Antrittsvorlesung

Frankfurt, October 2010

Nuclear Matter between Heaven and Earth: The QCD Phase Diagram

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GOETHE UNIVERSITÄT FRANKFURT AM MAIN

- Introduction to QCD and lattice simulations
- Phase diagram: the many faces of QCD
- Computational difficulties: we are still at the very beginning

supported by

Bundesministerium für Bildung und Forschung

Deutsche Forschungsgemeinschaft DFG

HIC for FAIR Helmholtz International Center

QCD according to Google...

QCD OF AMERICA
The Dental and Vision Benefit Program

Application: QCD

Category: Audio

brings the user a huge arsenal of quality features, and uses simplicity as its decoy. QCD is truly the most unique player available.

- The EOS 20D has a rear QCD (Quick Control Dial) which can be used to set exposure compensation, flash compensation, shutter speed or aperture

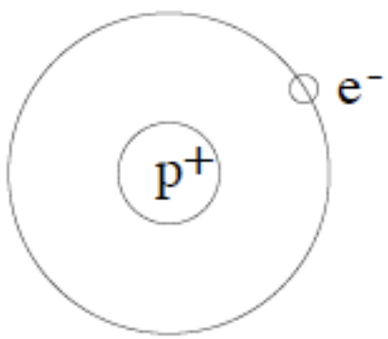
Quantum Chromodynamics, theory of strong interactions

$$\mathcal{L}_{\text{QCD}} = \frac{1}{4g^2}\,\mathrm{Tr}\,F_{\mu\nu}F_{\mu\nu} + \sum_{i=1}^{3}\bar{\psi}_i[\gamma_\mu D_\mu + m_i]\psi_i$$

$$m_u \sim 3\text{MeV}, \quad m_d \sim 6\text{MeV}, \quad m_s \sim 120\text{MeV} \;\Rightarrow\; N_f \approx 2+1$$

weak vs. strong coupling:

QED



$M_e = 0.5$ MeV

$M_p = 938$ MeV

$E_{bind} = 13.6$ eV

hydrogen (e.m. force)

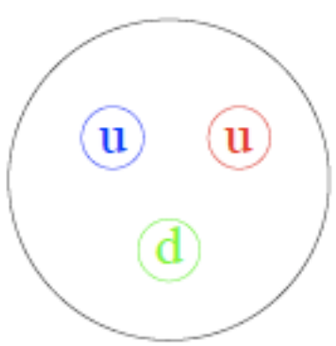
$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$$

photons → gluons

e,p → quarks

gauge group U(1) → gauge group SU(3)

QCD



$M_u \sim 3$ MeV

$M_d \sim 6$ MeV

$M_p = 938$ MeV

proton (strong force)

$$\alpha_s = \frac{g^2}{4\pi} \approx 1$$

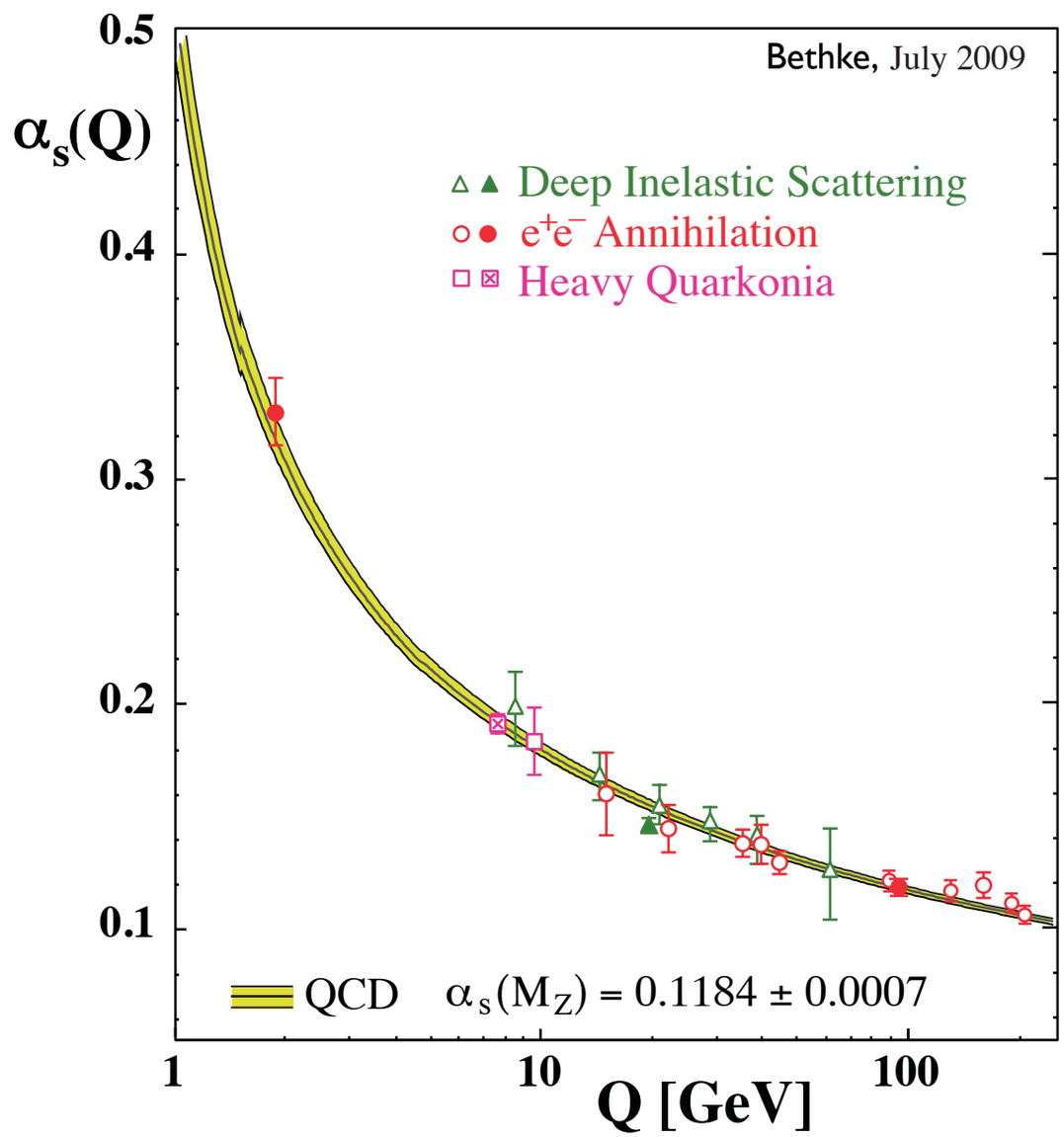
⇒ **Confinement, non-perturbative**

gluon self-interaction!

Running coupling + asymptotic freedom

Coupling strength changes with energy scale: Gross, Politzer, Wilczek Nobel 2004

non-perturbative,
nuclear physics



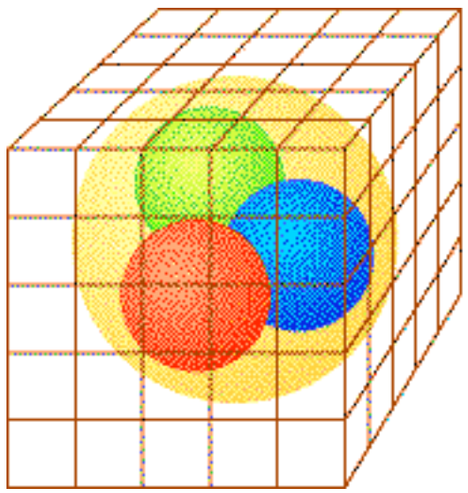
perturbative (QED-like),
verified in
high energy scattering!

energy scale of physical process

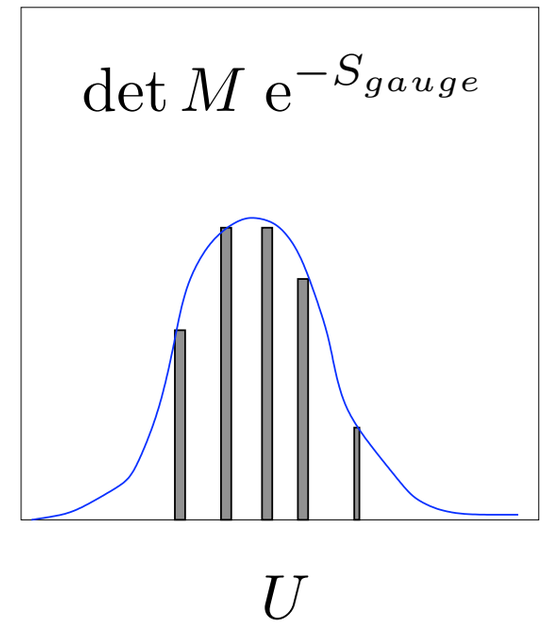
Lattice gauge theory + Monte Carlo method

QCD partition fcn: $$Z = \int DU \prod_f \det M(\mu_f, m_f; U)\, \mathrm{e}^{-S_{gauge}(\beta;U)}$$

links=gauge fields



$4d : N_s^3 \times N_t$



lattice spacing a<< hadron << L !

typically $> 10^8 - 10^{10}$ dim. integral

Monte Carlo, importance sampling

Euclidean time:

$$T = \frac{1}{aN_t}$$

Continuum limit: $N_t \to \infty, a \to 0$

Phase diagram: $N_t = 4, 6$

$a \sim 0.3, 0.2$ fm

Light fermions expensive, cost$(\det M) \sim \dfrac{1}{m_q^n}$

Simulate with different quark masses, extrapolate to physical point

Lattice gauge theory + Monte Carlo method

QCD partition fcn: $$Z = \int DU \prod_f \det M(\mu_f, m_f; U)\, \mathrm{e}^{-S_{gauge}(\beta;U)}$$

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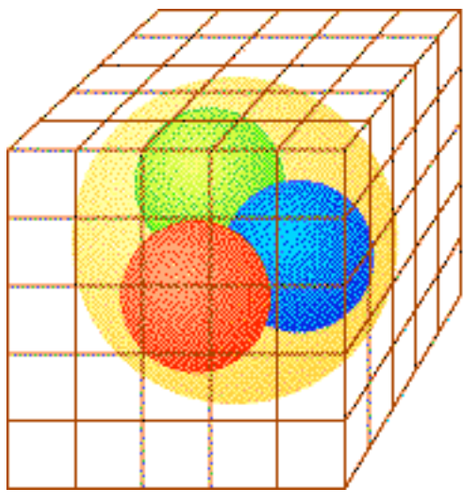
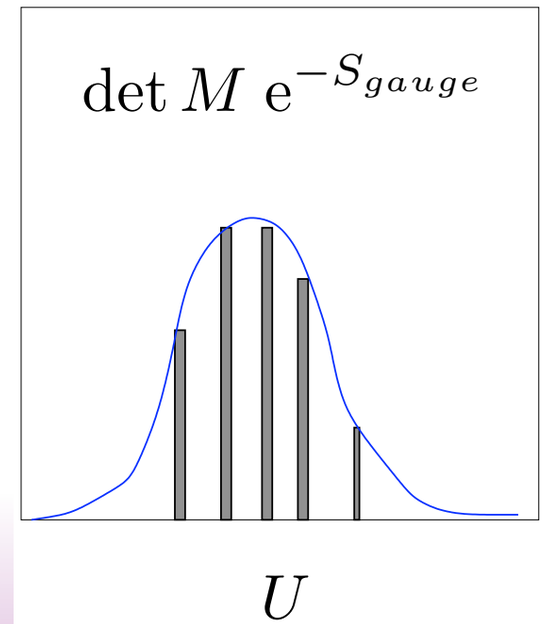


$4d : N_s^3 \times N_t$



lattice spacing a<< hadron << L !

typically $> 10^8 - 10^{10}$ dim. integral

SUPERCOMPUTING!

soon
LOEWE - CSC
at Goethe-Uni

Euclidean time:
$$T = \frac{1}{aN_t}$$

ase diagram: $N_t = 4, 6$

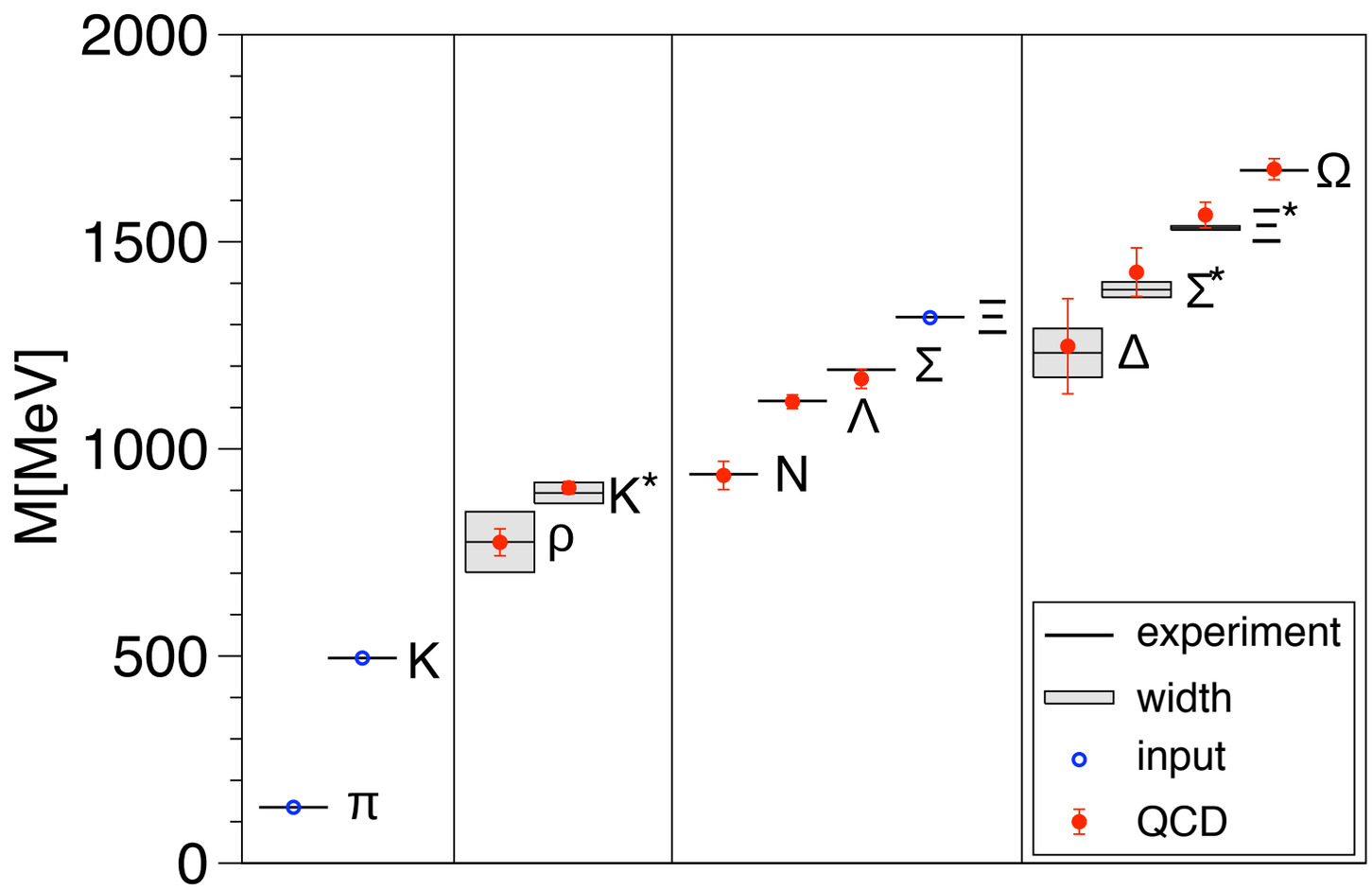
$a \sim 0.3, 0.2$ fm

Light fermions expensive, cost$(\det M) \sim \frac{1}{m_q^n}$

Simulate with different quark masses, extrapolate to physical point

Nuclear matter as we know it: light hadron spectrum from the lattice

BMW collaboration (Budapest, Marseille, Wuppertal) 2010



mesons=
quark anti-quark states

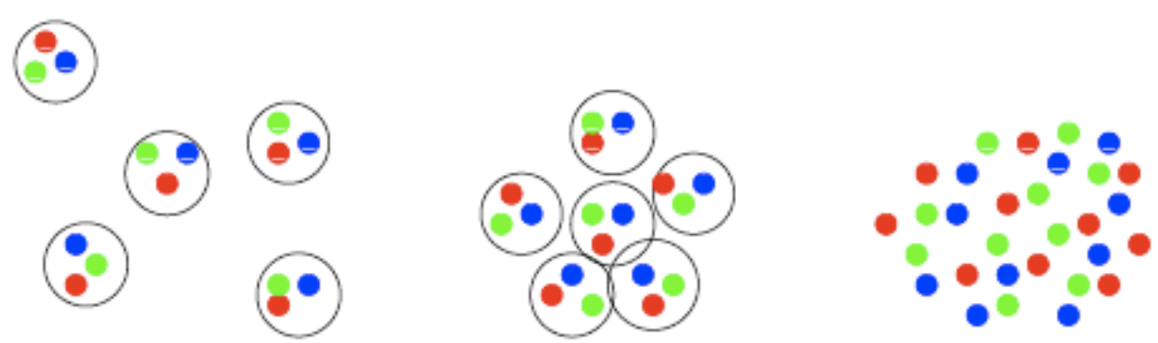
QCD is correct theory for strong interactions also at low energy!

QCD at high temperature/density: change of dynamics

asymptotic freedom $\alpha_s(p \to \infty) \to 0$

T, μ_B

Phase transitions?



Hadrongas

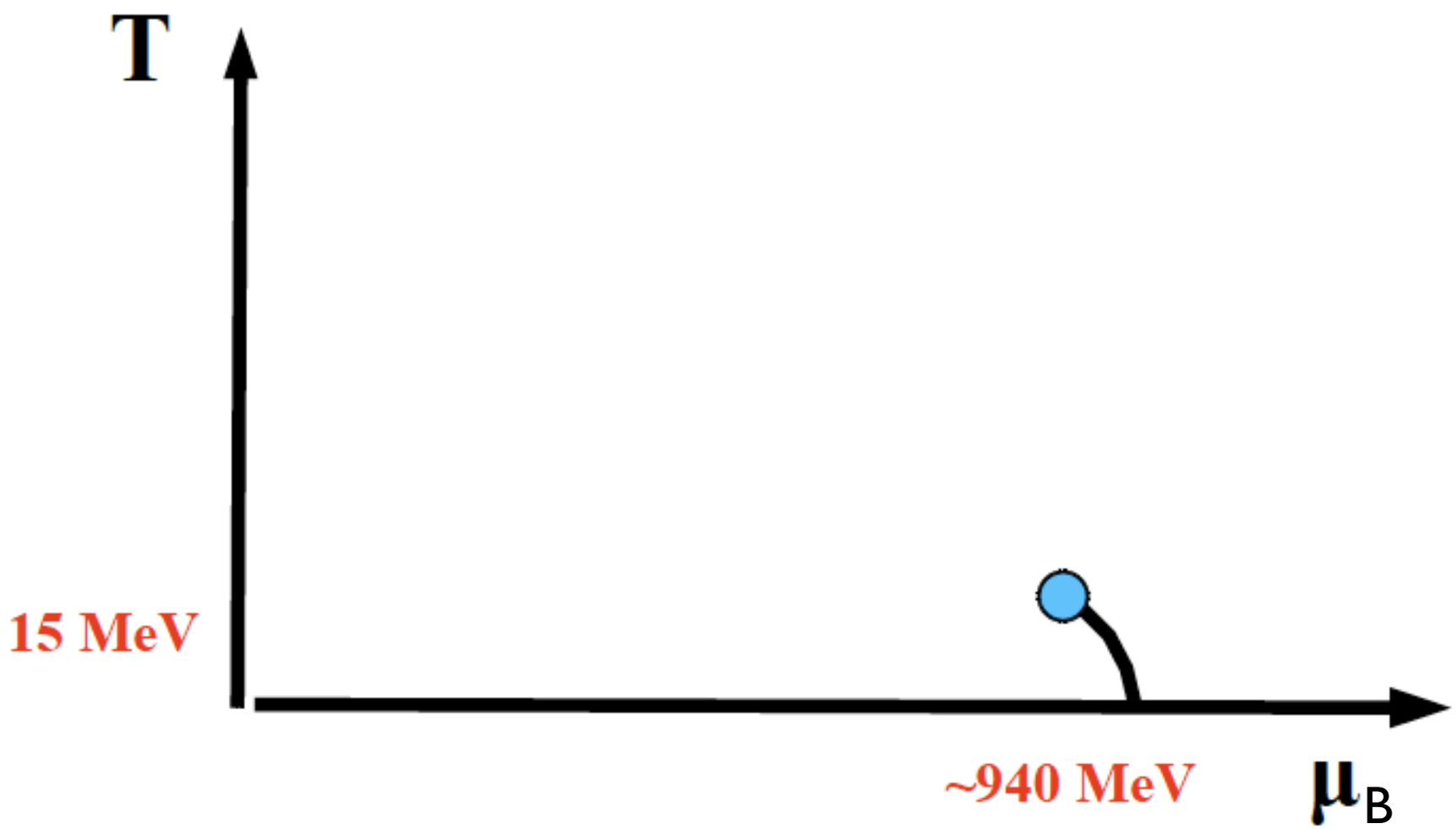
Quark-Gluon-Plasma

Chiral symmetry: broken (nearly) restored

Order parameters: $\langle \bar{\psi}\psi \rangle, \langle \psi\psi \rangle$

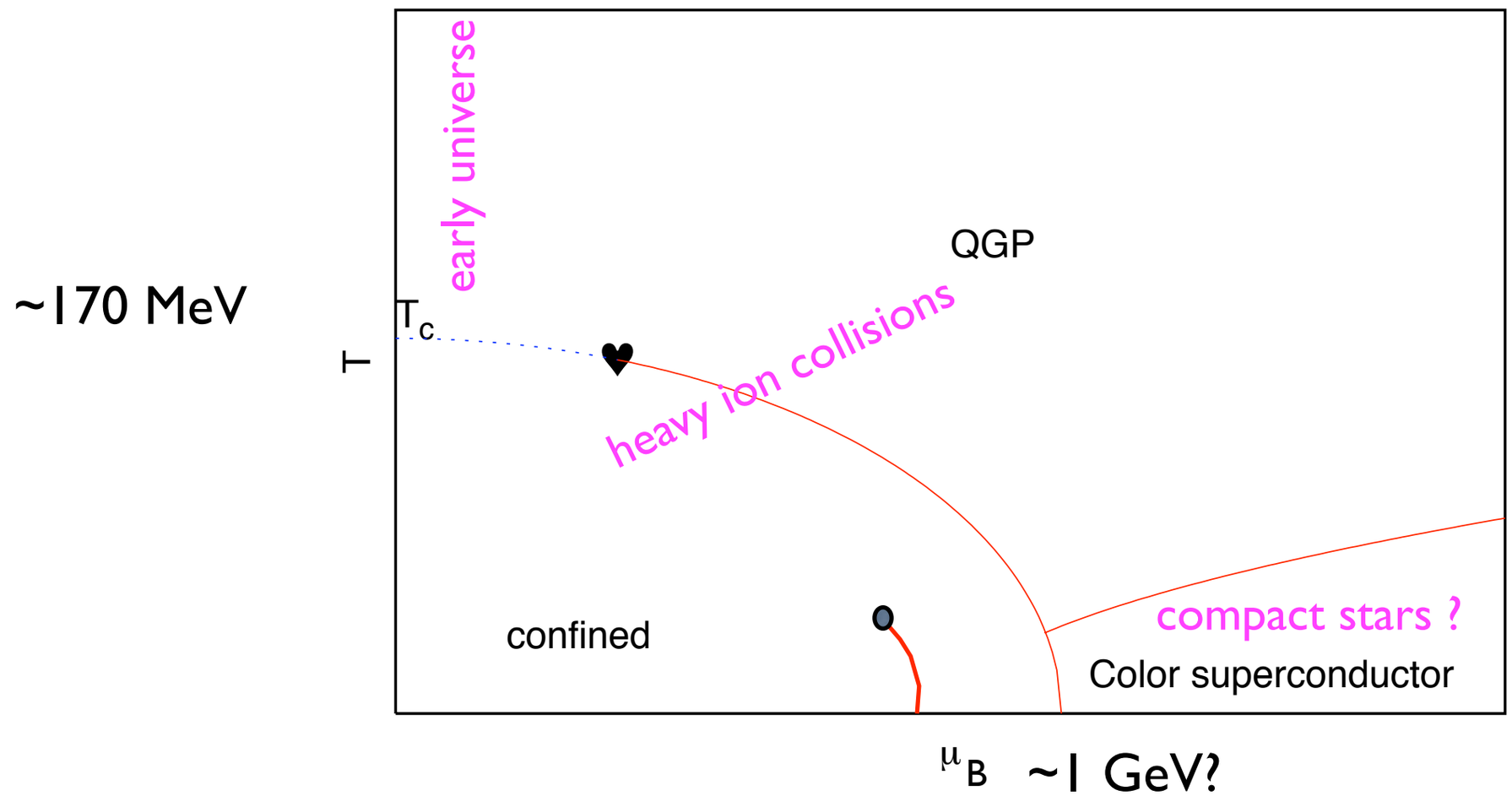
chiral condensate , Cooper pairs

The QCD phase diagram established by experiment:



Nuclear liquid gas transition with critical end point

QCD phase diagram: theorist's view (science fiction)

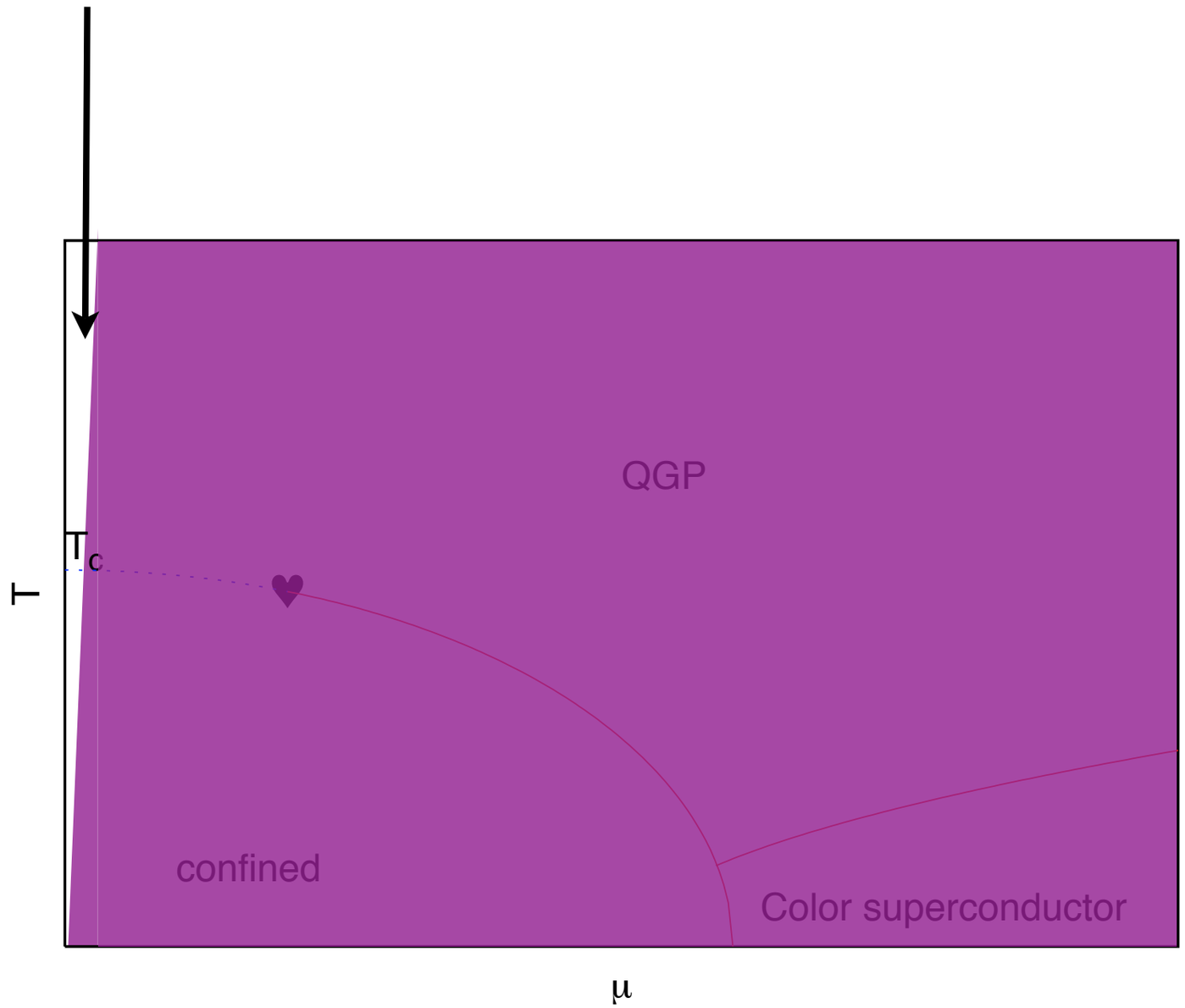


Until 2001: no finite density lattice calculations, sign problem!

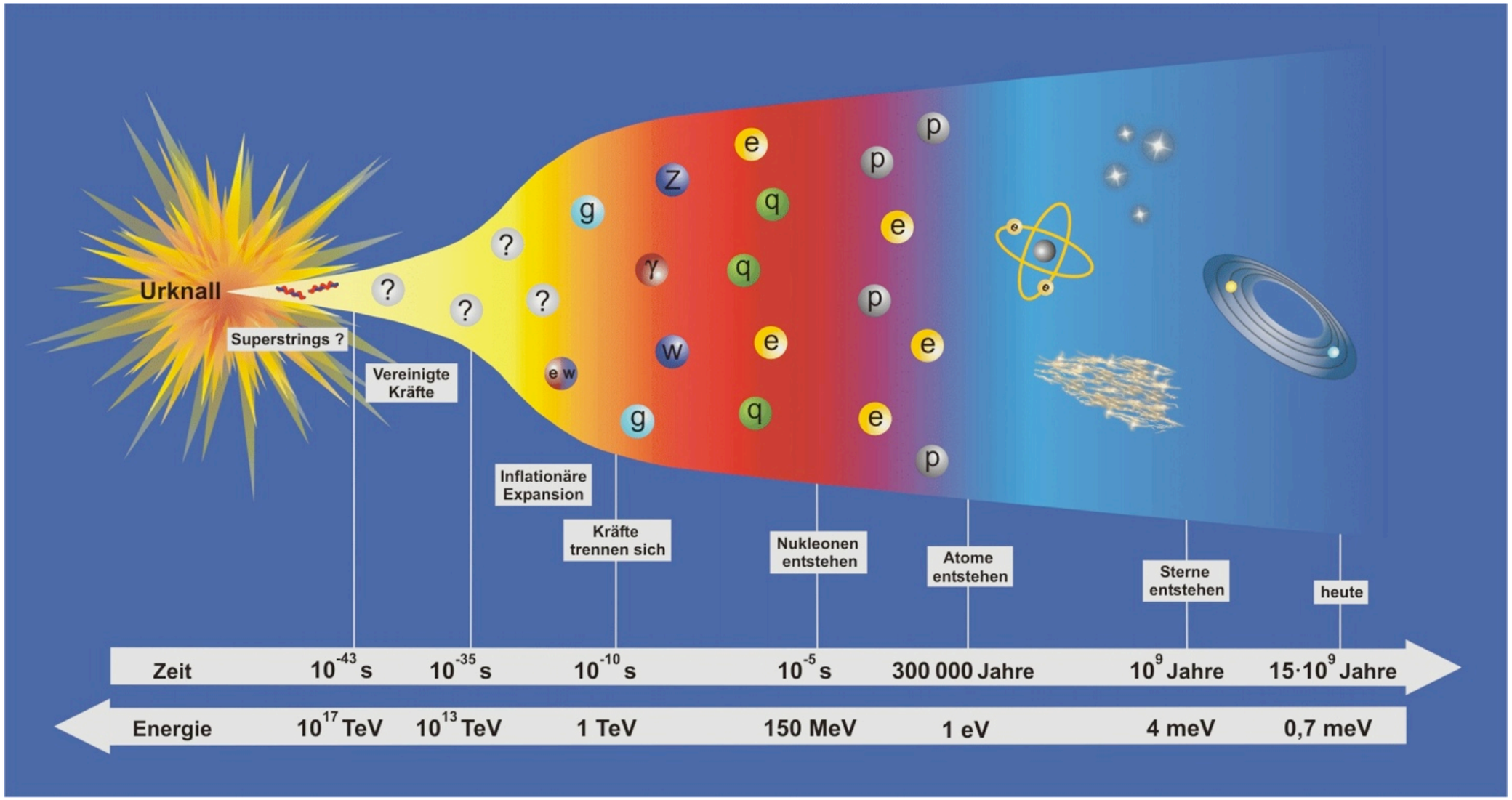
Expectation based on simplifying models (NJL, linear sigma model, random matrix models, ...)

Check this from first principles QCD!

Zero and nearly zero baryon density



QCD in the early universe



QCD phase transition

QGP + other particles

Expansion: pressure of photons + matter particles

- **Pressure from non-interacting particles with g degrees of freedom:**

Relativistic Boson, $m \ll T$	$\times$ *(Fermion)*	*Non-relativistic*, $m \gg T$
$p_r = g\frac{\pi^2}{90}T^4$	$\left(\frac{7}{8}\right)$	$p_{nr} = gT\left(\frac{mT}{2\pi}\right)^{\frac{3}{2}}\exp(-m/T)$

- **Pressure from interacting particles:**

Define effective numbers of d.o.f. for energy & entropy densities:

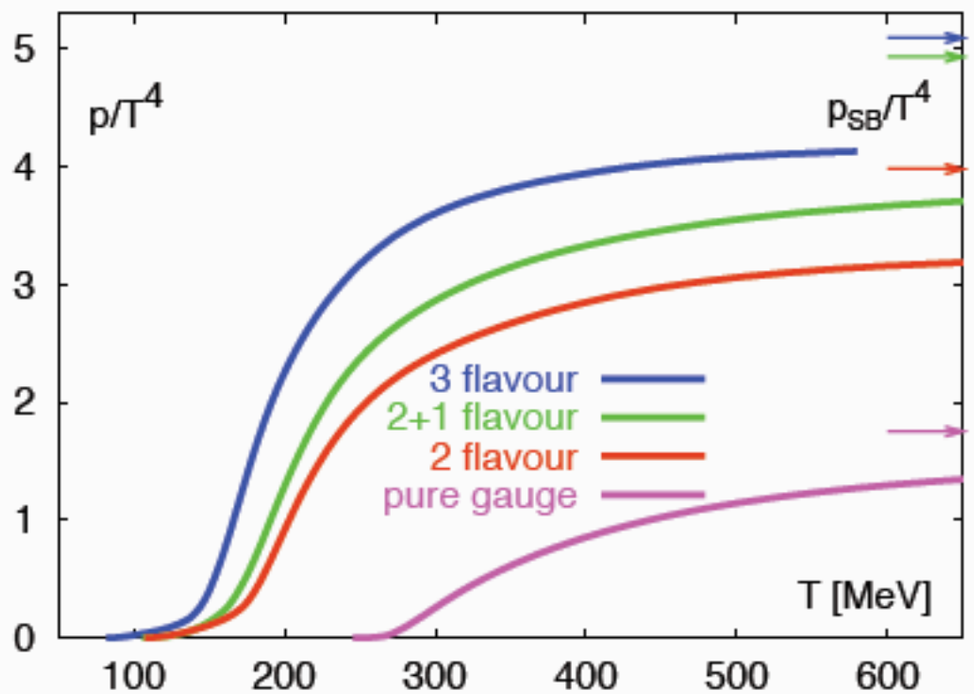
$$\rho = \frac{\pi^2}{30}g_{\rm eff}(T)T^4, \quad s = \frac{2\pi^2}{45}h_{\rm eff}(T)T^3,$$

$\Rightarrow$ Calculate from **Quantum Field Theory**

The QCD equation of state: deconfinement

staggered p4-improved, $N_\tau = 4$

Bielefeld



compare with ideal gas:

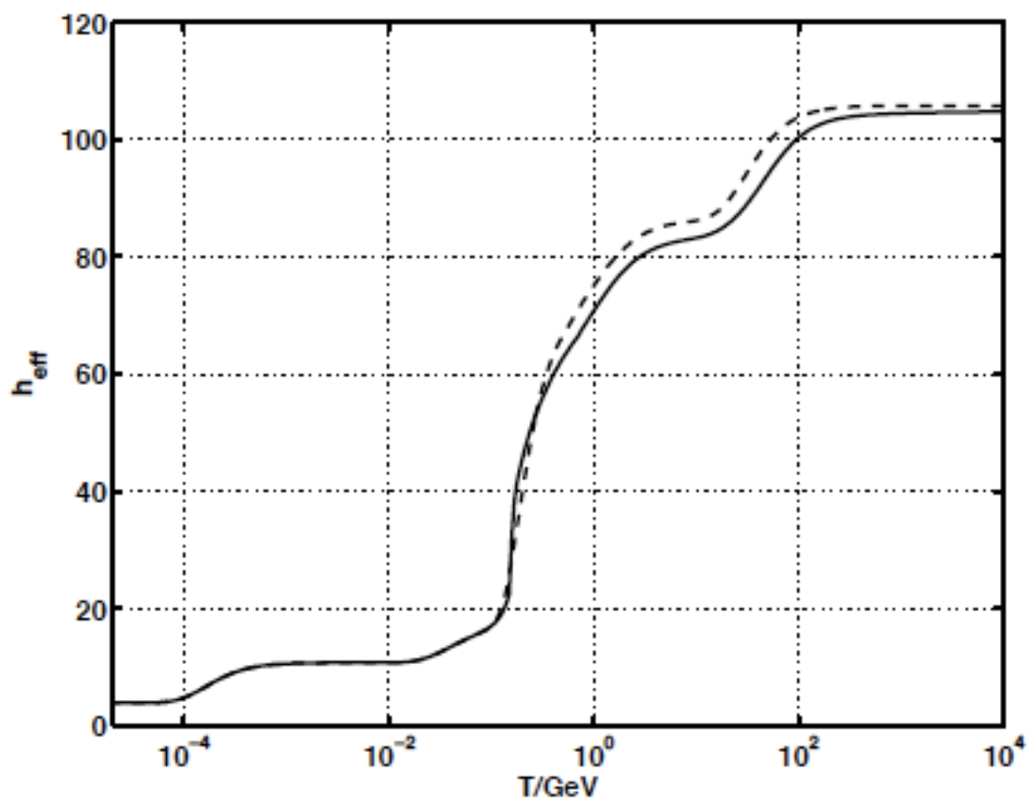
$$\frac{\epsilon_{SB}}{T^4} = \frac{3p_{SB}}{T^4} = \begin{cases} 3\frac{\pi^2}{30} & , T < T_c \\ (16 + \frac{21}{2}N_f)\frac{\pi^2}{30} & , T > T_c \end{cases}$$

$T > T_c$: more degrees of freedom, but significant interaction!

→ sQGP or `almost ideal' gas....?

Continuum extrapolation with physical quarks in progress

SM degrees of freedom with lattice QCD-EoS



Hindmarsh, O.P. 05

$$h_{\rm eff}(T)$$

Full line: free particles

Dashed line: corrected for QCD interactions

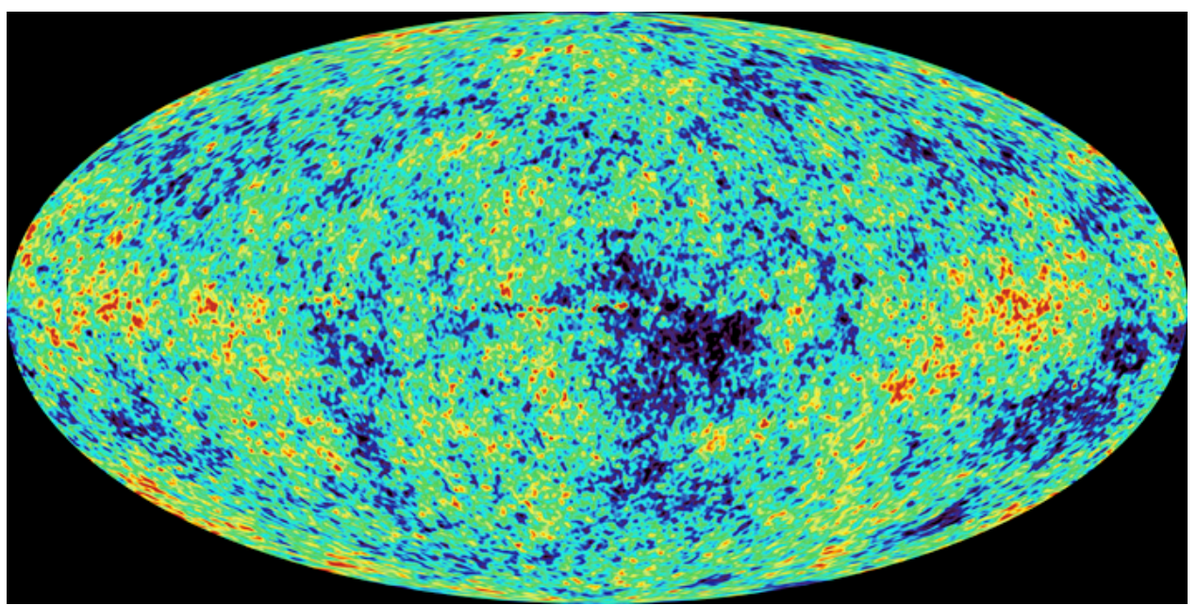
Important for relic densities of WIMPS

Weakly Interacting Massive Particles: dark matter candidates!

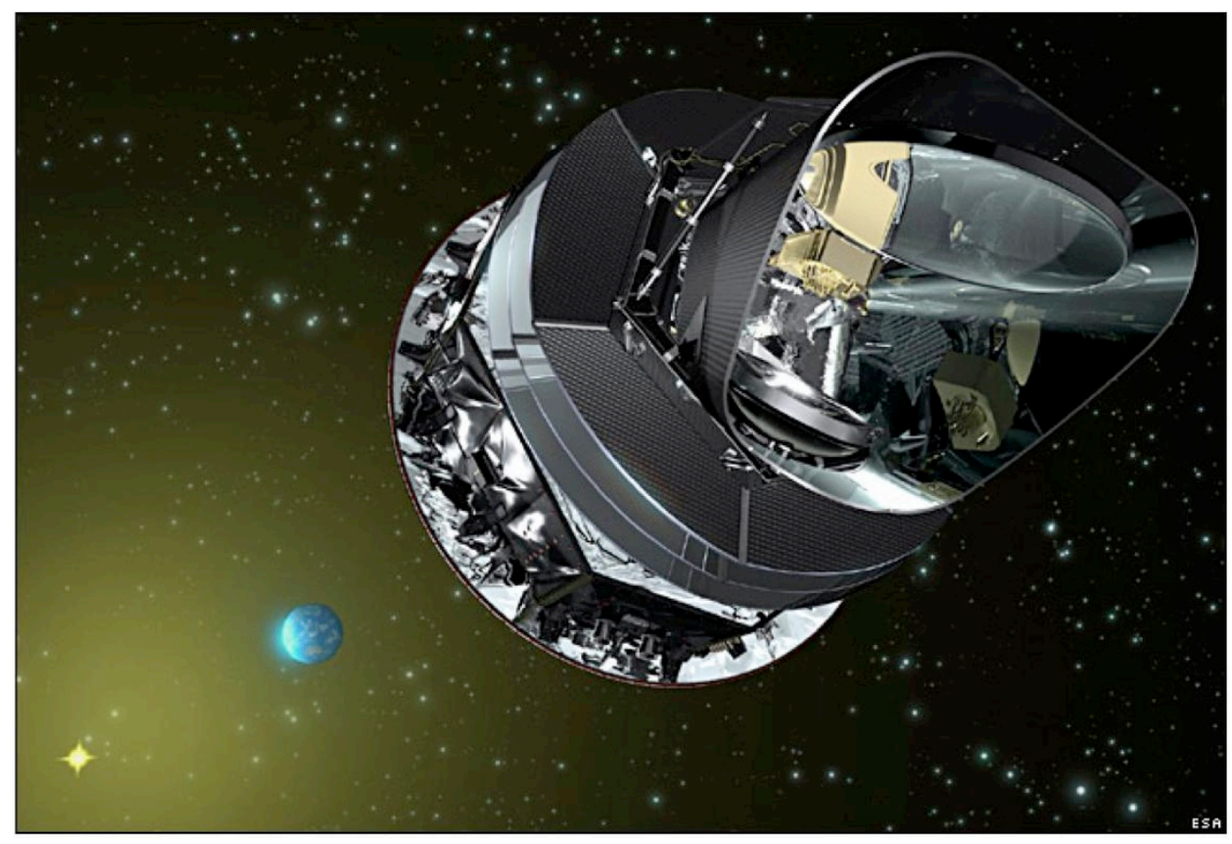
Energy density of WIMPs depends on g_{eff}, h_{eff} of Standard Model

➡ increase of WIMPs by ~2-3% from QCD corrections

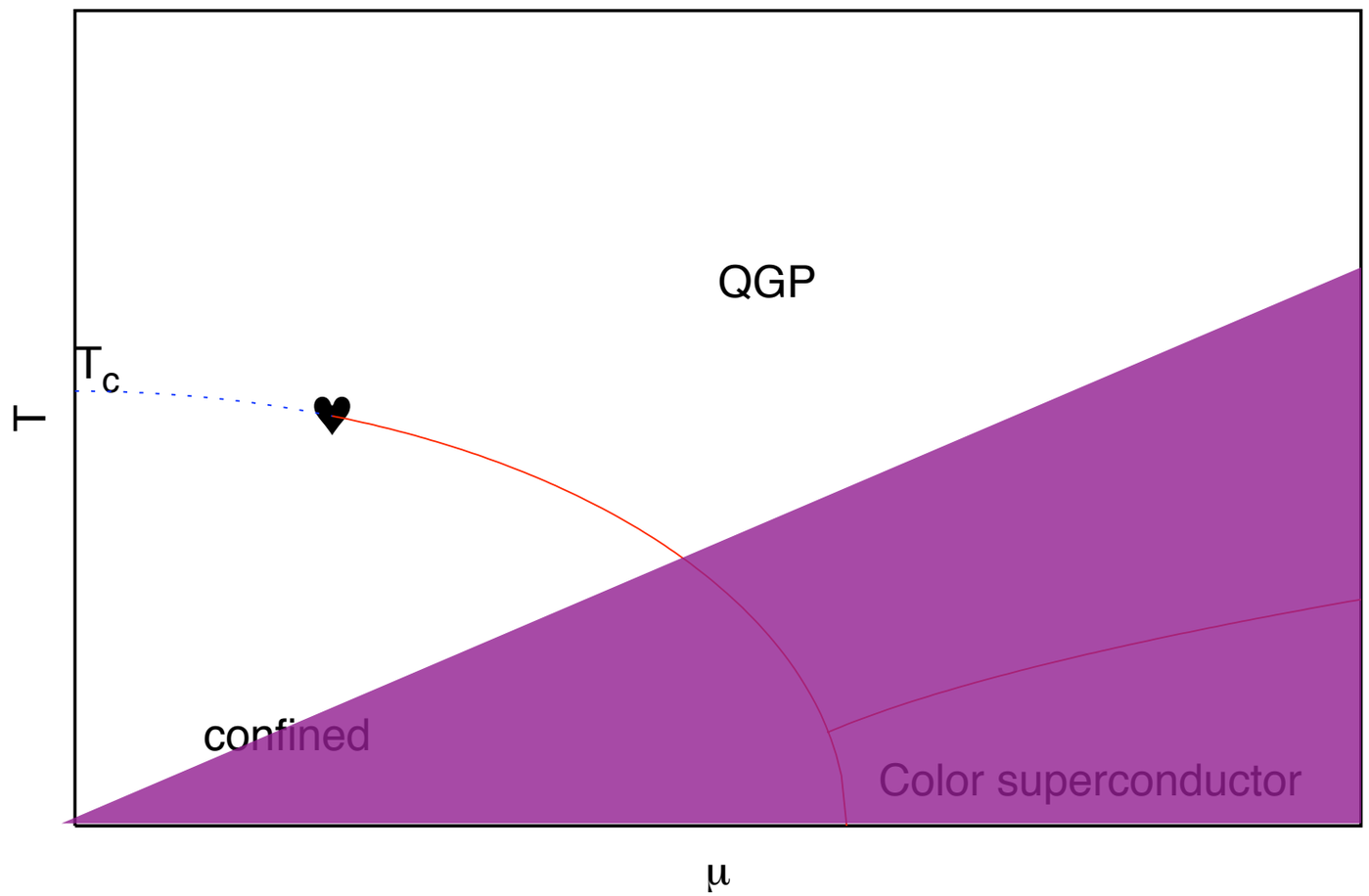
Hindmarsh, O.P. 05



Planck satellite promises to constrain relic densities to better than 1%!

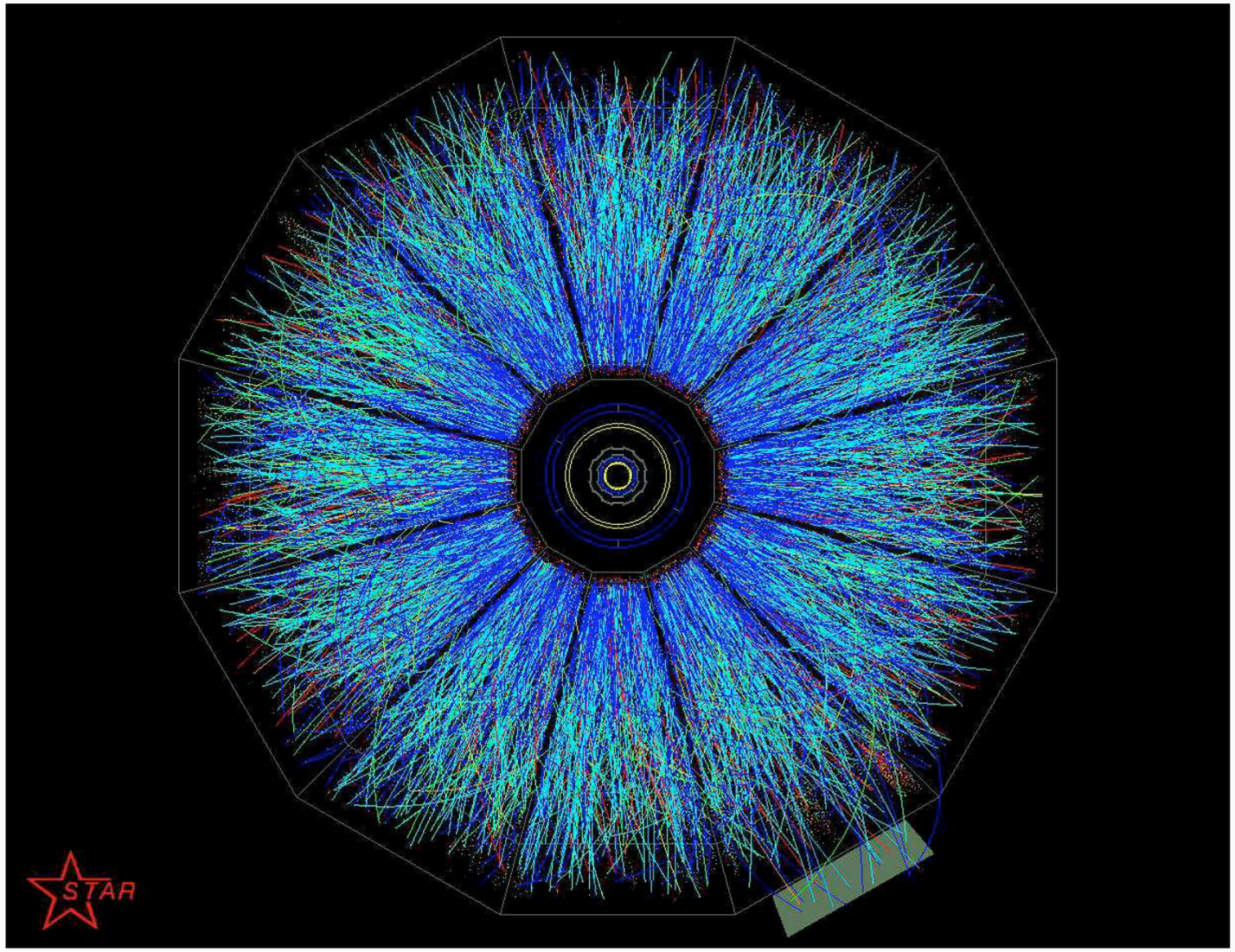


The calculable region of the phase diagram $\mu < T$



- Upper region: equation of state, screening masses, quark number susceptibilities etc. under control
- Here: phase diagram itself, most difficult!

Thermal QCD in experiment

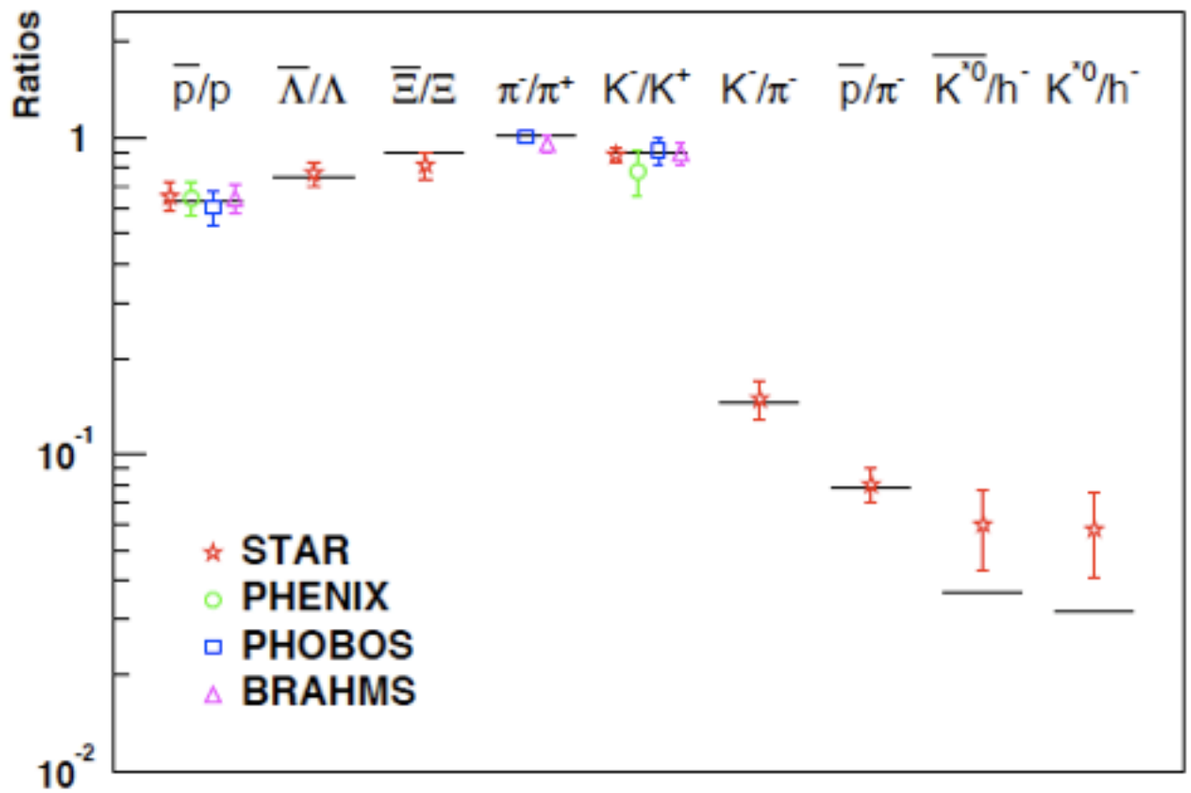


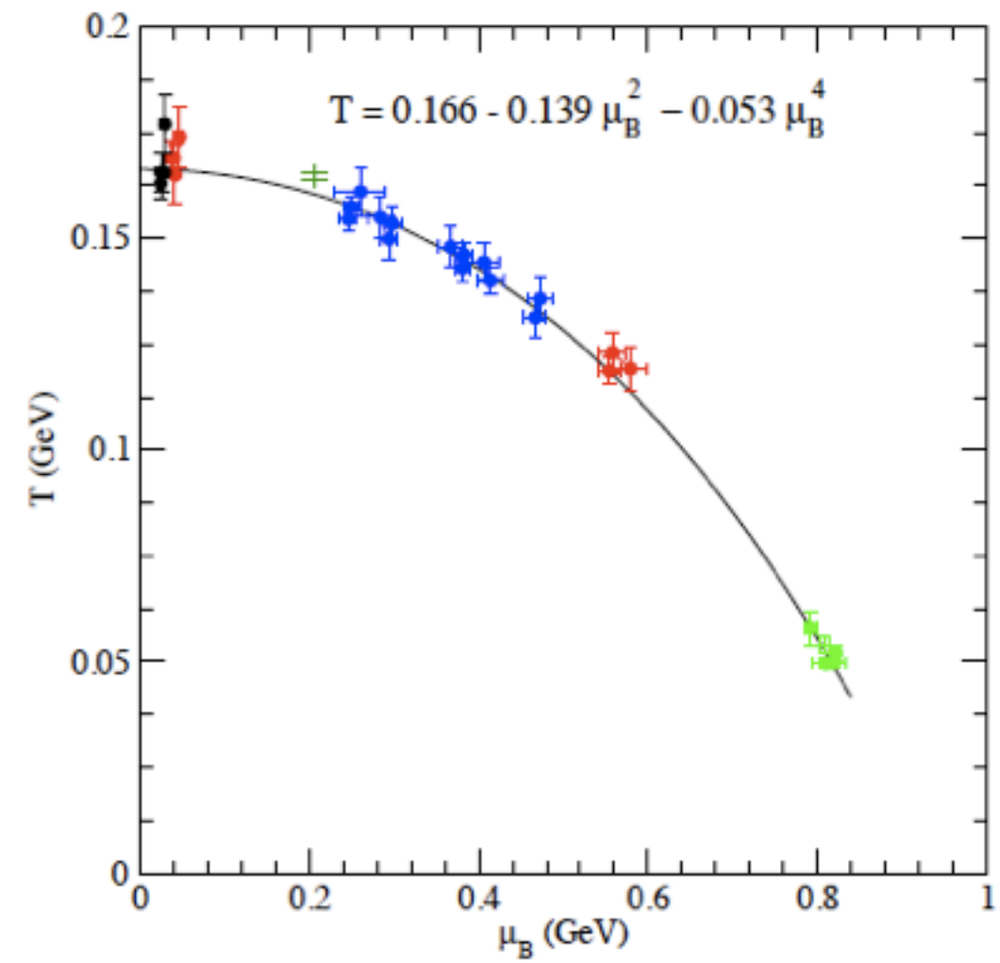
heavy ion collision experiments at RHIC, LHC, GSI.... have finite baryon density!

Deconfinement on Earth:

Free the Quarks!!!

Phase boundary from hadron freeze-out?





- At fixed collision energy $\sqrt{s}$, abundances well fitted by Boltzmann distribution (T, μ_B)
- T(freeze-out) $\leq T_c$ but very close ?

Braun-Munzinger et al

Theory: how to calculate p.t., critical temperature

deconfinement/chiral phase transition $\longrightarrow$ quark gluon plasma

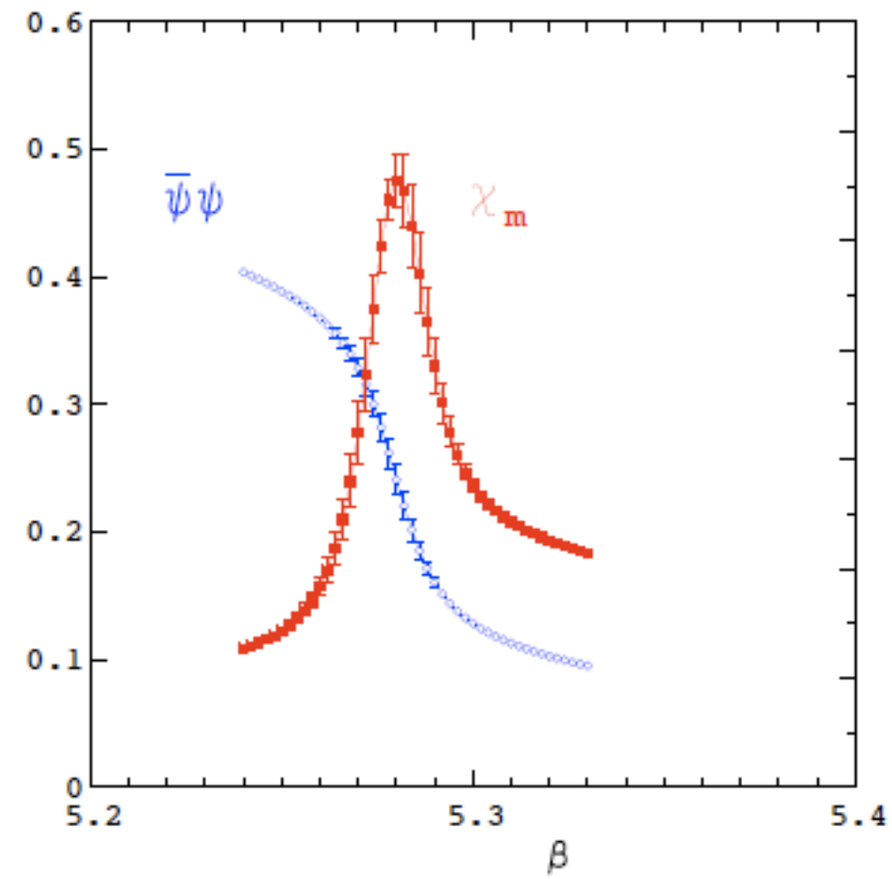
"order parameter":

chiral condensate $\langle\bar{\psi}\psi\rangle$

generalized susceptibilities:

$$\chi = V(\langle\mathcal{O}^2\rangle - \langle\mathcal{O}\rangle^2)$$

$$\Rightarrow \chi_{max} = \chi(\beta_c) \quad \Rightarrow T_c$$



lattice coupling β, viz. T

only pseudo-critical on finite V!

Order of transition:

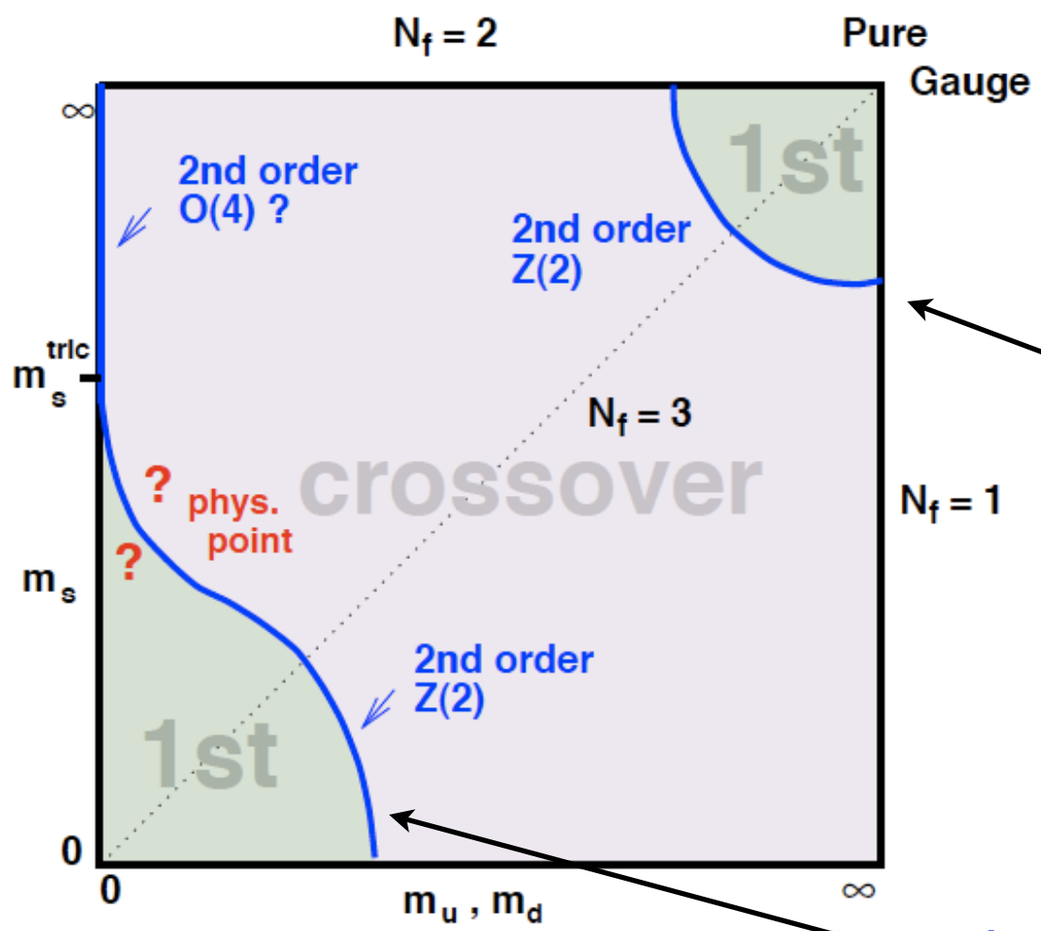
finite volume scaling

$$\chi_{max} \sim V^{\sigma}$$

$\sigma = 1$	1st order
σ = crit. exponent	2nd order
$\sigma = 0$	crossover

The order of the p.t., arbitrary quark masses $\mu = 0$

deconfinement p.t.:
breaking of global $Z(3)$ symmetry



deconfinement critical line

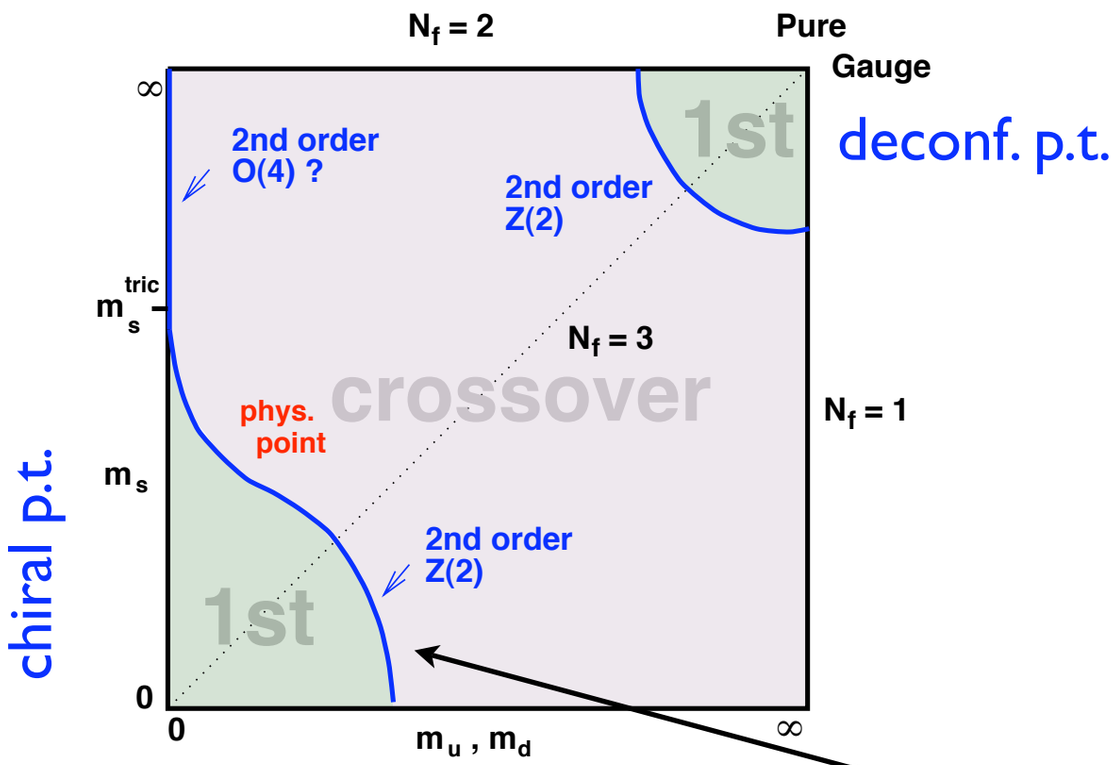
chiral critical line

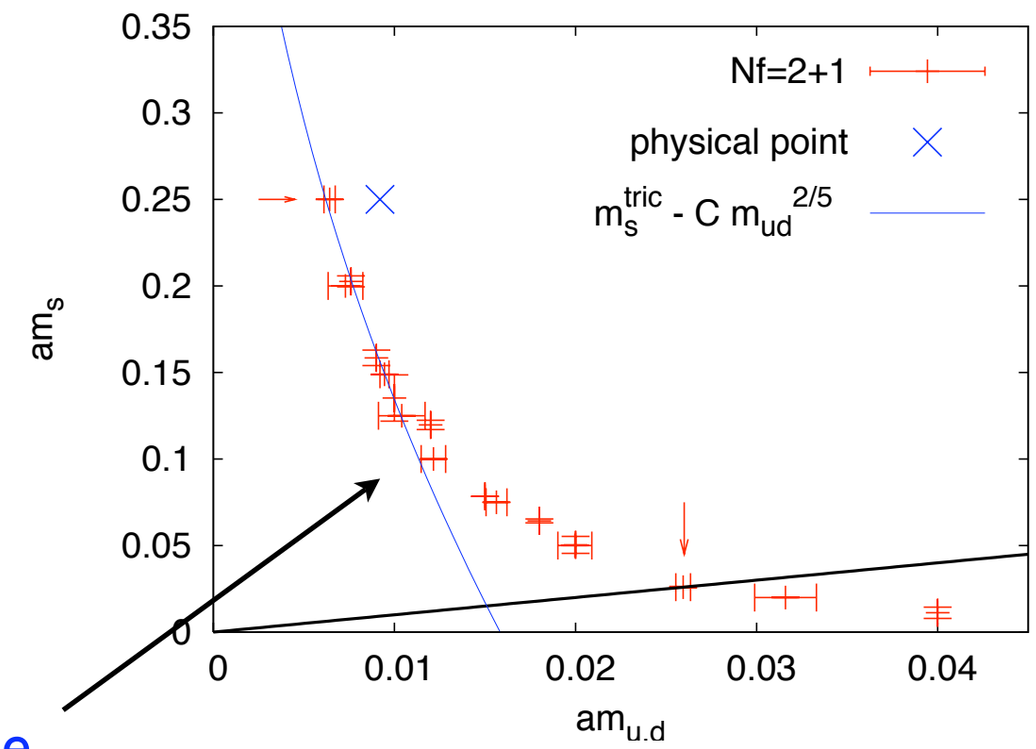
chiral p.t.
restoration of global symmetry in flavour space

$$SU(2)_L \times SU(2)_R \times U(1)_A$$

anomalous

Order of p.t., arbitrary quark masses $\mu = 0$





chiral critical line

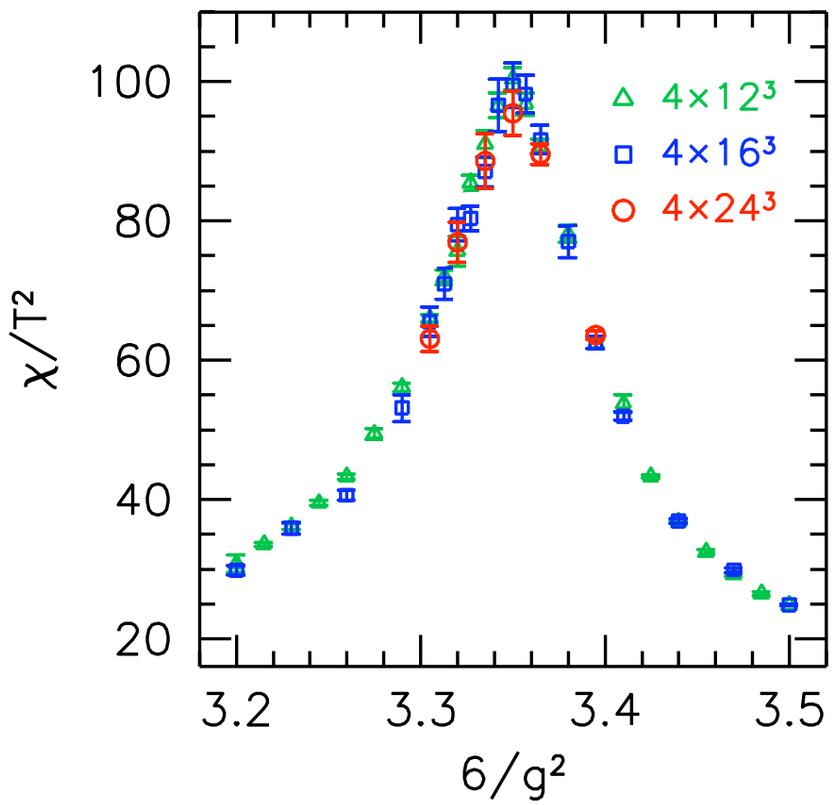
- chiral critical line on $N_t = 4, a \sim 0.3$ fm

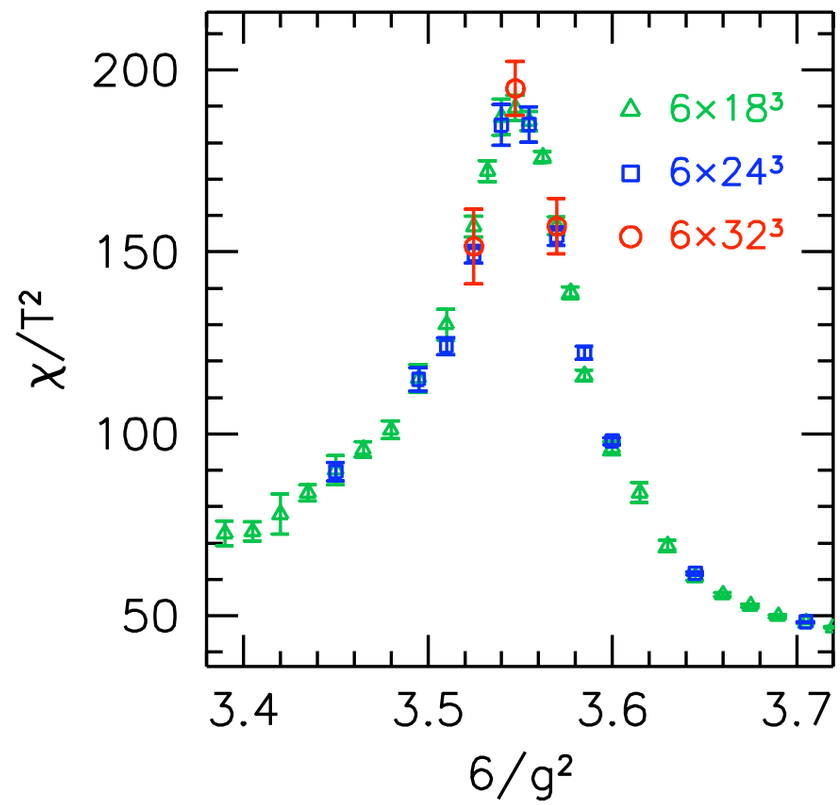
de Forcrand, O.P. 07

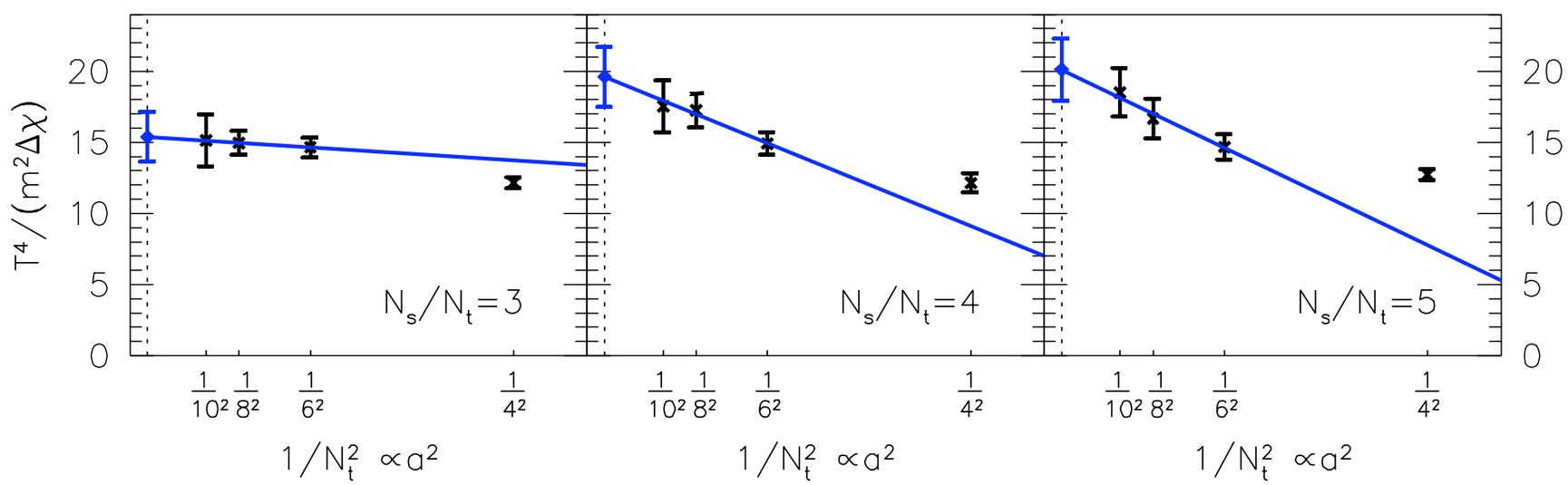
The nature of the transition for phys. masses

Aoki et al. 06

...in the staggered approximation...in the continuum...is a crossover!







The 'sign problem' is a phase problem

$$Z = \int DU \, [\det M(\mu)]^f \, \mathrm{e}^{-S_g[U]}$$

importance sampling requires
positive weights

Dirac operator: $\not{D}(\mu)^\dagger = \gamma_5 \not{D}(-\mu^*)\gamma_5$

$\Rightarrow \det(M)$ complex for SU(3), $\mu \neq 0$

$\Rightarrow$ real positive for SU(2), $\mu = i\mu_i$

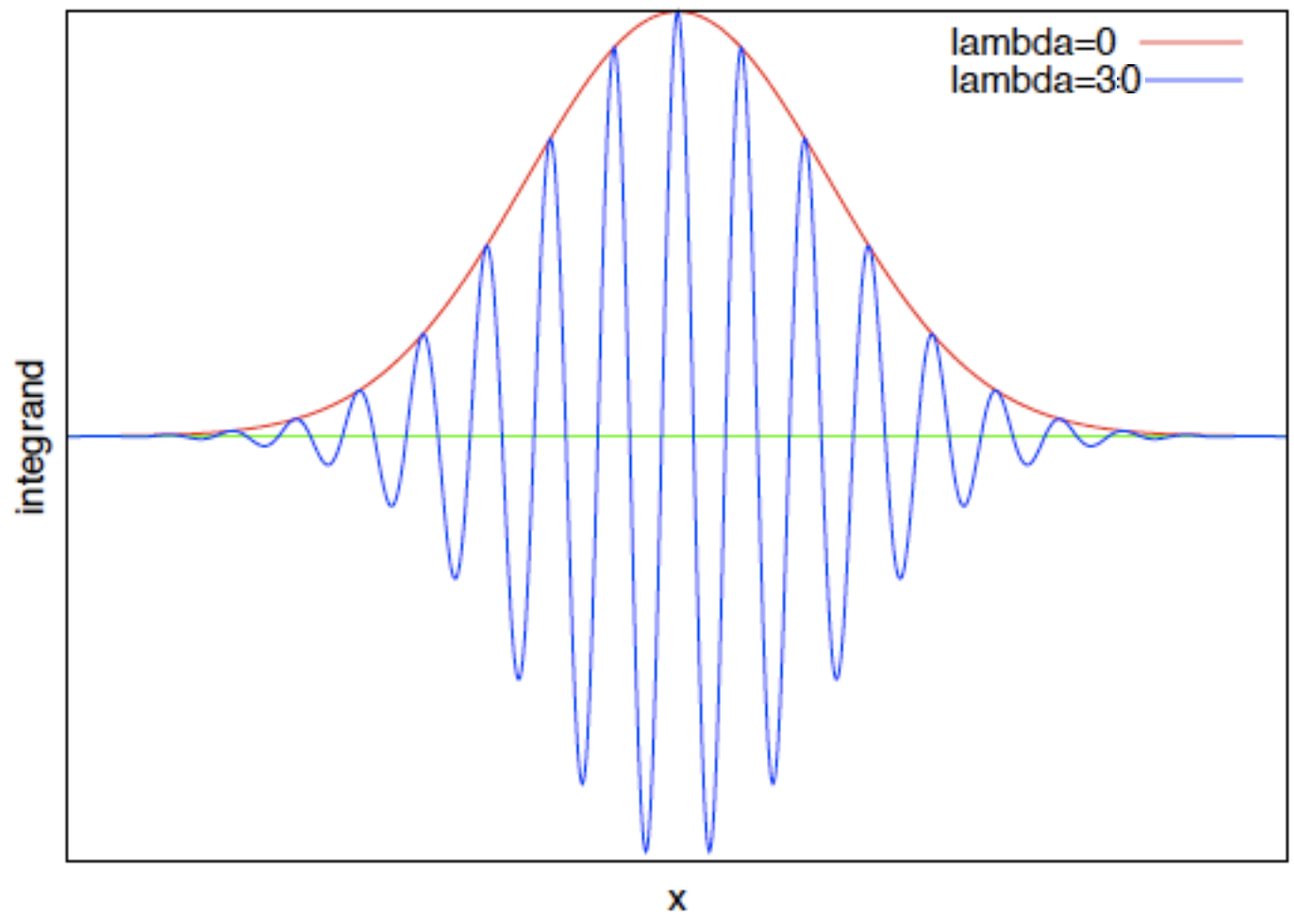
$\Rightarrow$ real positive for $\mu_u = -\mu_d$

N.B.: all expectation values real, imaginary parts cancel,
but importance sampling config. by config. impossible!

Same problem in many condensed matter systems!

1 dim. illustration

- Example: $Z(\lambda) = \int dx \exp(-x^2 + \mathbf{i}\lambda\mathbf{x})$



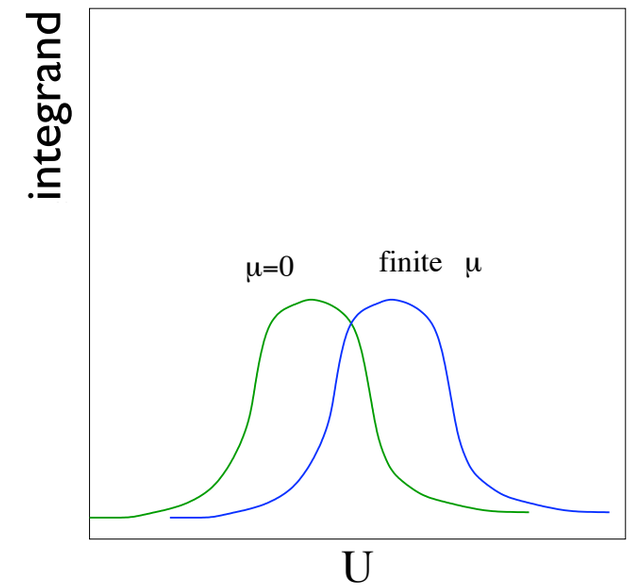
- $Z(\lambda)/Z(0) = \exp(-\lambda^2/4)$: exponential cancellations

Finite density: methods to evade the sign problem

- Reweighting: $$Z = \int DU \, \det M(0) \frac{\det M(\mu)}{\det M(0)} \, \mathrm{e}^{-S_g}$$

  ~exp(V) statistics needed, overlap problem

  use for MC ↑ calculate ↑



- Taylor expansion:

$$\langle O \rangle(\mu) = \langle O \rangle(0) + \sum_{k=1} c_k \left(\frac{\mu}{\pi T} \right)^{2k}$$

coeffs. one by one, convergence?

- Imaginary $\mu = i\mu_i$: no sign problem, fit by polynomial, then analytically continue

$$\langle O \rangle(\mu_i) = \sum_{k=0}^{N} c_k \left(\frac{\mu_i}{\pi T} \right)^{2k}, \qquad \mu_i \to -i\mu$$

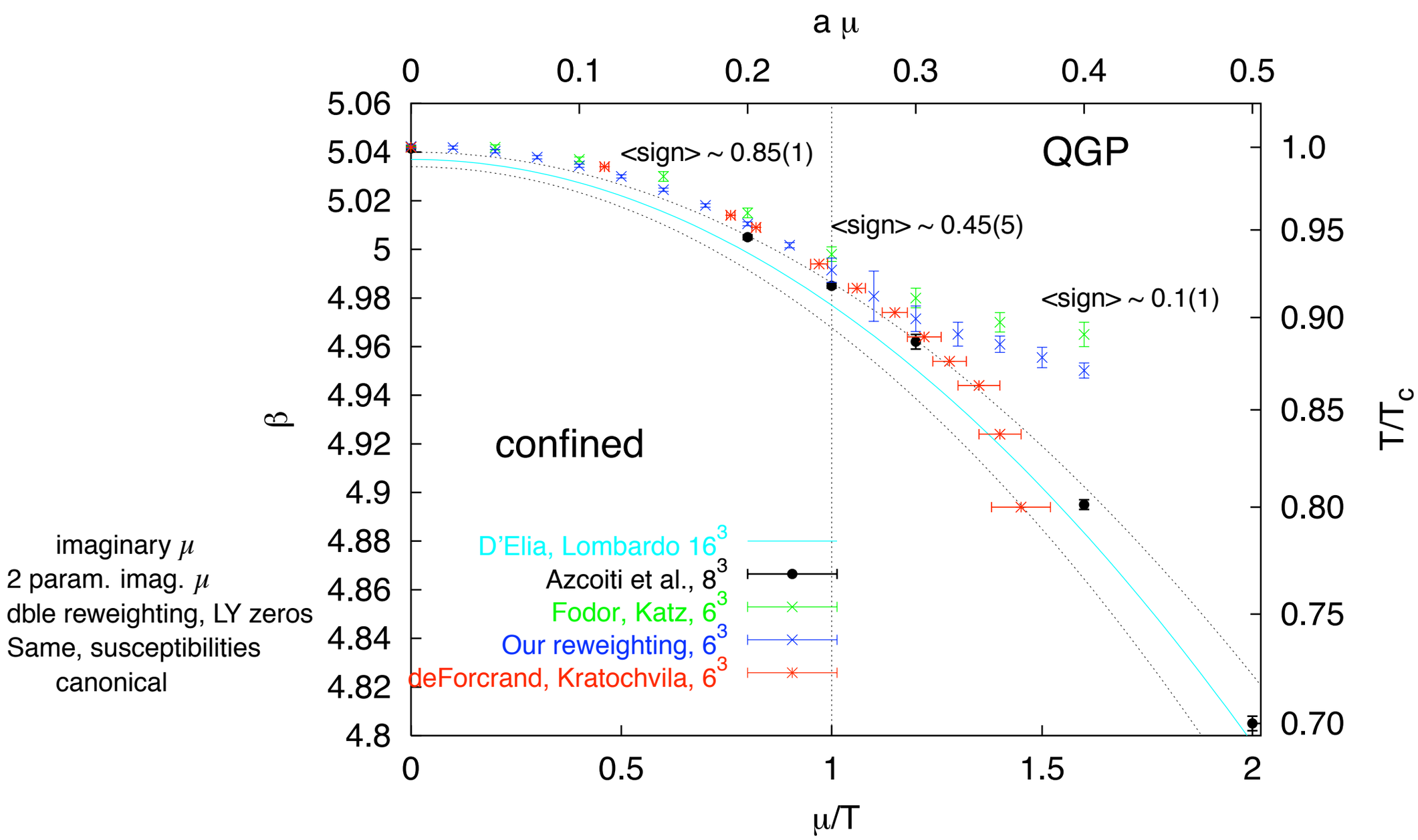
requires convergence for anal. continuation

All require $\mu/T < 1$!

The good news: comparing $T_c(\mu)$

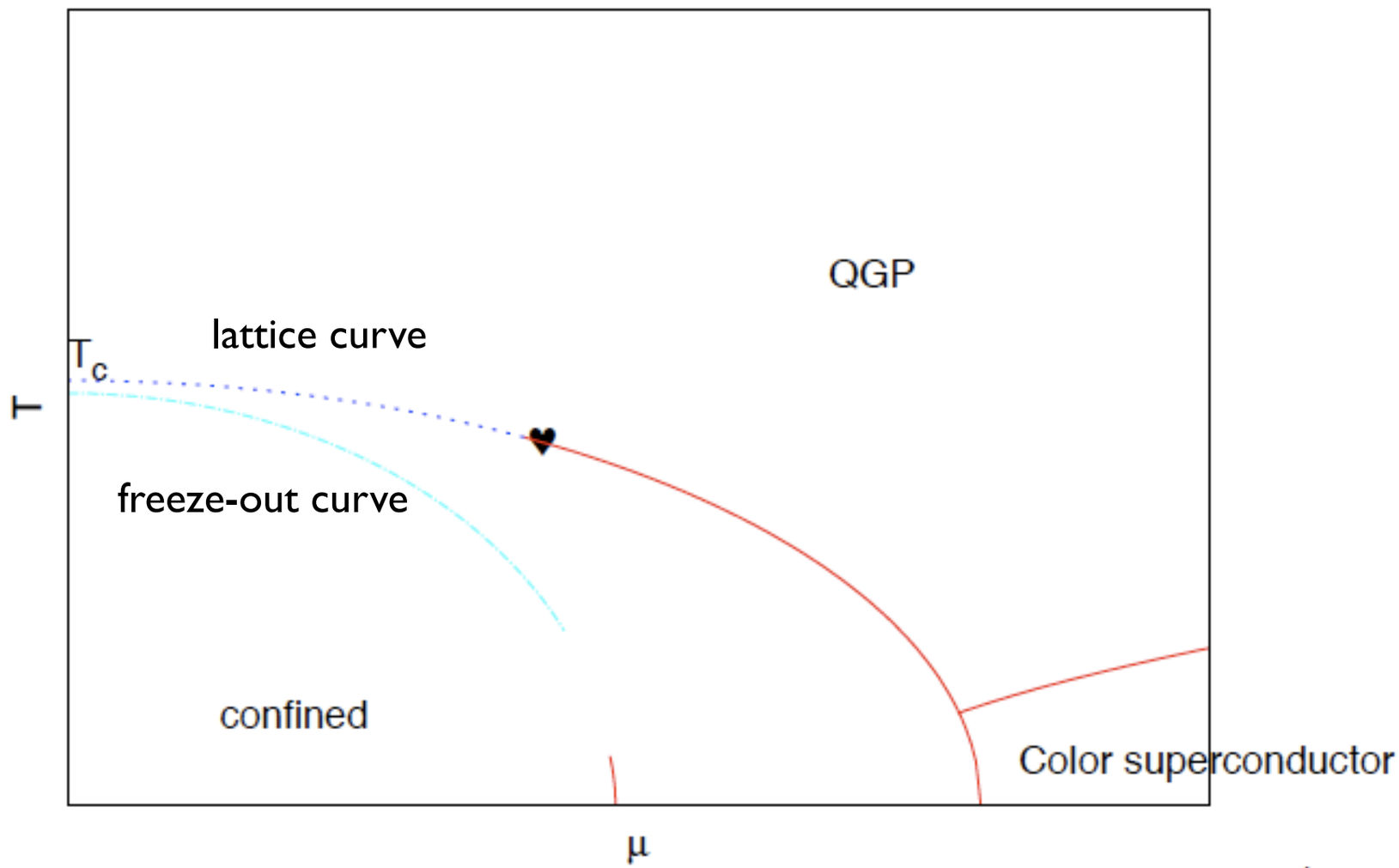
de Forcrand, Kratochvila 05

$N_t = 4, N_f = 4$; same actions (unimproved staggered), same mass



Agreement for $\mu/T \lesssim 1$

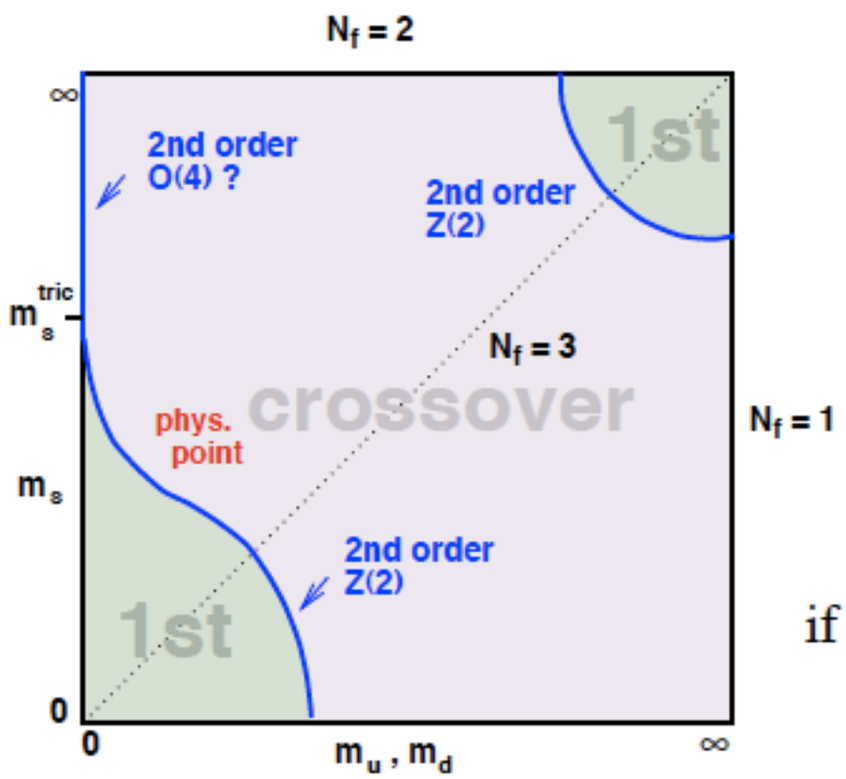
Comparison with freeze-out curve so far



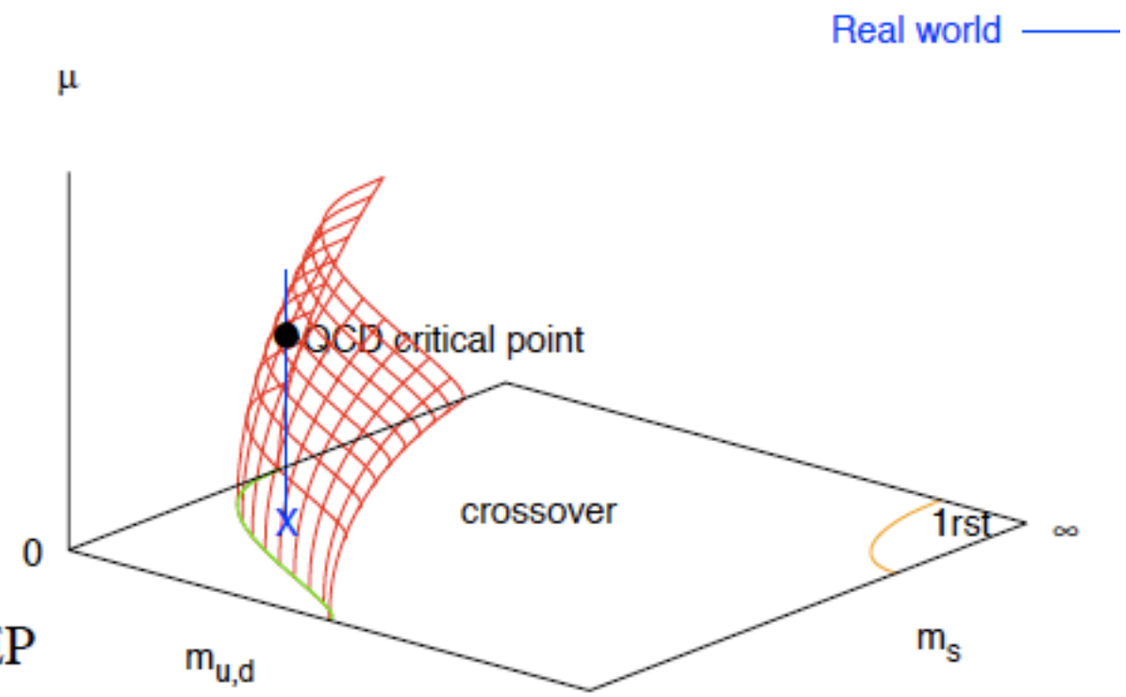
$T_c(\mu)$ considerably flatter than **freeze-out** curve (factor ~ 3 in $\left.\frac{d^2 T_c}{d\mu^2}\right|_{\mu=0}$)

(But not yet extrapolated to continuum; in progress)

Much harder: is there a QCD critical point?



if *chiral* CEP



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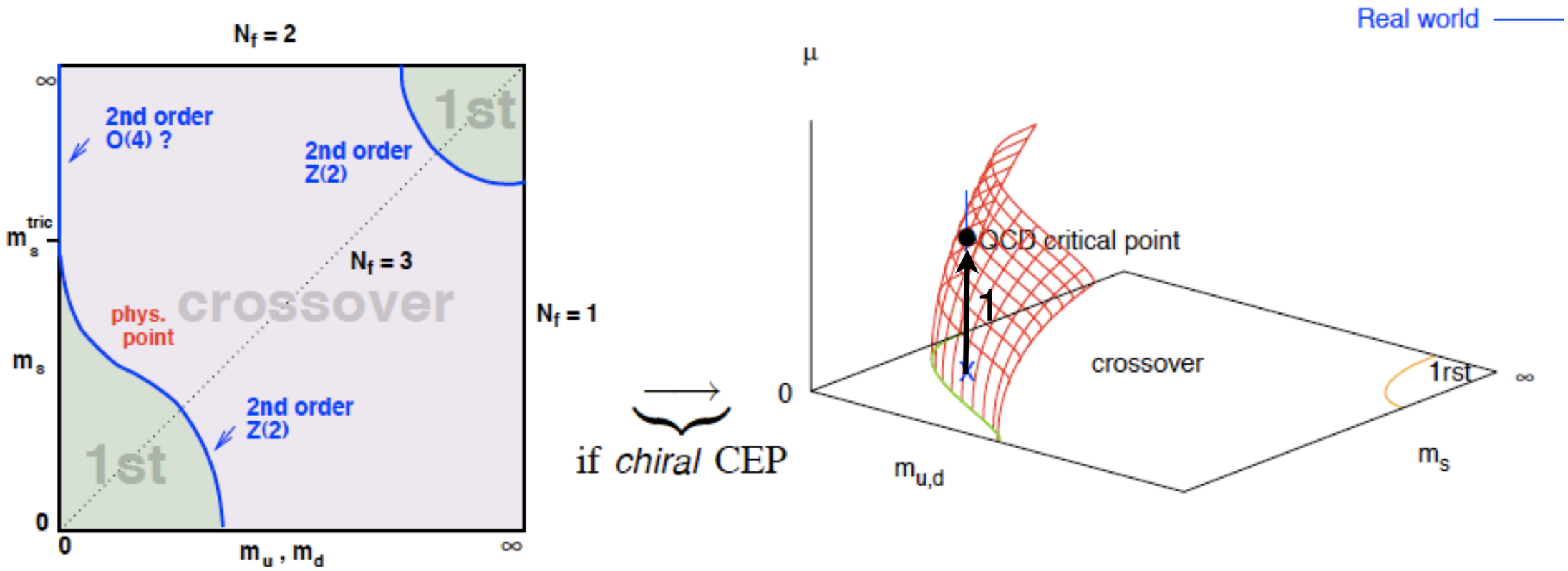
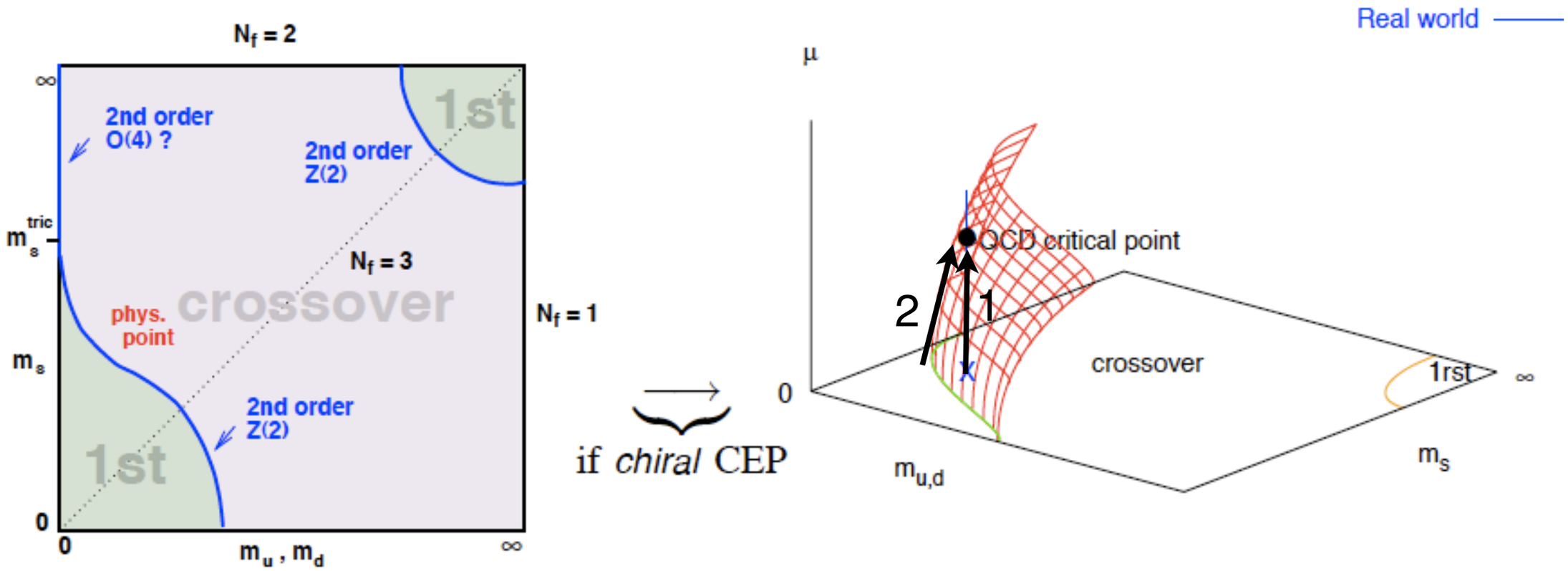


Two strategies:

1 follow vertical line: $m = m_{\text{phys}}$, turn on μ

Much harder: is there a QCD critical point?



Two strategies:

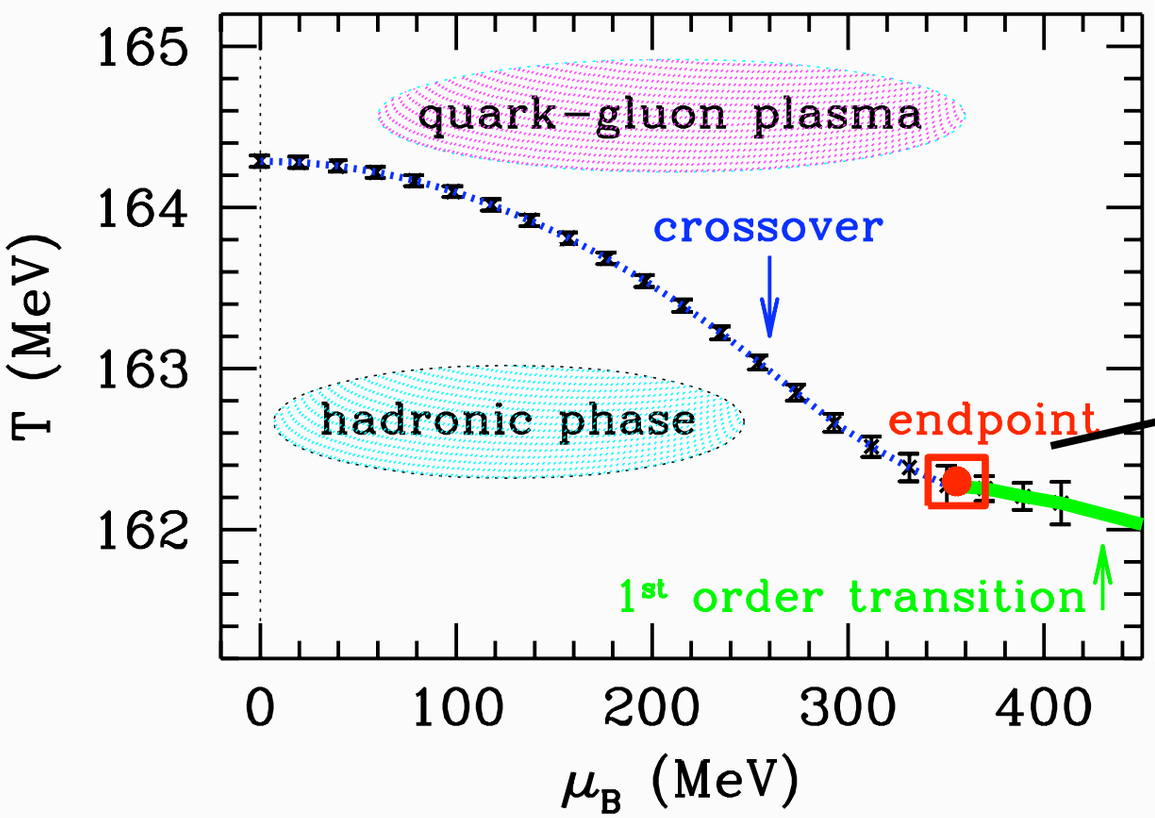
1 follow vertical line: $m = m_{\text{phys}}$, turn on μ

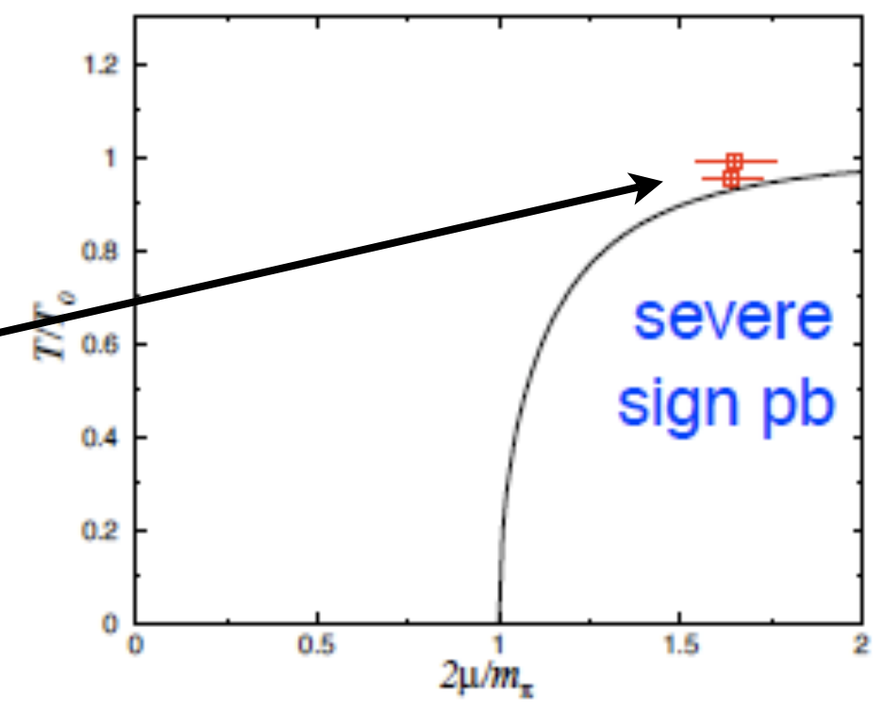
2 follow critical surface: $m = m_{\text{crit}}(\mu)$

Approach 1: CEP from reweighting

Fodor, Katz 04

$N_t = 4, N_f = 2 + 1$ physical quark masses, unimproved staggered fermions





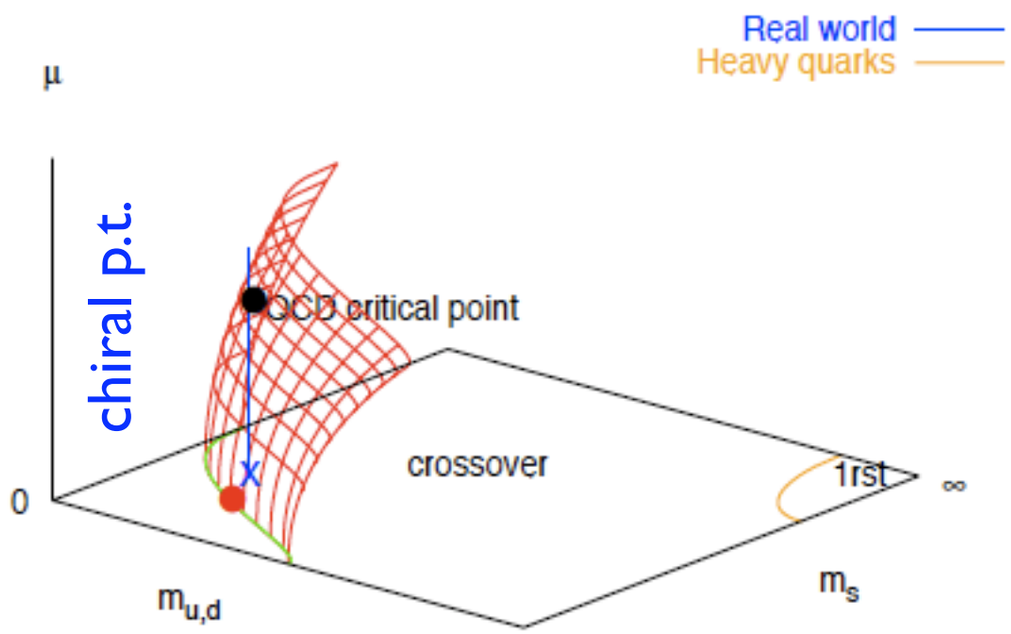
$(\mu_E^q, T_E) = (120(13), 162(2))$ MeV

abrupt change: physics or problem of the method?

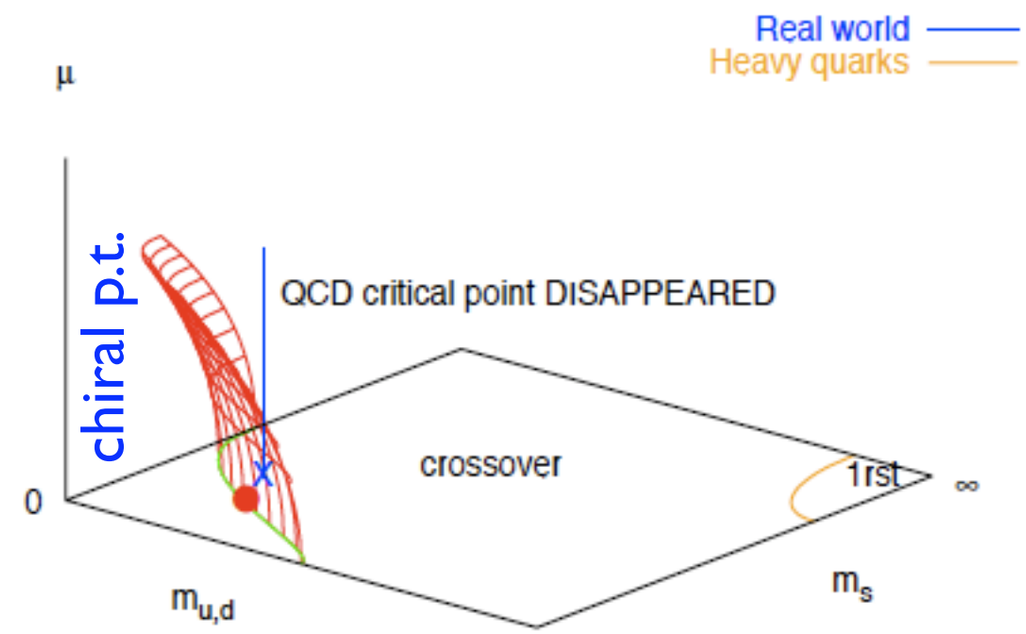
(entire curve generated from one point!)

Splittorff 05;
Han, Stephanov 08

Approach 2: follow chiral critical line → surface



$c_1 > 0$



$c_1 < 0$

$$\frac{m_c(\mu)}{m_c(0)} = 1 + \sum_{k=1} c_k \left(\frac{\mu}{\pi T}\right)^{2k}$$

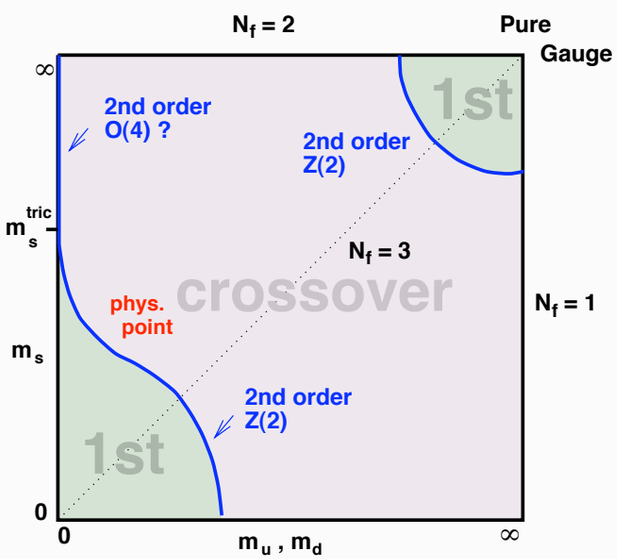
1. Tune quark mass(es) to $m_c(0)$: 2nd order transition at $\mu = 0$, $T = T_c$
   known universality class: 3d Ising

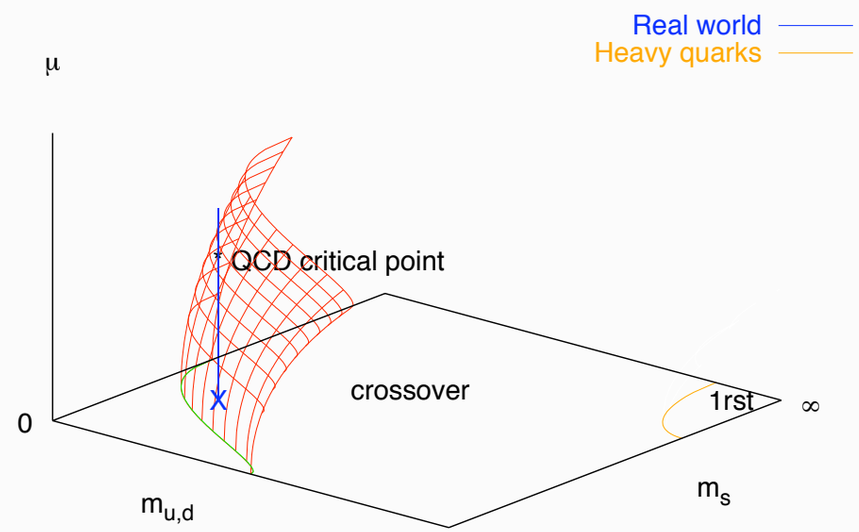
2. Measure derivatives $\frac{d^k m_c}{d\mu^{2k}}|_{\mu=0}$:

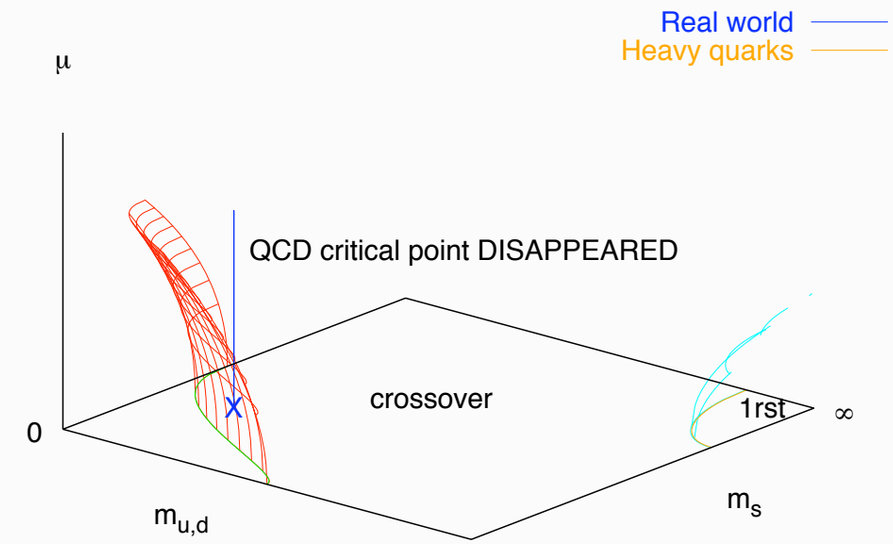
   Turn on imaginary μ and measure $\frac{m_c(\mu)}{m_c(0)}$

de Forcrand, O.P. 08,09

Finite density: chiral critical line ⟶ critical surface







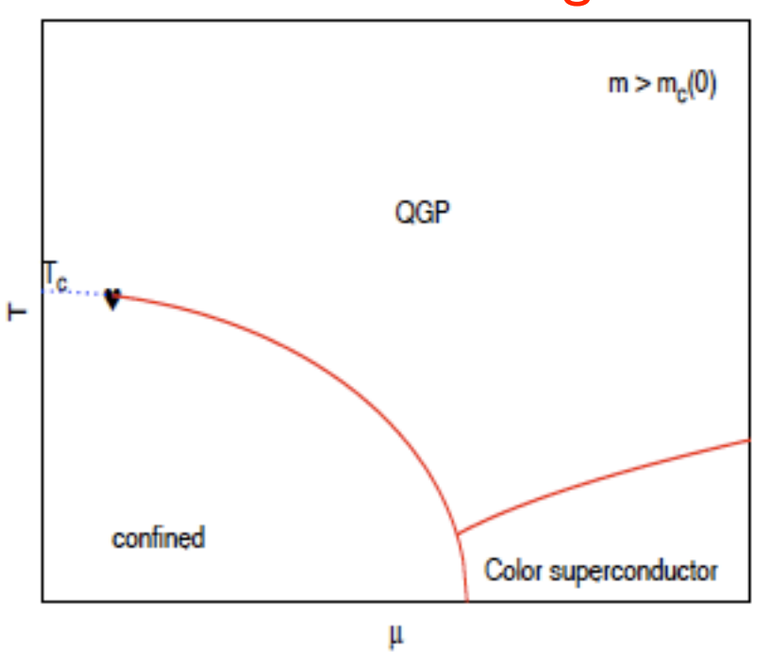
$$\frac{m_c(\mu)}{m_c(0)} = 1 + \sum_{k=1} c_k \left(\frac{\mu}{\pi T}\right)^{2k}$$

$c_1 > 0$

$c_1 < 0$

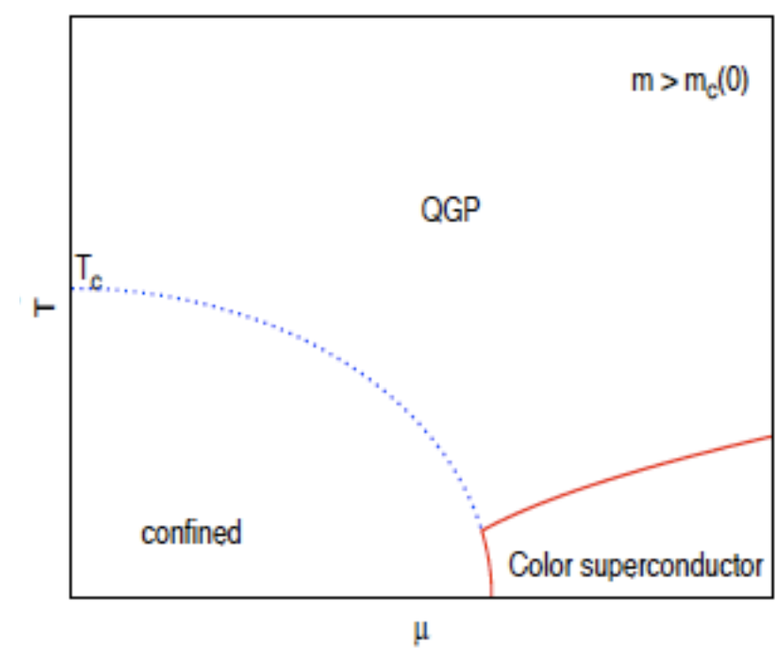
Standard scenario

transition strengthens

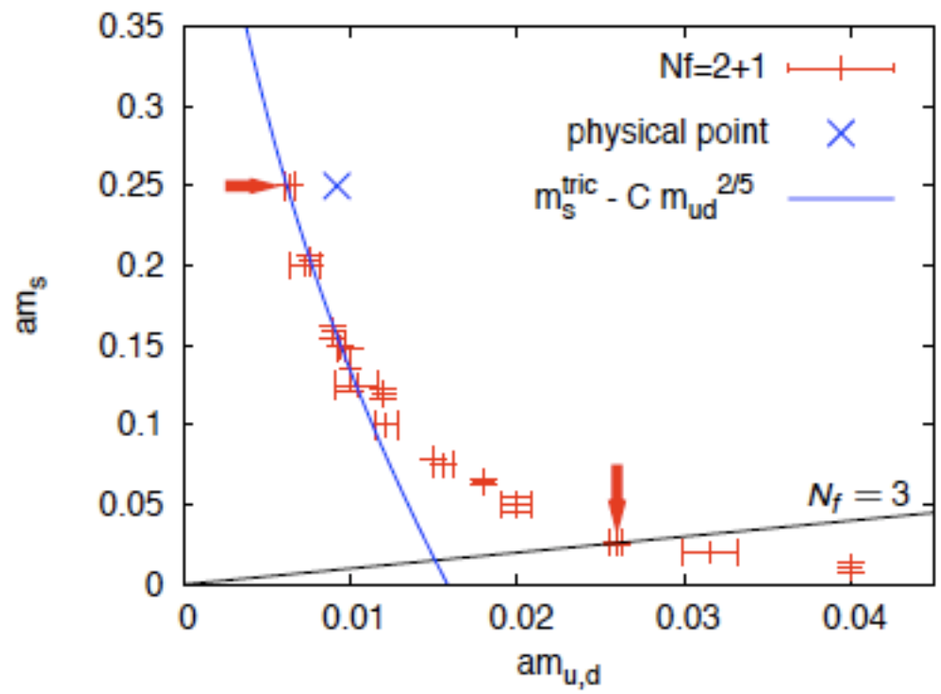


Exotic scenario

transition weakens



Curvature of the chiral critical surface



$N_f = 3$

$N_f = 2+1,\ m_s = m_s^{\rm phys}$

consistent $8^3 \times 4$ and $12^3 \times 4$, $\sim 5 \times 10^6$ traj.

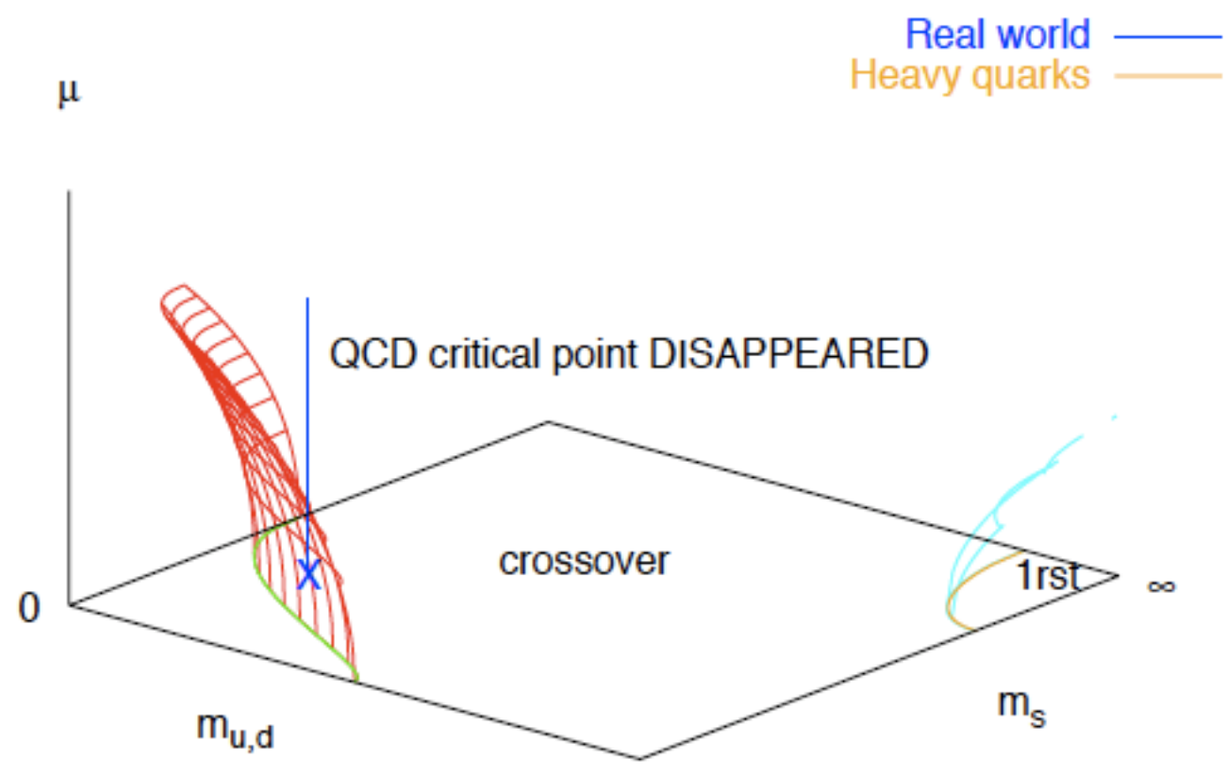
$$\frac{m_c(\mu)}{m_c(0)} = 1 - 3.3(3)\left(\frac{\mu}{\pi T}\right)^2 \underbrace{- 47(20)\left(\frac{\mu}{\pi T}\right)^4}_{\text{8th derivative of P}} - \ldots$$

$16^3 \times 4$, Grid computing, $\sim 10^6$ traj.

$$\frac{m_c^{u,d}(\mu)}{m_c^{u,d}(0)} = 1 - 39(8)\left(\frac{\mu}{\pi T}\right)^2 - \ldots$$

de Forcrand, O.P. 08,09

On coarse lattice exotic scenario: no chiral critical point at small density



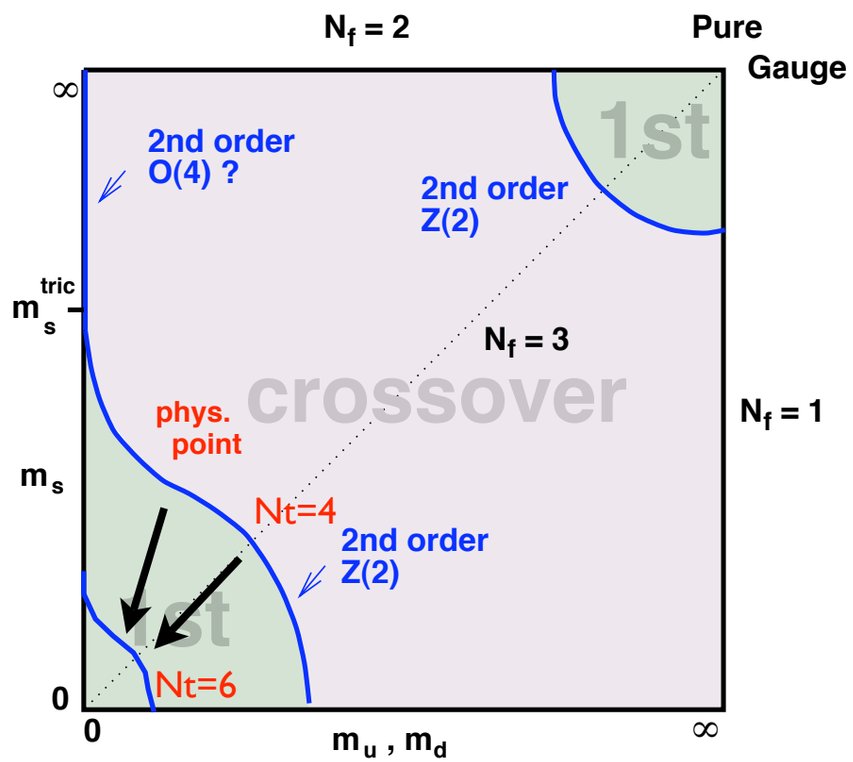
Weakening of p.t. with chemical potential also for:

-Heavy quarks — de Forcrand, Kim, Takaishi 05

-Light quarks with finite isospin density — Kogut, Sinclair 07

-Electroweak phase transition with finite lepton density — Gynther 03

Towards the continuum: $N_t = 6, a \sim 0.2$ fm

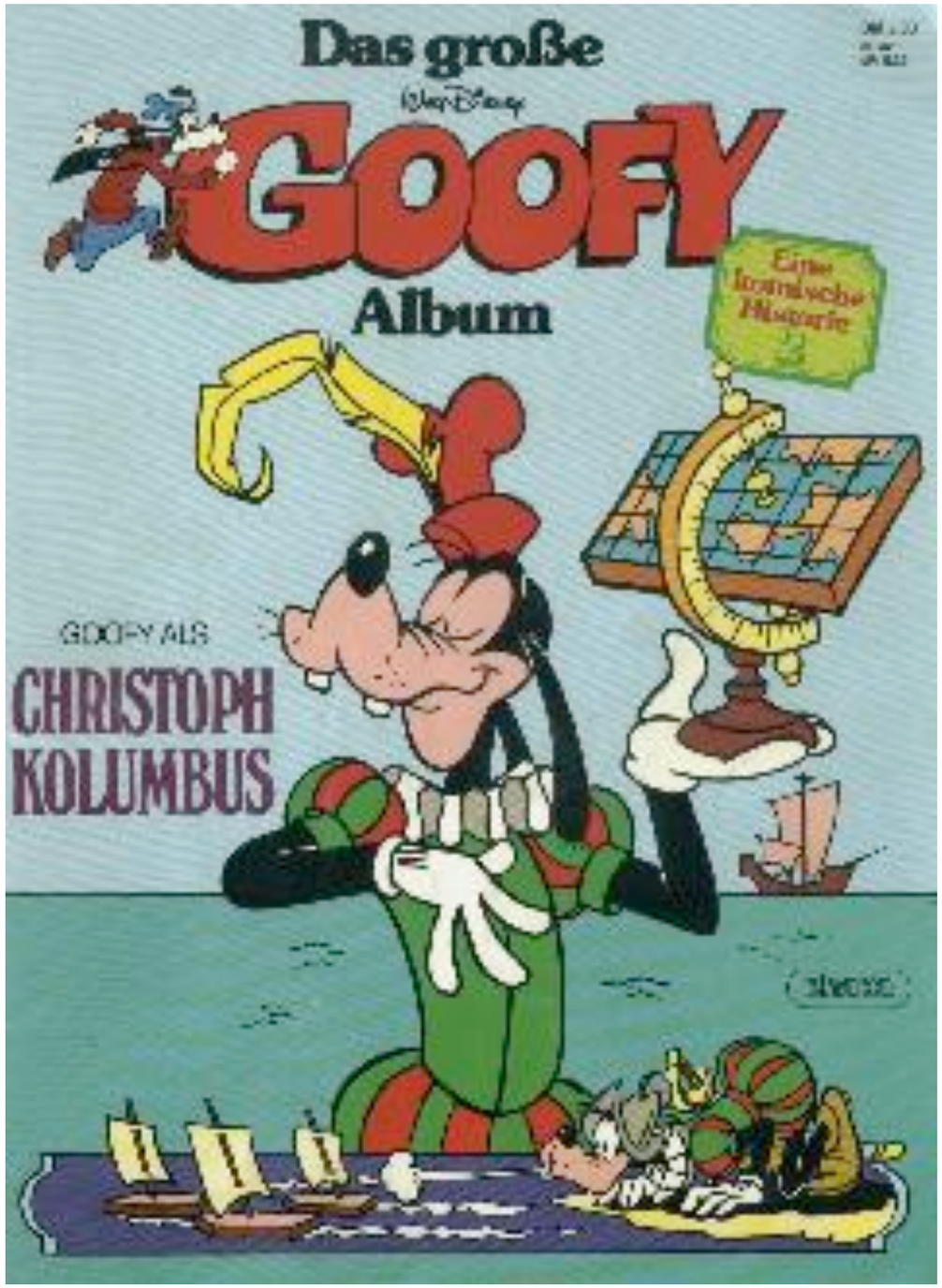


de Forcrand, Kim, O.P. 07
Endrodi et al 07

- Physical point deeper in crossover region as $a \to 0$
- Cut-off effects stronger than finite density effects!
- Preliminary: curvature of chiral crit. surface remains negative de Forcrand, O.P. 10

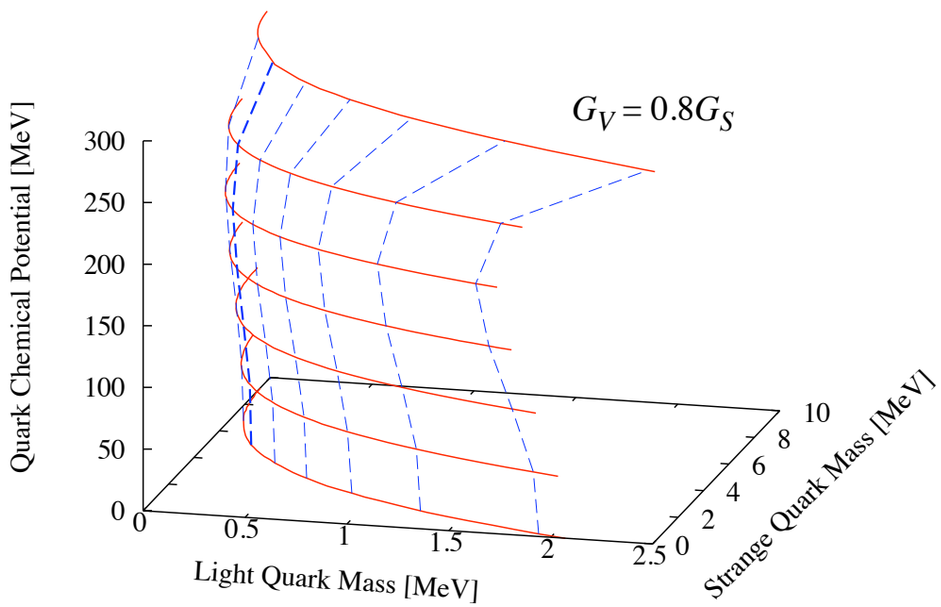
Un-discovering a critical point feels like...

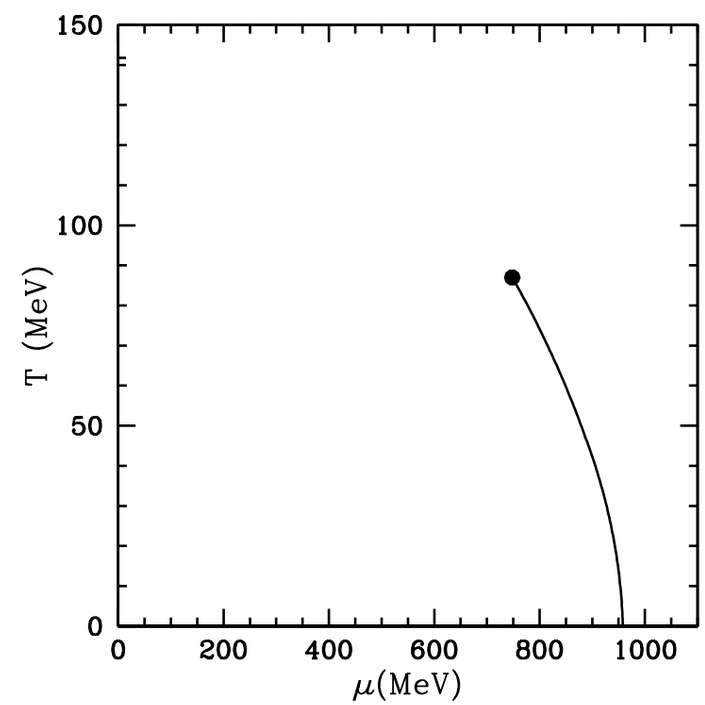
Un-discovering a critical point feels like...



Recent model studies with similar results

Fukushima 08, Bowman, Kapusta 09



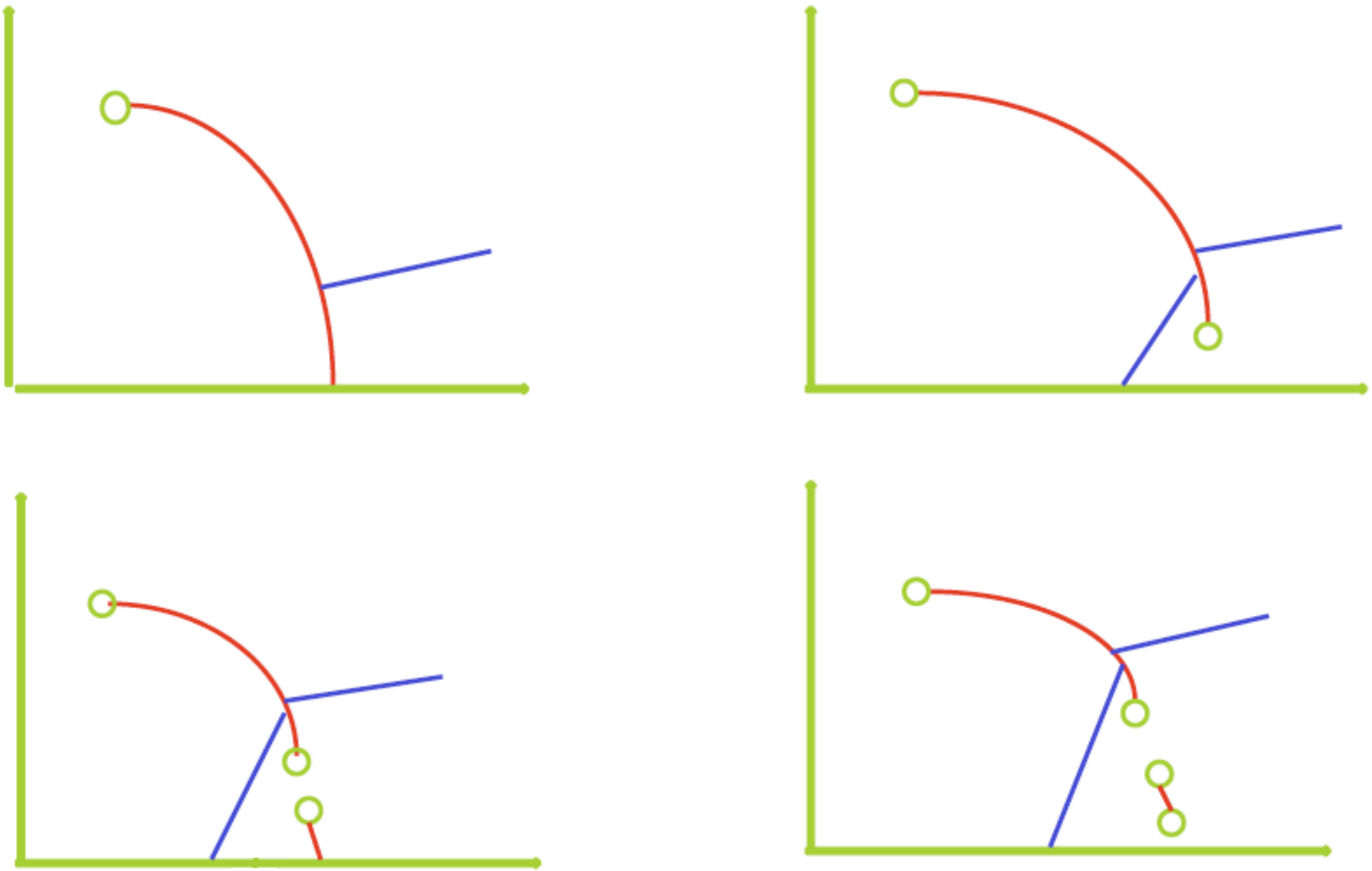


NJL-Polyakov loop model with vector-vector interaction, quark meson model

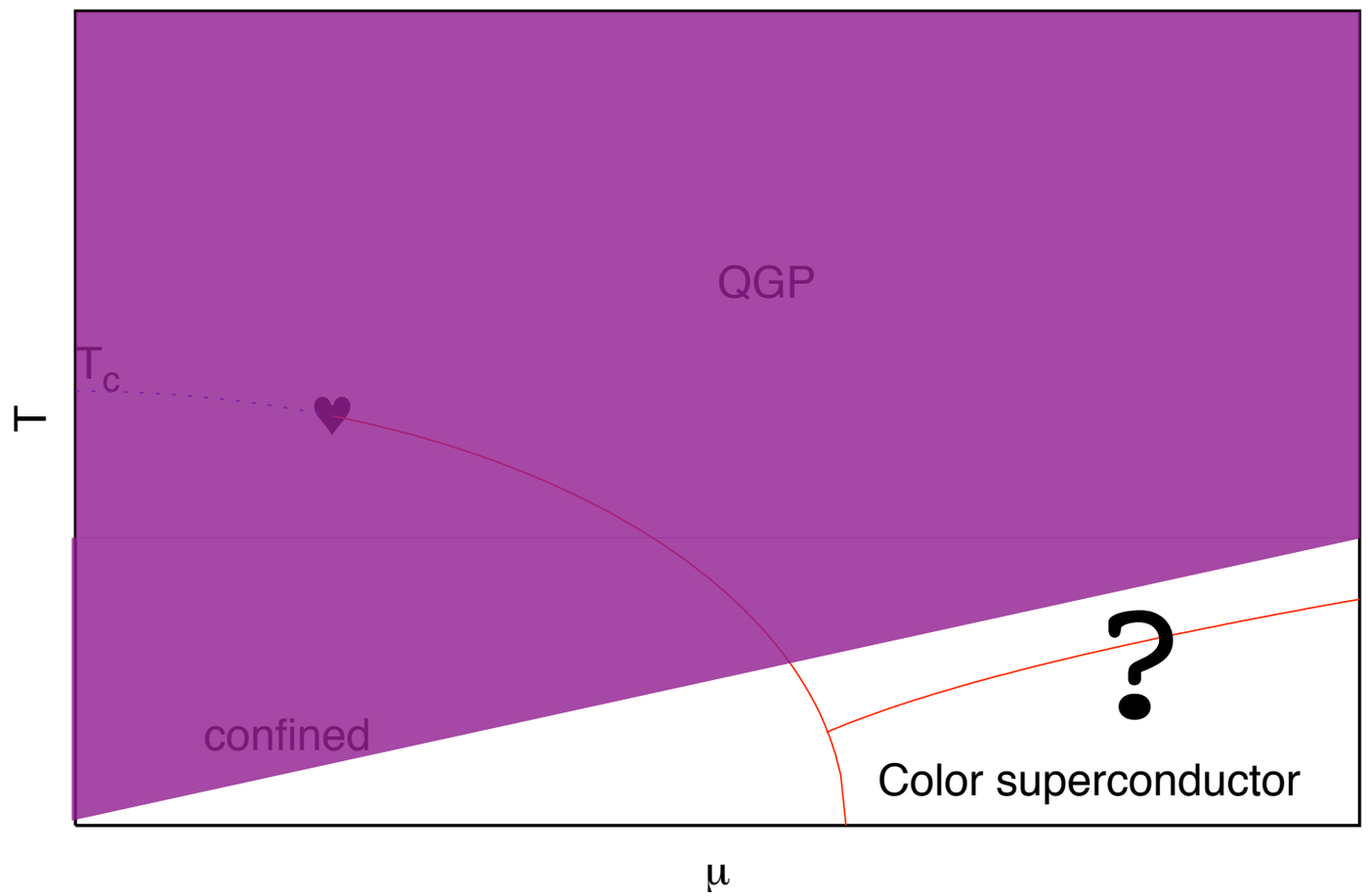
Qualitative behaviour similar to exotic lattice scenario
Critical end point at higher density!

Others with more (and non-chiral) critical points...

NJL with vector interactions, Zhang, Kunihiro, Fukushima 09
Ginzburg-Landau approach Baym et al. 06
for quark condensates,
beyond mean field methods... Ferroni, Koch, Pinto 10

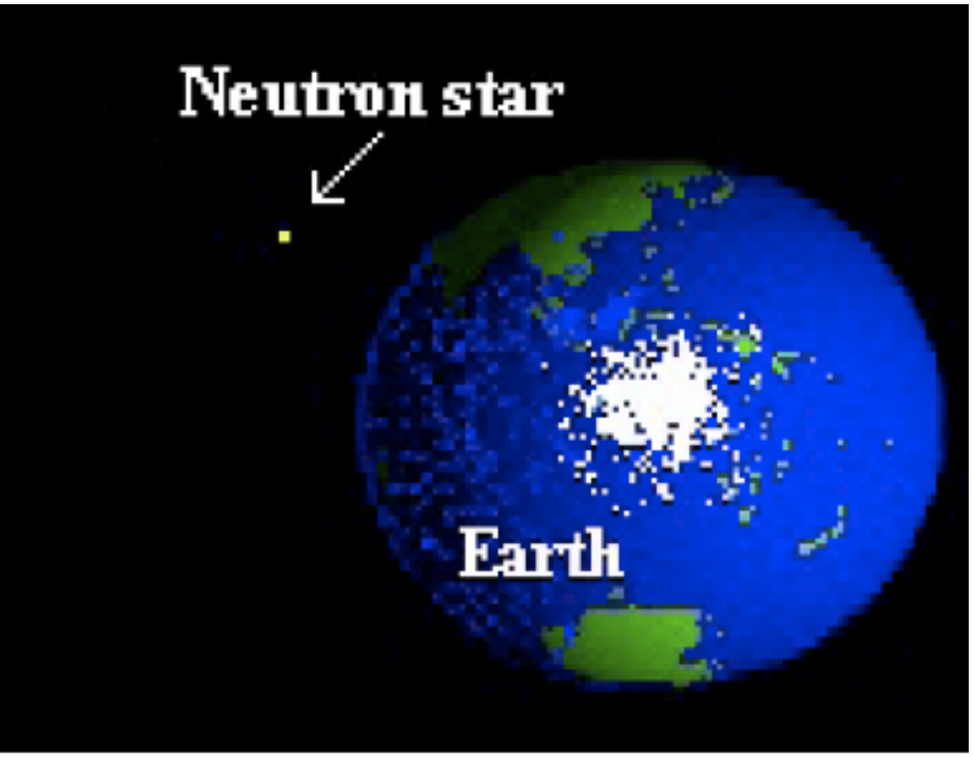


Outlook: what about high density physics?



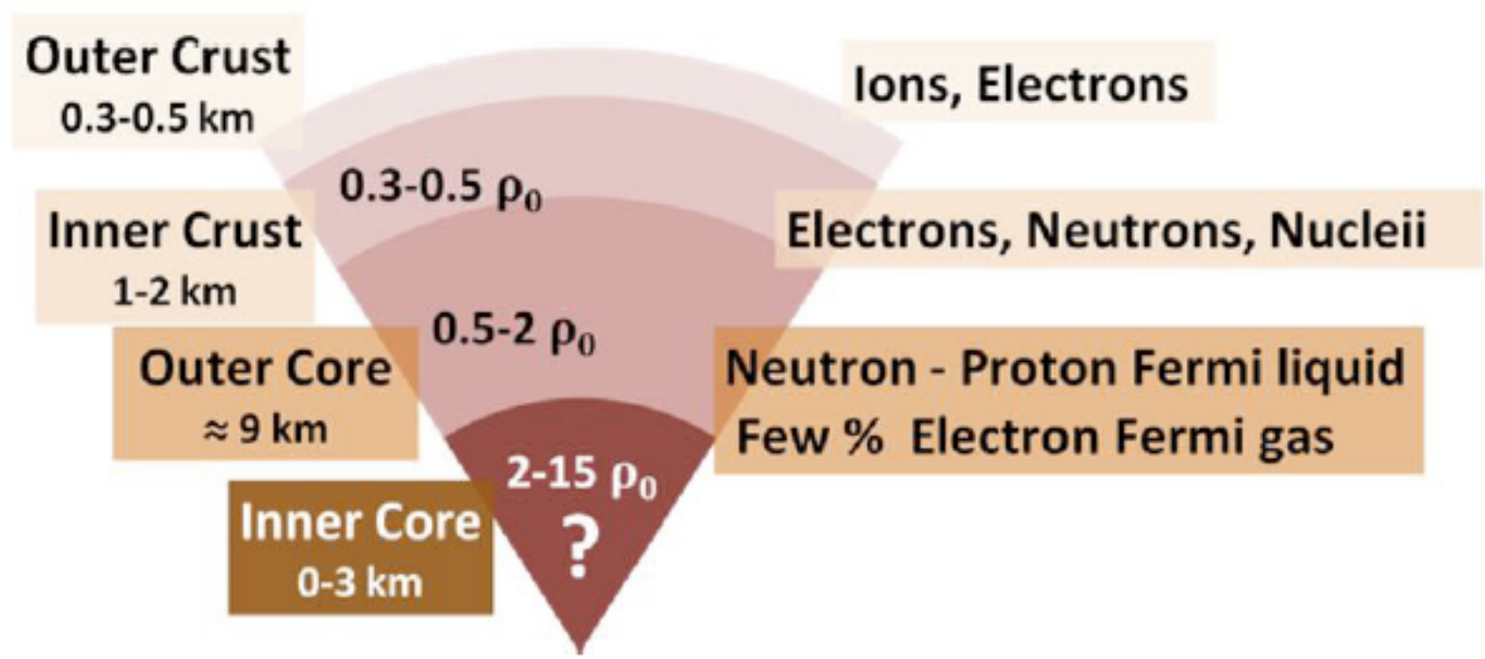
- Required input for physics of compact stars, phase diagram + EoS needed
- Sign problem strong, methods for $\mu < T$ fail

Back to heaven: what are compact stars made of?

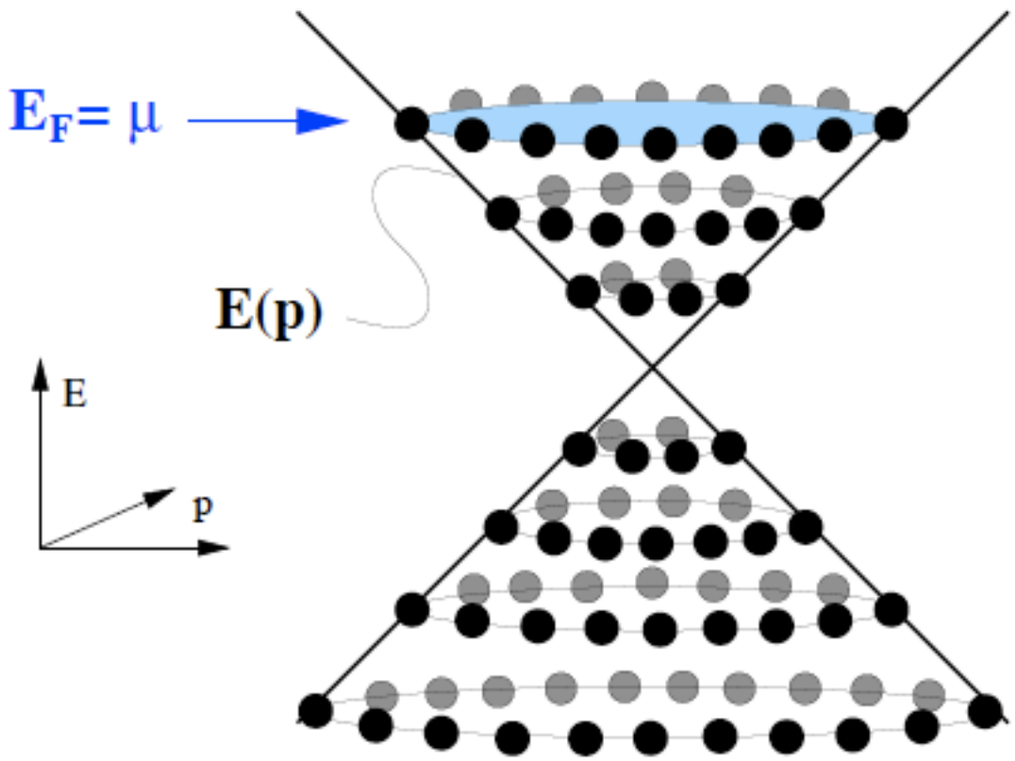


Radius ~ 10-12 km
Mass ~ 1.2-1.5 x Solar Mass

ρ_0 : nuclear density



- **Superconductivity: Cooper pairing of fermions**



free energy $\Omega = E - \mu N$

- no interactions: add fermion at $E = \mu$ without cost
- attractive interaction: add pair with gain
- pairs condense $\rightarrow$ superconductivity

This Bardeen-Cooper-Schrieffer (BCS) argument holds for electrons in metal, ^{3}He atoms, ...,
and quarks in quark matter at small coupling (infinite density)

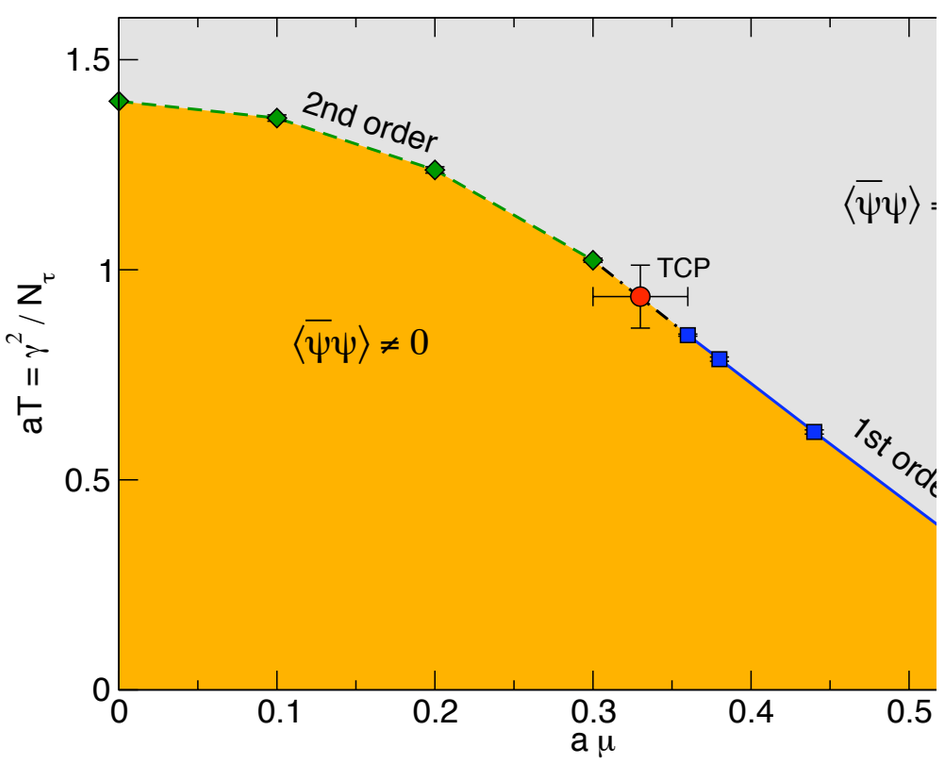
What happens at realistic densities/couplings??

Effective theories!

Split problem in perturbative (analytical) and non-perturbative (numerical) part

Large densities: perturbative; eff. theories for $\mu \to \infty$ exist, improve to NLO, discretise

Intermediate densities: lattice, strong coupling expansion in $\frac{1}{\alpha_s}$
eff. theory for infinite coupling exists, can be evaluated numerically



Liquid-gas transition at infinite coupling, m=0
de Forcrand, Fromm 09

Deconfinement transition for heavy quarks
Langelage, Lottini, O.P. 10

Perturbative refinement to large but finite couplings possible!

Conclusions

- QCD is verified fundamental theory of nuclear matter
- Hot/dense QCD important: nuclear, astro- and particle physics, heavy ion collisions
- Phase diagram: on coarse lattices no chiral critical point for $\mu < T$
- Still at the beginning: large discretisation and quark mass dependence, high densities need new methods
- Uncharted territory: do QCD critical points exist?