Does time matter?
Asymptotically Safe Lorentzian Gravity

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Quantum Gravity:
- No experimental data available!
- But many theories:
  - String Theory
  - Loop Quantum Gravity
  - Causal Dynamical Triangulations (CDT)
  - Random Gravity (RG)
  - Asymptotic Safety (AS)
  - ...
- Are they describing the same physics?
- Or do we have to choose different formulations?
- Comparison of Asymptotic Safety to CDT ≈ AS with causal structure?
- RG ≈ AS with broken Lorentz symmetry?

Foliation of Spacetime
- time slicing \( t(x) \) such that \( \partial_t \) is future-directed, timelike VF
- \( \Sigma_x \) = spatial slices with coordinate \( x^a \)
- parameterized curve \( \gamma(t) \) with \( y(t) = y(t) \)
- \( n^a = \frac{\gamma'(t)}{\sqrt{-g^{ab}\gamma(t)\gamma'(t)}} \)
- \( n^a \approx \frac{\sqrt{-g^{ab}\gamma(t)\gamma'(t)}}{\gamma'(t)} \)
- change of coordinates: \( x^t \rightarrow (r, y) \)
  \( dr^2 = \gamma_{
\mu} d\nu^\mu d\nu^\nu + \gamma_{\mu} d\nu^\nu d\nu^\mu \)
  with induced spatial metric: \( \gamma_{\mu\nu} = \gamma_{ab}\gamma_{\mu a}\gamma_{\nu b} \)
- theory described by \( (N^a, N^\nu) \) instead of \( \gamma_{ab} \)
- \( C = 1 \) = Euclidean instead of \( -C = 1 \) = Lorentzian

Exact RG Equation
Effective average action \( \Gamma_{\langle\rangle} \) interpolates between the microscopic action \( \langle A \rangle \) and the effective action \( A = 0 \):

\[ \partial_t \Gamma_{\langle\rangle} = \frac{1}{2} \text{Str} \left( \left( \Gamma^{(2)} + R_0 \right)^{-1} \partial_t R_0 \right) \]

Practical computations:
- Expansion of \( \Gamma_{\langle\rangle} \):
  \[ \Gamma_{\langle\rangle} = \sum_i a_i O_i \]
  \( O_i \in \{ \phi^\dagger, \phi, (\partial_\mu \phi)^\dagger \} \)
- Flow of \( \phi \) given by \( \beta \) functions: \( \partial_\mu \phi = -\lambda(\phi, \phi, \ldots) \)

Asymptotic Safety
A nonperturbatively renormalizable theory can be achieved by using an UV fixed point of the RG flow:
- Fixed Point: Vanishing of all \( \beta \) functions.

\[ \partial \phi = B_i (\phi, \phi, \ldots) \]
- In the vicinity of a fixed point \( \phi_0 \): linearized flow equations:
  \[ \partial \phi = \tilde{B}_i (\phi, \phi, \ld\ldots) \]
- Critical exponents \( \lambda_i \) : positive eigenvalues of \( B_i \).
- \[ \text{Hadamard} \rightarrow 0 \] (irrelevant directions) \( \rightarrow \) blue arrows.
- \[ \text{Hadamard} \rightarrow \infty \] (irrelevant directions) \( \rightarrow \) green arrows.
- Relevant directions determine the number of physical parameters to be fixed.
- Irrelevant directions determine the number of predictable physical parameters.

Truncation

General Einstein-Hilbert transition \( \text{see [3]} \)

\[ S = \frac{1}{16\pi G_N} \int d^dx \sqrt{-g} \left( -R + 2\Lambda \right) \]

dimensional constant \( \Lambda \) and \( (p - 1) \) scalar, scalar curvature \( R \)

Fixed Points
Compact Time Scenario
- \( T = \frac{1}{\hbar} \) constant \( k_\hbar m_\hbar = -m_\hbar \Rightarrow m_\hbar = 0 \)
- In this limit all trapezoidal functions decrease
- \( 0.00316 < \Lambda < 0.00319 \) \( T = \frac{1}{\hbar} \)

Non-Compact Time Scenario
- \( \text{choose} \ m = \hbar = \text{const} \ (e.g. \ 2\hbar) \) \( T \sim \frac{1}{\hbar} \)
- \( \text{all trapezoidal terms stay finite} \)
- \text{fixed point exists for both signatures}

Flow towards IR
- \( \text{Euclidean flow} \)
- \( \text{Star} \)

Geometric Cutoff
- compactly time direction on a circle \( \text{circumference} T \)
- \( \text{diagonal} \) transformation in time direction

\[ \phi(x) = \sum_{\nu = 0}^\infty \phi(\nu) \left( e^{2\nu T} \right) \Rightarrow \phi(x) = \frac{1}{T} \int_0^T \phi \left( \frac{x}{T} e^{2\nu T} \right) \]
- flow equations (similar Kaluza-Klein masses: \( m = \hbar \))
- kinetic \( k_\hbar \gamma_{\mu\nu} = \gamma_{\mu\nu}(\phi, \phi, \ldots) \)
- \( k_\hbar a_{\mu\nu} = \gamma_{\mu\nu}(\phi, \phi, \ldots) \)
- \( k_\hbar d_{\mu\nu} = \gamma_{\mu\nu}(\phi, \phi, \ld\ldots) \)
- \( k_\hbar x^2 > 0 \) (hyperbolic functions)
- \( k_\hbar x^2 > 0 \) \( \Rightarrow \) trapezoidal functions

References