Fluid dynamic calculations with HYDRA

Transport Meeting

Jakob Lohr

25. 01. 2024

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Supervisors: Prof. Dr. Carsten Greiner, Jan Fotakis
lohr@itp.uni-frankfurt.de

Overview

1. Theory
2. Code
3. Tests
4. Transport Coefficients
5. Baryon Stopping
6. Summary, Outlook

Fundamental equations of relativistic hydrodynamics

energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

charge conservation of charge q

$$\partial_\mu N^\mu_q = 0 \qquad e.g. \quad \partial_\mu N^\mu_B = 0$$

normalised fluid 4-velocity

$$u_\mu u^\mu = 1$$

projector orthogonal to u^μ

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

Energy-Momentum Tensor and Charge Currents

energy-momentum tensor

$$T^{\mu\nu} = eu^\mu u^\nu - 2W^{(\mu}u^{\nu)} - (P_{eq} + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

charge 4-current of charge q

$$N_q^\mu = n_q u^\mu + V_q^\mu$$

Landau matching

$$e = e_{eq}, \qquad n_q = n_{q,eq}$$

choice of frame

$$T^{\mu\nu}u_\nu = eu^\mu \quad \Rightarrow \quad W^\mu = 0$$

Equations of Motion

energy conservation equation

$$\dot{e} = -e\theta - (P_{eq} + \Pi)\theta + \pi^{\mu\nu}\sigma_{\mu\nu}$$

momentum conservation equations

$$(e + P_{eq} + \Pi)\dot{u}^{\mu} = \nabla^{\mu}(P_{eq} + \Pi) - \Delta^{\mu}_{\alpha}\partial_{\nu}\pi^{\alpha\nu}$$

charge conservation equations

$$\dot{n}_q = -n_q\theta - \partial_{\mu}V^{\mu}_q$$

Multiple Conserved Charges

Navier-Stokes limit

$$\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu}, \quad V_q^{\mu} = \sum_{q'}^{\{B,Q,S\}} \kappa_{qq'} \nabla^{\mu} \frac{\mu_{q'}}{T}, \quad \Pi = -\zeta\theta$$

Diffusion matrix gains multiple entries in multi component description

charge diffusion equation of motion

$$\sum_{q}^{\{B,Q,S\}} \tau_{q'q} \dot{V}_q^{\langle\mu\rangle} + V_{q'}^{\mu}$$

$$= \sum_{q}^{\{B,Q,S\}} \kappa_{q'q} \nabla^{\mu} \frac{\mu_q}{T} - \sum_{q}^{\{B,Q,S\}} \tau_{q'q} V_{q,\nu} \omega^{\nu\mu} - \sum_{q}^{\{B,Q,S\}} \delta_{VV}^{(q'q)} V_q^{\mu} \theta + [\ldots]$$

Multiple Conserved Charges

Navier-Stokes limit

$$\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu}, \quad V_q^\mu = \sum_{q'}^{\{B,Q,S\}} \kappa_{qq'} \nabla^\mu \frac{\mu_{q'}}{T}, \quad \Pi = -\zeta\theta$$

Diffusion matrix gains multiple entries in multi component description

charge diffusion equation of motion

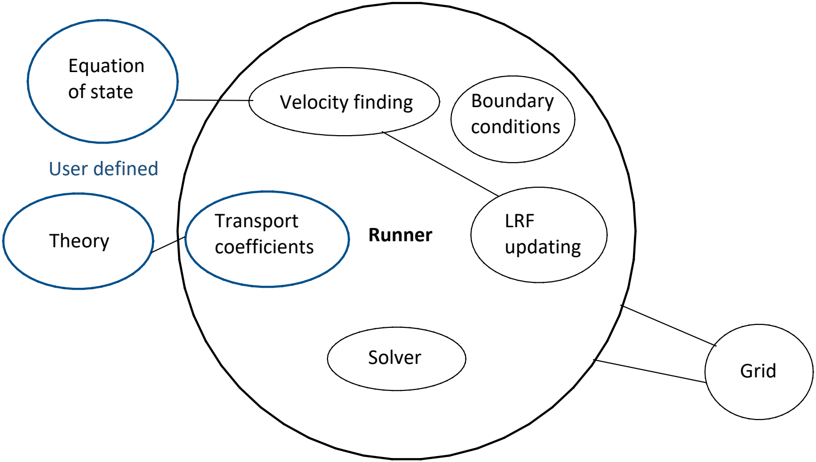
$$\sum_{q}^{\{B,Q,S\}} \tau_{q'q} \dot{V}_q^{\langle\mu\rangle} + V_{q'}^\mu$$

$$= \sum_{q}^{\{B,Q,S\}} \kappa_{q'q} \nabla^\mu \frac{\mu_q}{T} - \sum_{q}^{\{B,Q,S\}} \tau_{q'q} V_{q,\nu}\omega^{\nu\mu} - \sum_{q}^{\{B,Q,S\}} \delta_{VV}^{(q'q)} V_q^\mu \theta + [\ldots]$$

Concept

- HYDRA is written in C++20
- use of multiple conserved charges
- reduction of redundancies
- modularity of problem specific components
  (e.g. user defined equation of state)
- eventually performance increase

Functionality



Bjorken test

boost inveriance

system is boost invariant along the z-direction

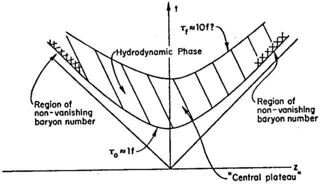
Milne coordinates

description in hyperbolic coordinates

$$ds^2 = -d\tau^2 + dx^2 + dy^2 + \tau^2 d\eta^2$$

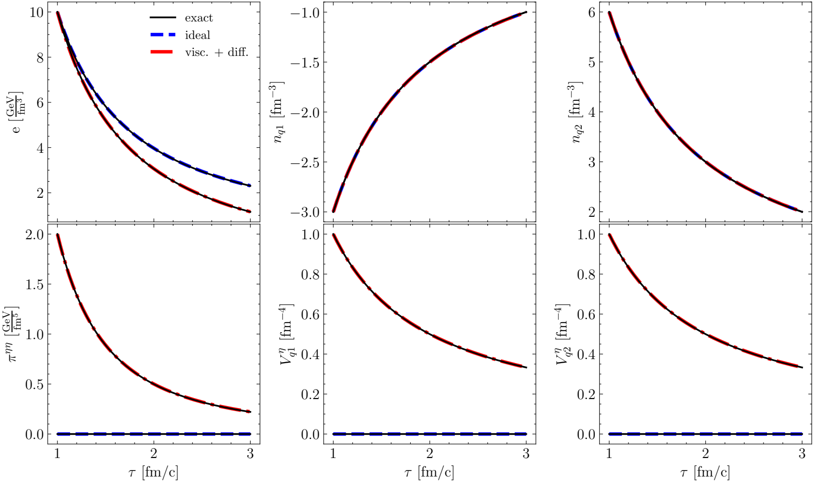
$$\tau = \sqrt{t^2 - z^2} \qquad \eta = \frac{1}{2} ln\left(\frac{t+z}{t-z}\right)$$

$$V_z = \frac{z}{t}$$

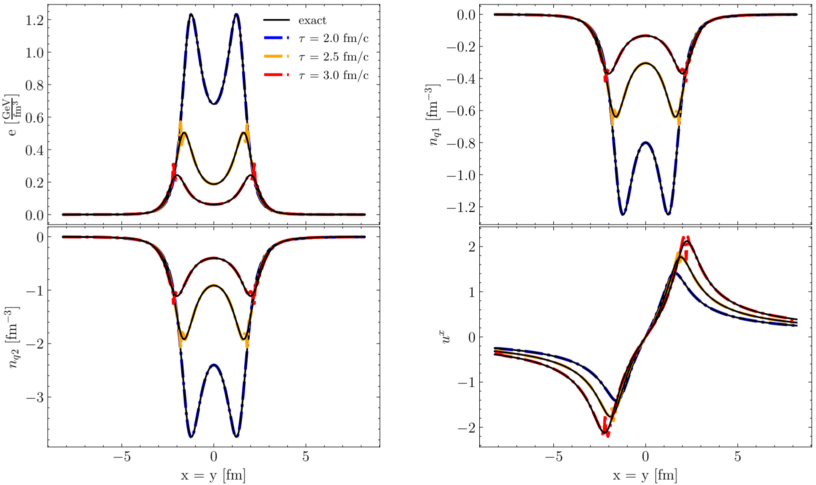


taken from J.D. Bjorken, Phys.Rev.D 27 (1983), 140-151

Bjorken test



Gubser test


exact
τ = 2.0 fm/c
τ = 2.5 fm/c
τ = 3.0 fm/c
e [GeV/fm³]
n_q1 [fm⁻³]
n_q2 [fm⁻³]
u^x
x = y [fm]
x = y [fm]

Riemann test

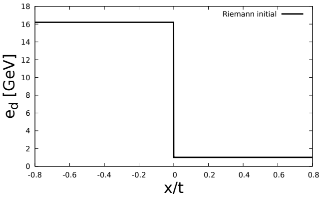
two different mediums

system consists of two initially seperated mediums

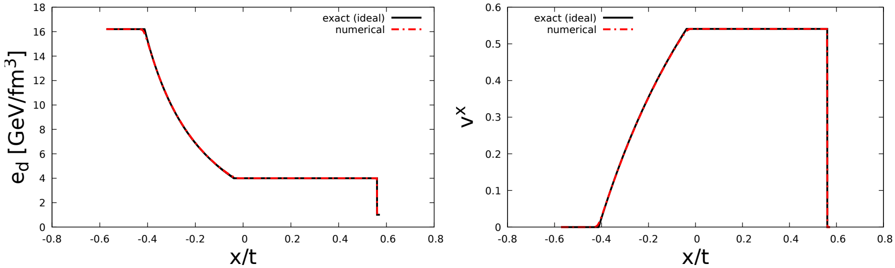
1+1 dimensional

we only consider the x-direction of the system

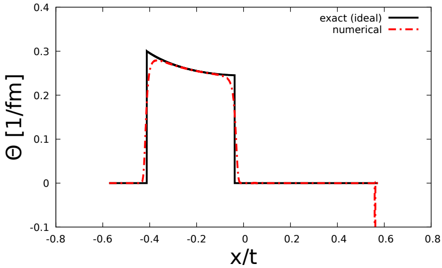
$$T_1 = 0.4\mathrm{GeV} \quad T_2 = 0.2\mathrm{GeV}$$

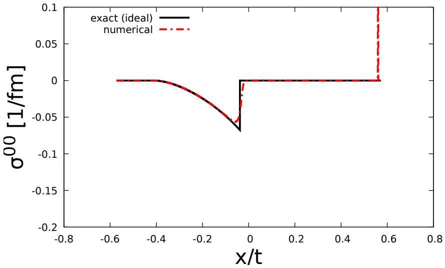


Riemann test



Riemann test





Transport Coefficients

inverse temperature and thermal potential

$$\beta = \frac{1}{T}, \qquad \alpha_i = \frac{\mu_i}{T}$$

total pressure

$$P = \sum_{i=1}^{N_{spec}} P_i, \qquad P_i = \frac{g_i}{\pi^2} e^{\alpha_i} T^4$$

charge concentrations

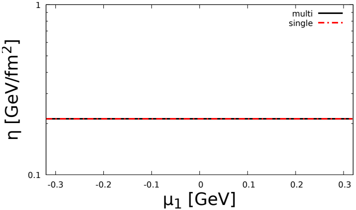
$$c_q = \sum_{i=1}^{N_{spec}} q_i \frac{P_i}{P}, \qquad c_{qq'} = \sum_{i=1}^{N_{spec}} q_i q_i^{'} \frac{P_i}{P}$$

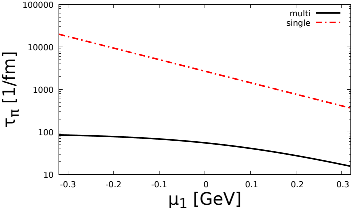
coefficients from [PRD D106, 036009 (2022)]

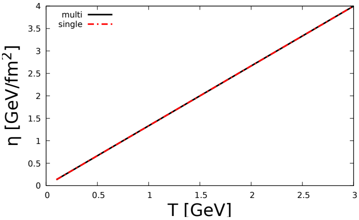
constant values for the following graphs

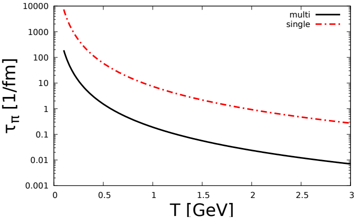
$$\mu_1 = \mu_2 = 0.4\mathrm{GeV} \quad T = 0.16\mathrm{GeV} \quad \sigma_{tot} = 1 fm^2$$

Transport Coefficients





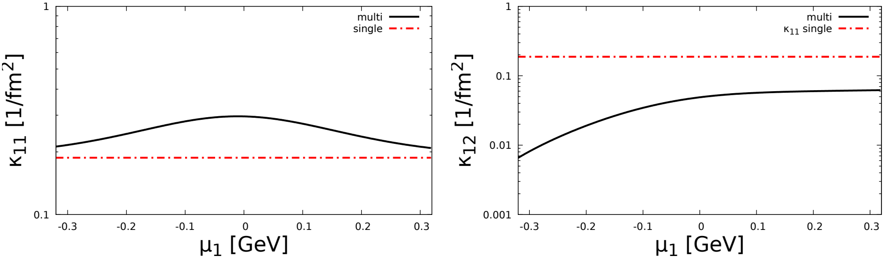




$$\eta = \frac{4}{3}\frac{1}{\sigma_{tot}\beta}$$

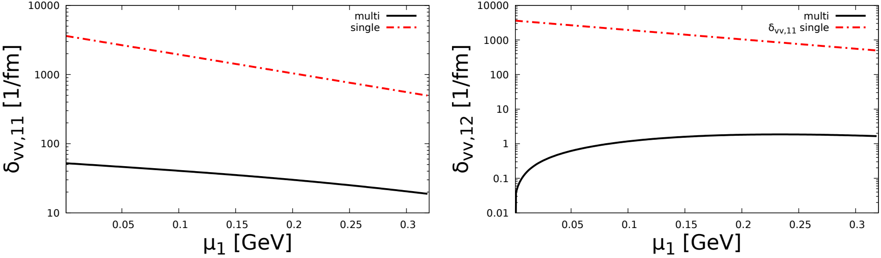
$$\tau_\pi = \frac{5}{3}\frac{1}{\sigma_{tot}\beta P}$$

Transport Coefficients



$$\kappa_{qq'} = \frac{8}{17\sigma_{tot}}\left(c_{qq'} - \frac{7}{128}c_q c_{q'}\right)$$

Transport Coefficients



$$\delta_{vv}^{(qq')} = \frac{1}{\beta \sigma_{tot}^2 P} \sum_{q''}^{\{B,Q,S\}} \left(\kappa^{-1}\right)_{q''q} \left[\frac{640}{867} c_{q'q''} - \frac{17551}{55488} c_{q'} c_{q''} + \frac{52}{289} \left(c_{q'q''} - c_{q'} c_{q''} \right) \right.$$

$$\left. \frac{4P}{\beta} \left(\tau_{00} + \frac{\beta}{4} \sum_{q'''}^{\{B,Q,S\}} \tau_{oq'''} c_{q'''} \right) \right] - \frac{4}{\beta \sigma_{tot}^2} \sum_{q''}^{\{B,Q,S\}} \left\{ \frac{\partial}{\partial \alpha_{q''}} \left[\sum_{q'''}^{\{B,Q,S\}} \left(\kappa^{-1}\right)_{q'''q} \right] \right\} + [\ldots]$$

System

Quark gas
: The considered fluid consists of up- down- and strange-quarks

Equation of state
: Ideal, ultrarelativistic

Constant crosssection
: Set to $1fm^2$

Constant diffusion coefficients
: Set to $1fm^2$

Initial Conditions

Data from McDIPPER [arXiv:2308.1171]

Calculated by Oscar Garcia-Montero

Averaged energy- and number-densities for quarks and gluons for 4000 events

3d tables in hyperbolic coordinates (η_s, x, y)

Initial conditions

Average over transversal plane

Averaged tabled data in (x, y) plane to obtain only η_s dependence transforamtion into B Q S densities

Transformation into BQS description:

Baryon density

$$n_B = \frac{1}{3}(n_u + n_d + n_s)$$

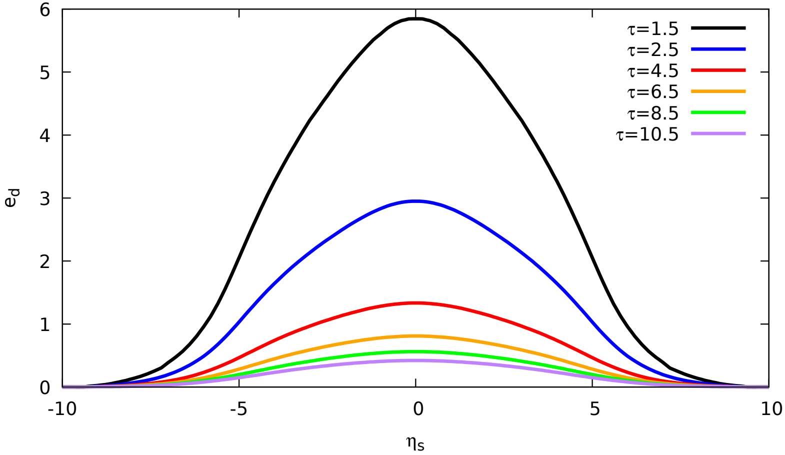
Electrical charge density

$$n_Q = \frac{1}{3}(2n_u - n_d - n_s)$$

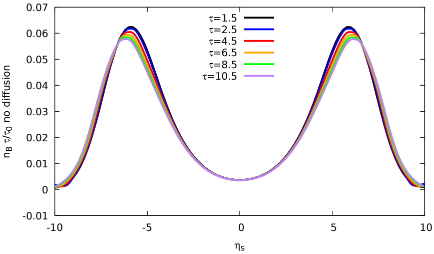
Strangeness density

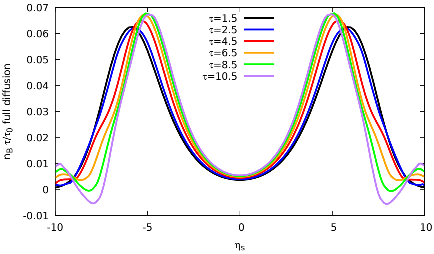
$$n_S = -n_s$$

Densities

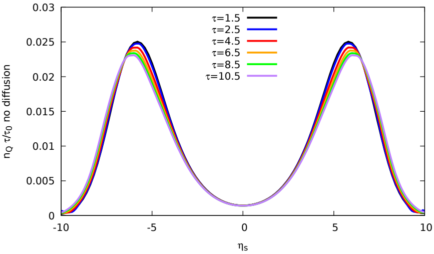


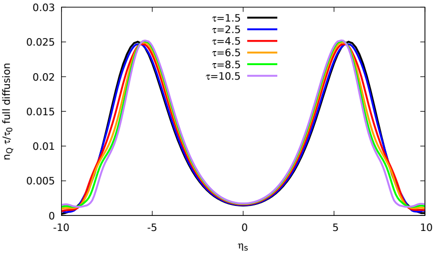
Densities


0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
-10
-5
0
5
10
$n_B\ \tau/\tau_0$ no diffusion
η_s
τ=1.5
τ=2.5
τ=4.5
τ=6.5
τ=8.5
τ=10.5


0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
-10
-5
0
5
10
$n_B\ \tau/\tau_0$ full diffusion
η_s
τ=1.5
τ=2.5
τ=4.5
τ=6.5
τ=8.5
τ=10.5

Densities


τ=1.5
τ=2.5
τ=4.5
τ=6.5
τ=8.5
τ=10.5
n_Q τ/τ_0 no diffusion
η_s
0.03
0.025
0.02
0.015
0.01
0.005
0
-10
-5
0
5
10


τ=1.5
τ=2.5
τ=4.5
τ=6.5
τ=8.5
τ=10.5
n_Q τ/τ_0 full diffusion
η_s
0.03
0.025
0.02
0.015
0.01
0.005
0
-10
-5
0
5
10

Summary, Outlook

Summary

- Implemented equations of motion from Fotakis et al [PRD D106, 036009 (2022)] for relativistic mixtures in (3+1)d
- Investigated transport coefficients of a simplistic ultrarelativistic, conformal mixture.
- Reviewed the diffusion towards midrapidity for a (1+1)d system

Outlook

- Calculate transport coefficients for massive systems
- Investigate the effects of baryon diffusion and coupled-charge transport for additional transport coefficients

kinetic theory

charge current

$$N_i^\mu = \int \frac{d^3\vec{k}_i}{(2\pi)^3 k_i^0} k_i^\mu f_{i,k} \qquad N_q^\mu = \sum_{i=1}^{N_{spec}} q_i N_i^\mu$$

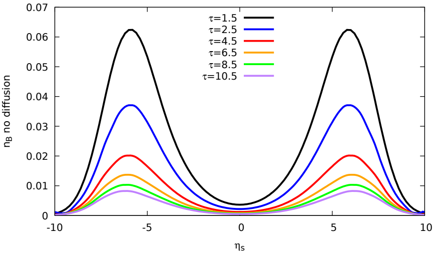
energy-momentum tensor

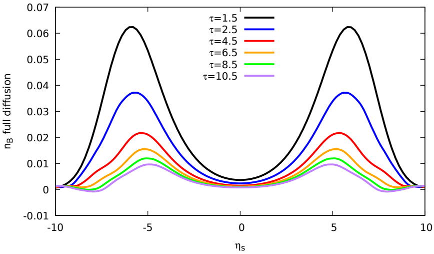
$$T_i^{\mu\nu} = \int \frac{d^3\vec{k}_i}{(2\pi)^3 k_i^0} k_i^\mu k_i^\nu f_{i,k} \qquad T^{\mu\nu} = \sum_{i=1}^{N_{spec}} T_i^{\mu\nu}$$

expansion in irreducible moments

$$f_k = f_{eq}\left[1 + \tilde{f_{eq}} \sum_{l=0}^{\infty} \sum_{n=0}^{N_l} \mathcal{H}_n^{(l)} \rho_n^{\mu_1 \dots \mu_l} k_{\langle \mu_1 \dots} k_{\mu_l \rangle}\right]$$

Densities


0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-10
-5
0
5
10
n_B no diffusion
η_s
τ=1.5
τ=2.5
τ=4.5
τ=6.5
τ=8.5
τ=10.5


0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-0.01
-10
-5
0
5
10
n_B full diffusion
η_s
τ=1.5
τ=2.5
τ=4.5
τ=6.5
τ=8.5
τ=10.5