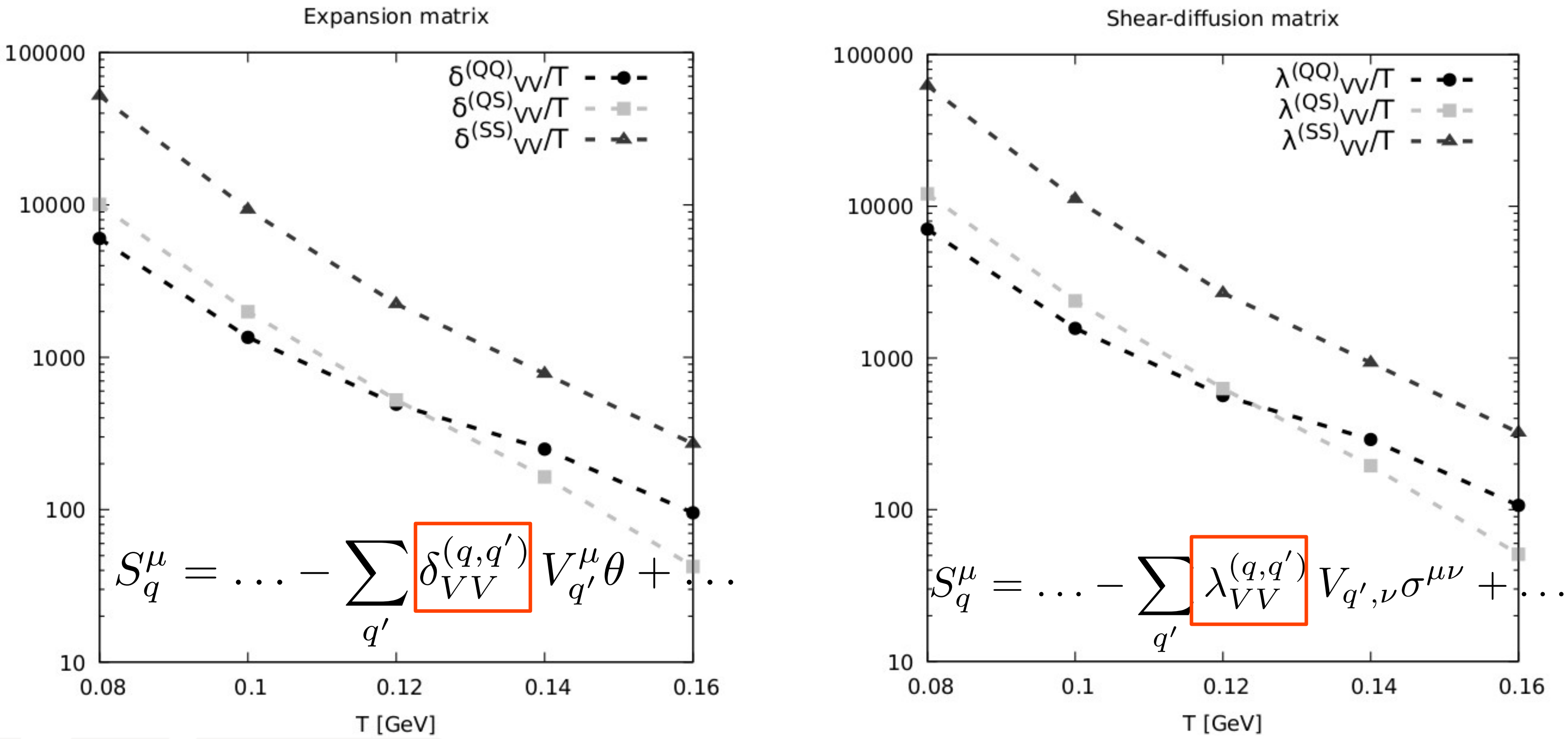

Conclusion

- Derived 2nd-order relativistic fluid dynamic theory for multi-component systems from the Boltzmann equation
- Transport coefficients given explicitly containing all information about particle interactions
- Mixed chemistry correlates particle flow → coupled charge-transport
- Consistency of equation of state, 1st- and 2nd-order transport coefficients is important!
- Implemented derived fluid dynamic theory in (3+1)D-hydro code **HYDRA**
- First preliminary calculations of second-order coefficients of massive mixture with more realistic cross sections with **TraCoLinR**
- **We plan to publish both codes**

Backup

Fluid dynamics of multi-component systems

Hydrodynamics: macroscopic effective field theory of thermal matter close to local equilibrium

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Conservation of Energy and Momentum: $\partial_\mu T^{\mu\nu} = 0$

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Conservation of charge: $\partial_\mu N_q^\mu = 0$

$$N_q^\mu = \sum_i q_i N^\mu = n_q u^\mu + V_q^\mu$$

$$10 + 4N_{\text{ch}} \text{ degrees of freedom}, \quad 4 + N_{\text{ch}} \text{ equations} \quad \rightarrow \quad 6 + 3N_{\text{ch}} \text{ unknowns}$$

What needs to be known:

- Equation of state $P_0 = P_0(\epsilon, n_q), \quad T = T(\epsilon, n_q), \quad \alpha_q = \mu_q/T = \alpha_q(\epsilon, n_q)$
- Equations of motion for dissipative fields & transport coefficients $\Pi, V_q^\mu, \pi^{\mu\nu}$
- Initial state
- Final state: freeze-out and δf-correction

Deriving fluid dynamics from kinetic theory

Denicol et al., PRD 85, 114047 (2012)

On basis of DNMR theory: derivation from the Boltzmann equation with method of moments

Fotakis, Molnar, Niemi, Rischke, Greiner, Phys. Rev. D 106 (2022), 036009

Also refer to: Monnai et al., Nucl. Phys. A847:283-314 (2010) or Kikuchi et al., PRC 92, 064909 (2015)

relativistic Boltzmann eq.

$$k_i^\mu \partial_\mu f_{i,\mathbf{k}} = \mathcal{C}_i[f_i]$$

→

2nd-order (multi-component) hydro

$$\dot{\Pi},\ \dot{V}_q^{\langle\mu\rangle},\ \dot{\pi}^{\langle\mu\nu\rangle}$$

Irreducible off-equilibrium moments obey Boltzmann eq.:

equilibrium, off-equilibrium

$$f_{i,\mathbf{k}} = f_{i,\mathbf{k}}^{(0)} + \delta f_{i,\mathbf{k}}$$

$$\rho_{i,n}^{\mu\nu} = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3 E_{i,\mathbf{k}}} E_{i,\mathbf{k}}^n k_i^{\langle\mu} k_i^{\nu\rangle} \delta f_{i,\mathbf{k}}$$

Problem: infinitely many coupled PDEs.

Aim: Truncate in a well-defined manner (“perturbation theory”)

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"Order-of-magnitude approximation":
relate off-equilibrium moments to the dissipative fields

$$\rho_{i,n}^{\mu\nu} = \frac{\eta_{i,n}}{\eta}\pi^{\mu\nu} + \mathcal{O}(2)$$

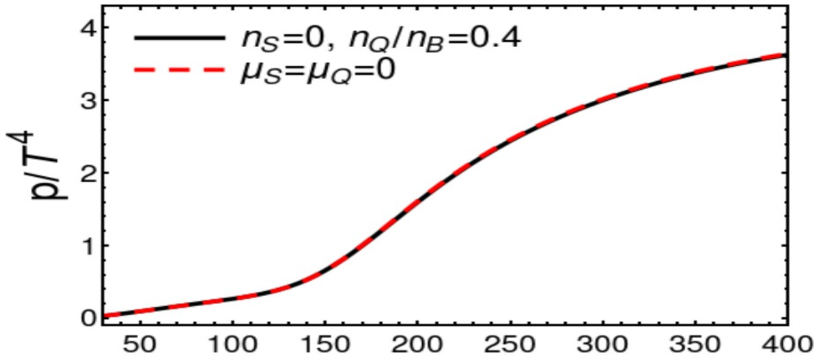
Counting scheme:

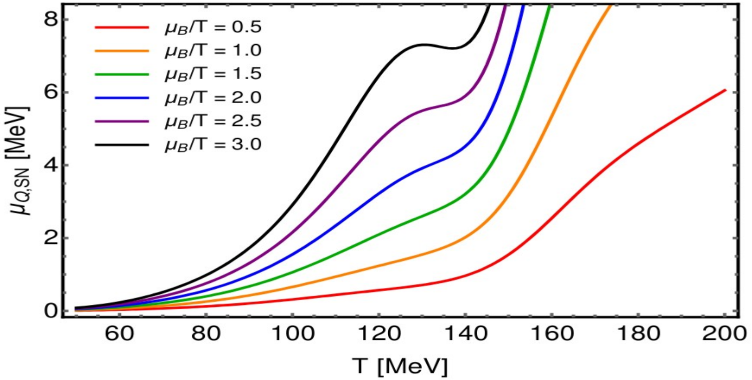
Gradients in velocity, temperature etc. $\sigma^{\mu\nu} \sim \mathcal{O}(1), \mathcal{O}(\mathrm{Kn})$
Dissipative fields $\pi^{\mu\nu} \sim \mathcal{O}(1), \mathcal{O}(\mathrm{Rn}^{-1})$

Equation of state with multiple conserved charges

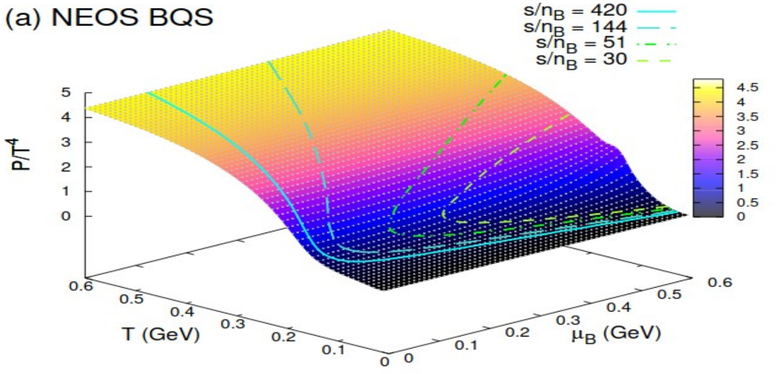
$$P_0(T) \quad \rightarrow \quad P_0(T, \mu_{\mathrm{B}}, \mu_{\mathrm{Q}}, \mu_{\mathrm{S}})$$

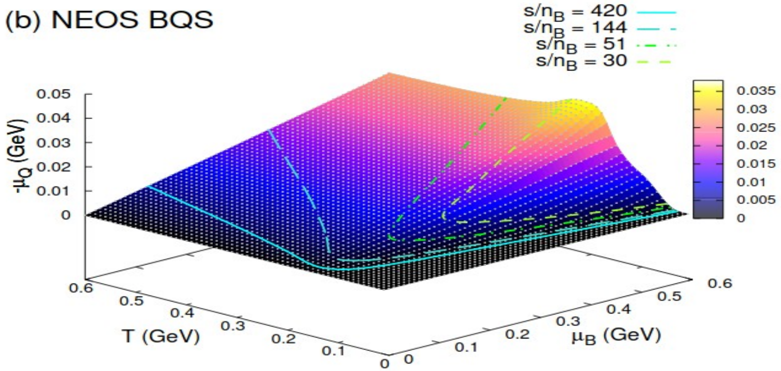
Noronha-Hostler et al., PRC 100, 064910 (2019)





Monnai et al., PRC 100, 024907 (2019)





Computation of transport coefficients
(Example: diffusion coefficients)

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On basis of DNMR theory: derivation from the Boltzmann equation with method of moments

Fotakis, Molnar, Niemi, Rischke, Greiner, Phys. Rev. D 106 (2022), 036009

relativistic Boltzmann eq.

$$k_i^\mu \partial_\mu f_{i,\mathbf{k}} = \mathcal{C}_i[f_i]$$

→

2nd-order (multi-component) hydro

$$\dot{\Pi},\ \dot{V}_q^{\langle\mu\rangle},\ \dot{\pi}^{\langle\mu\nu\rangle}$$

$$\mathcal{C}_{i,n-1}^{\langle\mu\rangle} \equiv \int \frac{\mathrm{d}^3\mathbf{k}_i}{(2\pi)^3 E_{i,\mathbf{k}}} E_{i,\mathbf{k}}^{n-1} k_i^{\langle\mu\rangle} \mathcal{C}_i[f_i]$$

$$= -\sum_{m=0}^{\infty} \sum_j \mathcal{C}_{ij,nm}^{(1)} \rho_{j,m}^\mu + \text{non-linear terms}$$

Entries of „collision matrix“ (for diffusive moments)

$$\kappa_{qq'} = \sum_{n=0}^{\infty} \sum_{i,j=1}^{N_{\text{species}}} \left(\mathcal{C}^{(1)}\right)^{-1}_{ij,0n} q_i \left(q'_j J_{j,n+1,1} - \frac{n_{q'}}{\epsilon + P_0} J_{j,n+2,1} \right)$$

Diffusion coefficient matrix! (equivalent to our PRL and PRD expression)

Greif, Fotakis et al., PRL 120, 242301 (2018)
Fotakis, Greif et al., PRD 101, 076007 (2020)
Fotakis, Soloveva et al, PRD 104, 034014 (2021)

Yet another hydro code - „Hydra“

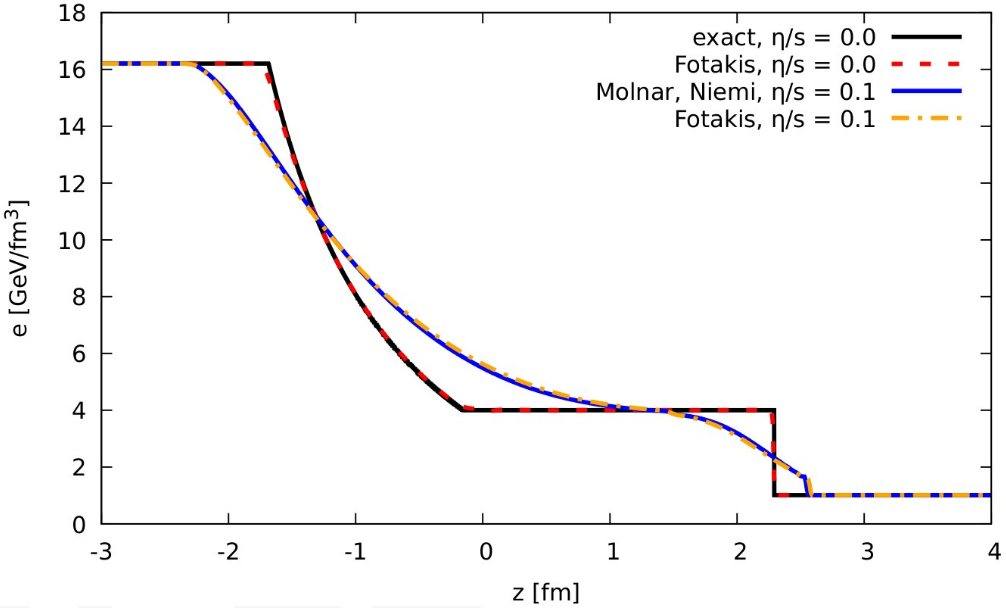
Core features:

- (3+1)D-hydro – optimized reduction to 2D and 1D
- (v)SHASTA solver
- Shear-stress and multiple conserved charges (2 charges)
- Ultrarelativistic, tabled and/or any user-defined equations of state
- DNMR theory, this theory, and/or any user-defined theory
- any (tabled, user-defined) transport coefficients
- Curve-linear geometry (so far Cartesian and Hyperbolic coordinates)
- state of the art unit and physical tests
- available in the CRC-TR211 collaboration soon (hopefully)

Yet another hydro code - „Hydra“

Core features:

- (3+1)D-hydro – optimized reduction to 2D and 1D
- (v)SHASTA solver



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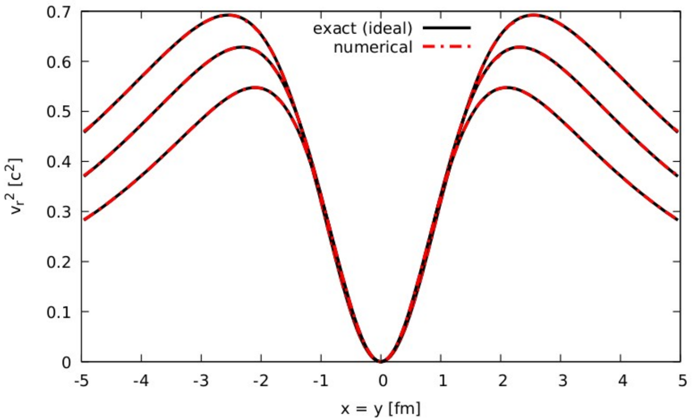


Equation of State - details

- Hadronic system including lightest 19 species

$$\pi^{\pm}, \pi^0, K^{\pm}, K^0, \bar{K}^0, p, \bar{p}, n, \bar{n}, \Lambda^0, \bar{\Lambda}^0, \Sigma^0, \bar{\Sigma}^0, \Sigma^{\pm}, \bar{\Sigma}^{\pm}$$

- Assume classical statistics and non-interacting limit

$$P_0(T, \{\mu_q\}) \equiv \frac{1}{3} \sum_{i=1}^{N_{\text{species}}} \int \frac{\mathrm{d}p^3}{(2\pi)^3 E_{i,p}} \left(E_{i,p}^2 - m_i^2\right) g_i \exp(-E_{i,p}/T + \sum_q q_i \alpha_q)$$

- Only assume baryon number and strangeness, neglect electric charge

- Tabulate state variables over energy density and net charge densities

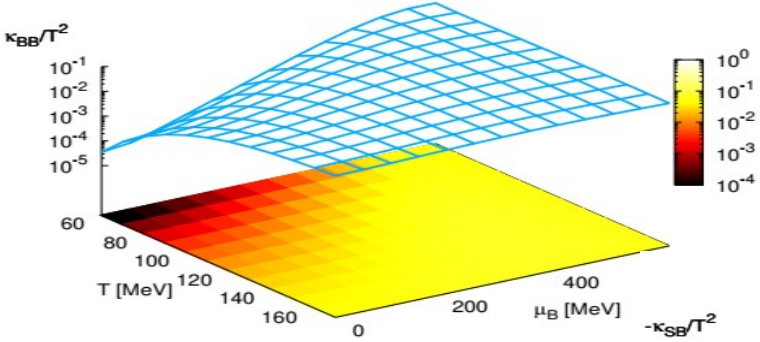
$$T \equiv T(\epsilon, \{n_q\}), \quad \mu_q \equiv \mu_q(\epsilon, \{n_q\}), \quad P_0 \equiv P_0(\epsilon, \{n_q\})$$

Diffusion coefficient matrix - details

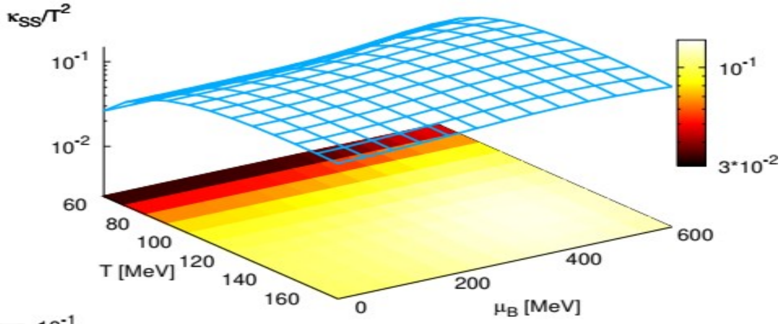
$$\begin{pmatrix} V_B^\mu \\ V_S^\mu \end{pmatrix} \sim \begin{pmatrix} \kappa_{BB} & \kappa_{BS} \\ \kappa_{SB} & \kappa_{SS} \end{pmatrix} \begin{pmatrix} \nabla^\mu \alpha_B \\ \nabla^\mu \alpha_S \end{pmatrix}$$

- Matrix is symmetric

L. Onsager, Phys. Rev. **37**, 405 (1931) & Phys. Rev. **38**, 2265 (1931)

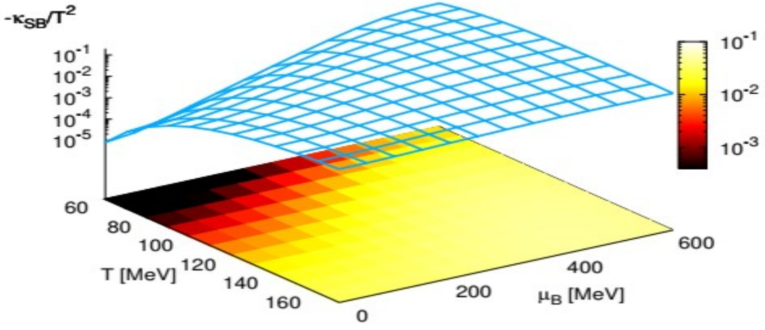


- Elastic isotropic cross sections from PDG, SMASH, GiBUU, UrQMD



κ_{SB} is negative and has similar magnitude as κ_{BB}

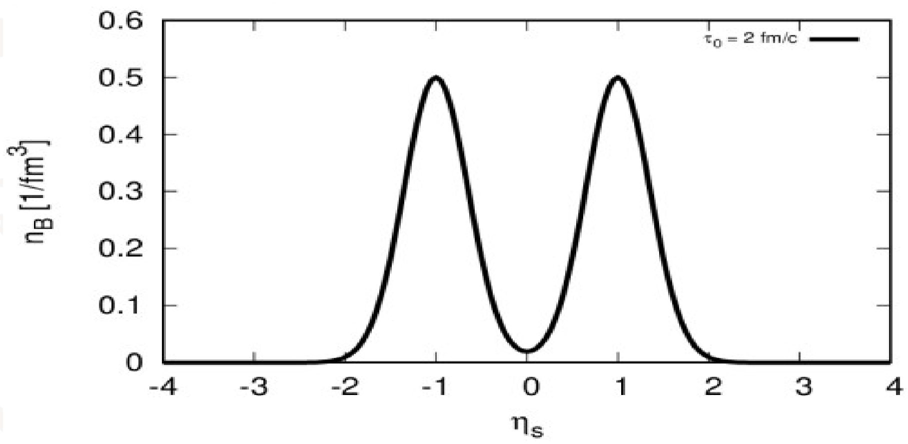
$\Longrightarrow$ significant coupling?

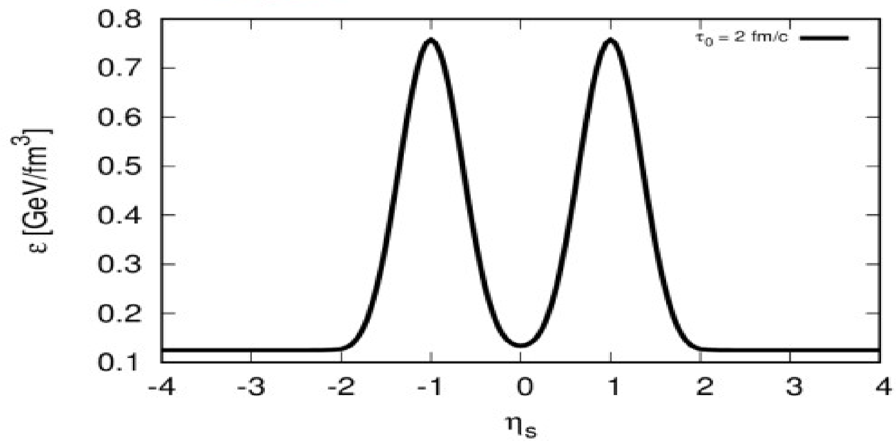


- Tabulate coefficient matrix over $T,\ \mu_B,\ \mu_S$
- $\mu_Q = 0$

Initial conditions - details

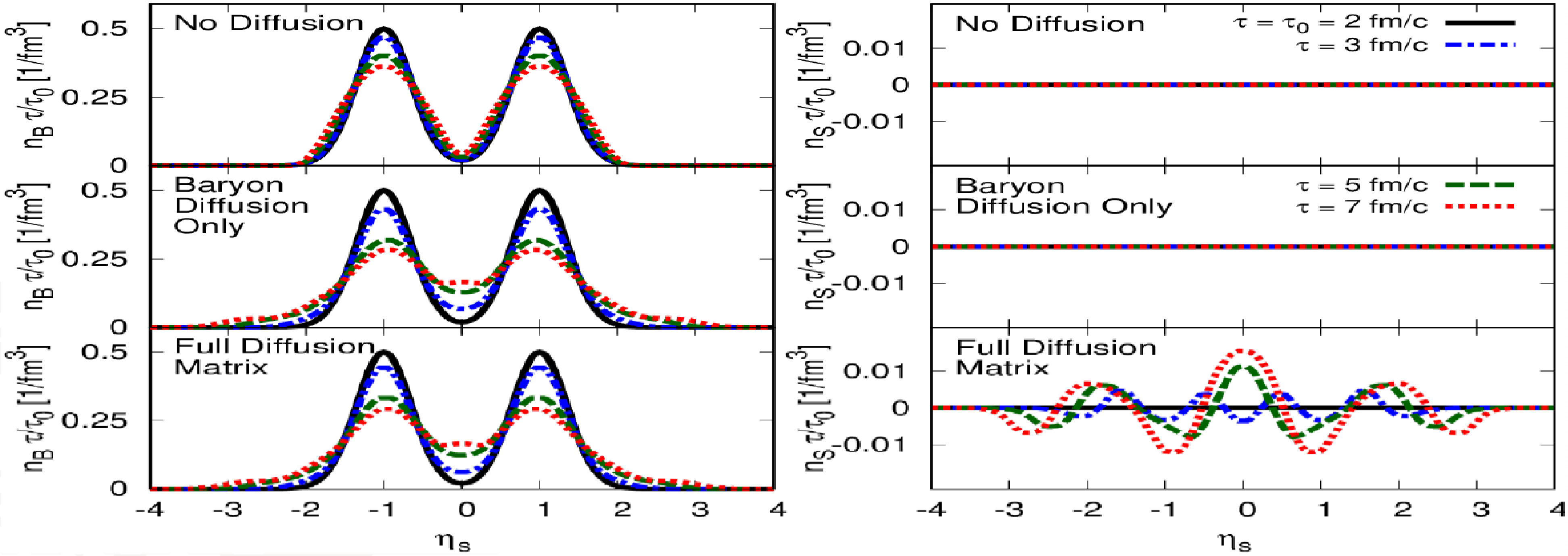
- $\tau_0 = 2\ \mathrm{fm}/c$
- Initially: no dissipation and only **Bjorken scaling flow**
- Temperature = 160 MeV
- Double-gaussian profile in net baryon number
- From **EoS**: get energy density





Coupled charge-transport

Greif, Fotakis et al., PRL 120, 242301 (2018)
Fotakis, Greif et al., PRD 101, 076007 (2020)



Simplistic case study: no viscosity, diffusion only, no 2nd-order terms, conserved B and S charge, classical, hadronic system (19 species), realistic binary elastic cross sections

Hydrodynamic (1+1)D-simulation

$$\Pi \equiv 0, \quad \pi^{\mu\nu} \equiv 0, \quad \tau_q \dot{V}_q^{\langle\mu\rangle} + V_q^\mu = \sum_{q'} \kappa_{qq'} \nabla^\mu \left(\frac{\mu_{q'}}{T}\right)$$

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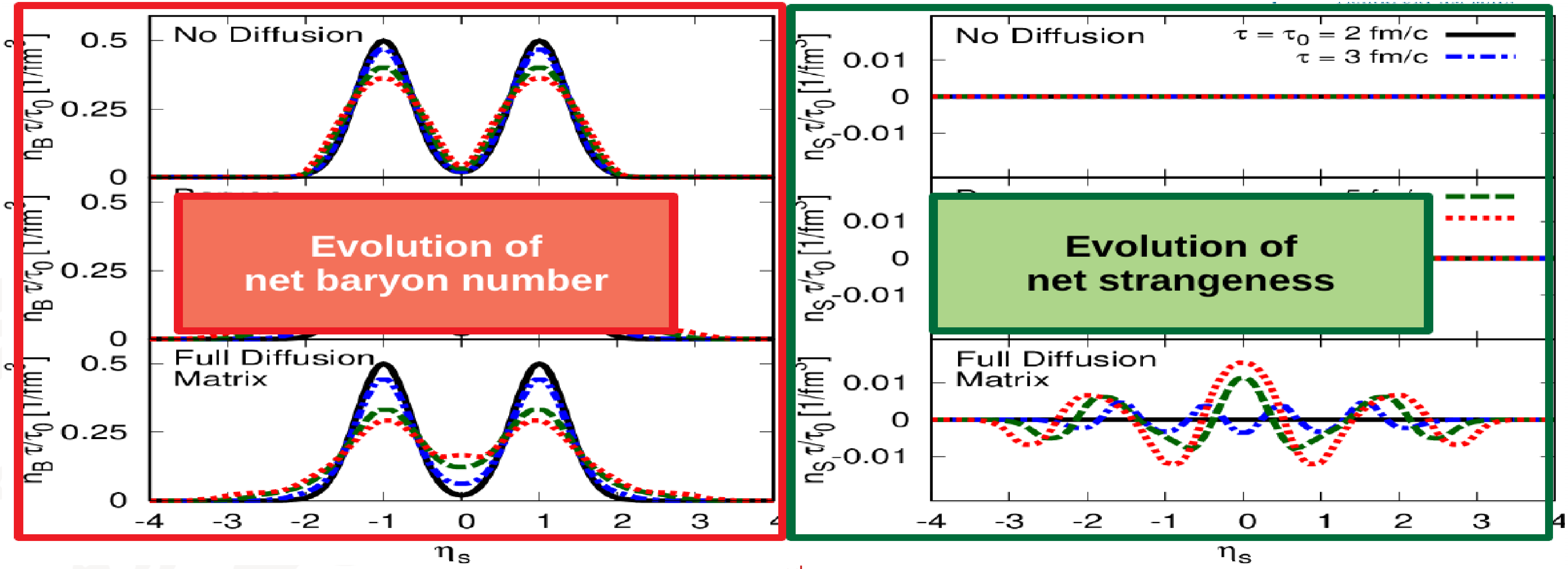

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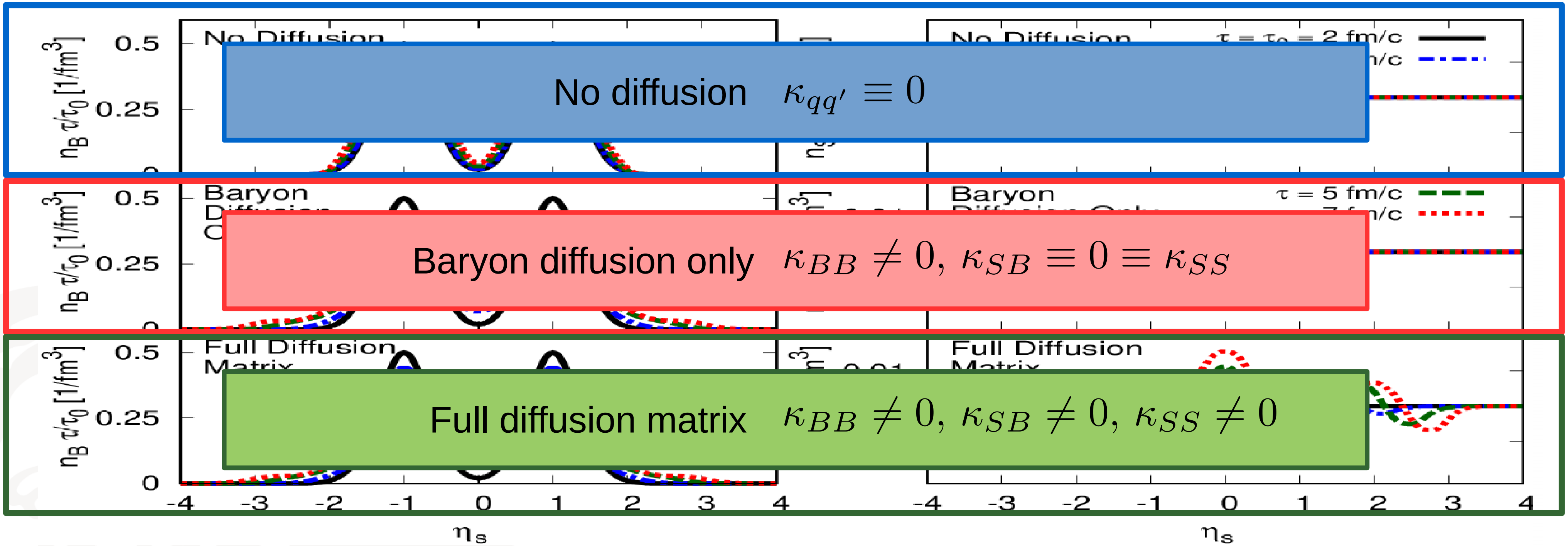

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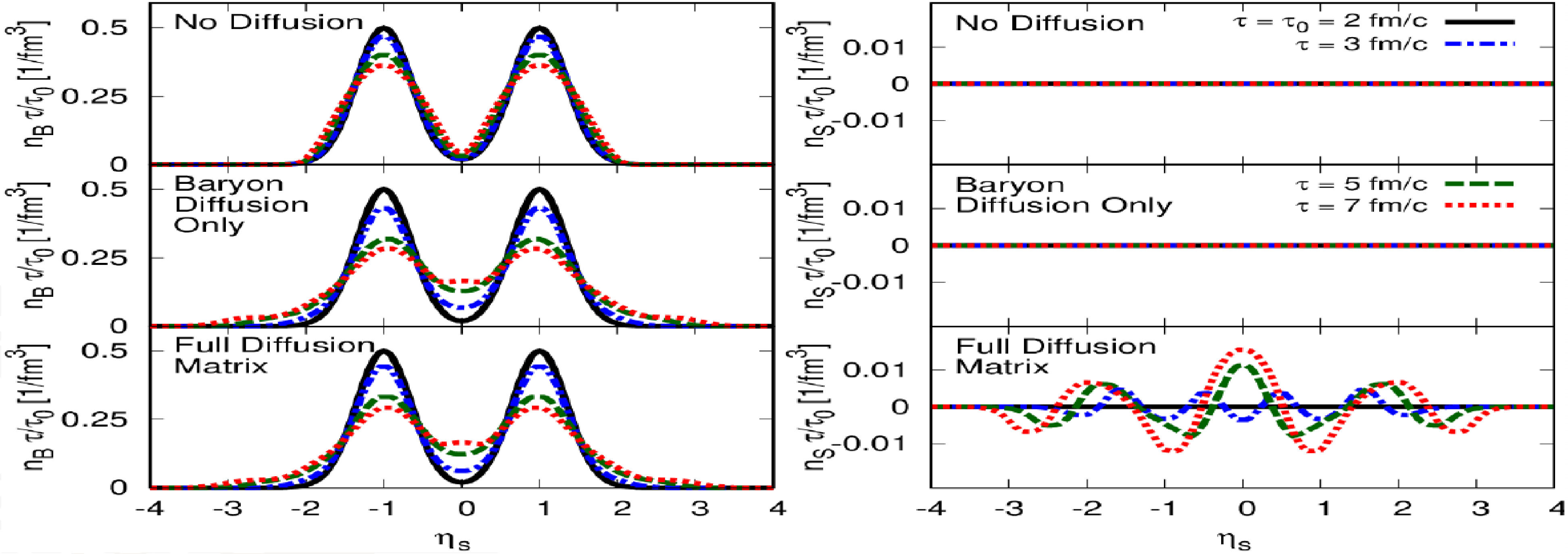

Mixed chemistry couples diffusion currents and introduces charge-correlation through EoS

e.g.:

$$\mu_S \equiv \mu_S(\epsilon, n_B, n_S)$$
$$\nabla^\mu \alpha_S \sim \nabla^\mu n_B$$

→ Generation of domains of non-vanishing local net charge (here net strangeness)!