Asymptotically Safe Lorentzian Gravity

Stefan Rechenberger

Uni Mainz

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arXiv:1102.5012v1 [hep-th]
with Elisa Manrique and Frank Saueressig
Outline

1. Motivation
2. CDT, Horava Gravity and Asymptotic Safety
3. Causal functional RG equation
4. Results
5. Conclusion
Motivation

Classical GR reaches its limits close to space-time singularities

- Black Holes
- Big Bang

Solution probably lies within a theory of Quantum Gravity
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different approaches to a theory of QG

- String Theory
- Loop Quantum Gravity
- Causal Dynamical Triangulations
- Horava Gravity
- **Asymptotic Safety**
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different approaches to a theory of QG

- String Theory
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which one is correct?
lack of experimental data  
⇒ nobody helps us to decide which is the best approach
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Best thing to do: compare different approaches
Causal dynamical triangulations (arXiv:1004.0352v1 [hep-th])

- discretization of gravitational path integral $\int \mathcal{D}g_{\mu\nu} e^{iS_{\text{grav}}}$
- summing over piecewise flat geometries
- modeling space-time geometries by gluing together simplices (higher dimensional generalizations of triangles)
- important: causal structure
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Horava Gravity (arXiv:0901.3775v2 [hep-th])

- different scaling of space and time
- UV: Lorentz invariance is broken
- IR: Lorentz invariance reestablished
- maybe connection to CDT due to global time foliation (arXiv:1002.3298v2 [hep-th])
Asymptotic Safety (Living Rev. Relativity 9, (2006))

- non-trivial fixed point (for UV completion)
- finite dimensional critical surface (for predictability)
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  - pure Einstein Hilbert action
  - f(R) gravity
  - gravity coupled to a scalar field
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- so far only Euclidean space-time has been studied
- Lorentzian space-times are necessary for comparison with CDT and HG
Causal functional RG equation

Starting point: Einstein Hilbert action

\[ S_{EH} = \frac{1}{16\pi G_N} \int d^D x \sqrt{\gamma} (-R + 2\Lambda) \]

- \( G_N \) ... Newton constant
- \( D \) ... space-time dimension
  \( (D = d + 1) \)
- \( \gamma \) ... metric
- \( R \) ... curvature scalar of space-time
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Motivation CDT, HG, AS

Causal FRGE

Results Conclusion

- split of space-time
  \[ M^D = M^d \times S^1 \]
- \( \Sigma \) ... spatial slices
- \( n^a \) ... vector orthonormal to \( \Sigma \)
- \( N \) ... lapse function
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- \( \sigma_{ij} \) ... spatial metric

\[
\gamma_{\mu\nu} = \begin{pmatrix}
\epsilon N^2 + N_i N^i & N_j \\
N_j & \sigma_{ij}
\end{pmatrix}
\]

\[
ds^2 = \epsilon N^2 d\tau^2 + \sigma_{ij} \left( dx^i + N^i d\tau \right) \left( dx^j + N^j d\tau \right)
\]
technical remarks

- choose background: $\bar{N} = 1$, $\bar{N}_i = 0$ and $\bar{\sigma}_{ij}$ unit sphere
- gauge fixing: $N = 0$ and $N_i = 0 \Rightarrow$ only spatial fluctuations
- purely spatial regulator: $R_k(\Delta)$ with $\Delta = \bar{\sigma}^{ij} \bar{D}_i \bar{D}_j$
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inserting this ansatz into the Wetterich equation

$$k \partial_k \Gamma_k = \frac{1}{2} \text{STr} \left[ k \partial_k \mathcal{R}_k \left( \Gamma_k^{(2)} + \mathcal{R}_k \right)^{-1} \right]$$

projection $\Rightarrow$ flow equations for $g_k$ and $\lambda_k$
\[ k \partial_k g_k = \beta_g(g, \lambda; m), \quad k \partial_k \lambda_k = \beta_\lambda(g, \lambda; m) \]

dimensionless Kaluza-Klein mass \( m = \frac{2\pi}{T_k} \)
Results

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carry out sums over Matsubara frequencies analytically:

\[ \sum_n \frac{1}{n^2 + x^2} = \frac{\pi}{x \tanh(\pi x)}, \quad x^2 > 0 \quad \text{(hyperbolic functions)} \]

\( x^2 < 0 \) analytic continuation leads to trigonometric functions
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in this limit all trigonometric functions diverge

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\( \Rightarrow \) NGFP only for \( g_* < 0 \) in Euclidean signature!
\[ m = \text{const.}(e.g. 2\pi) \implies T \propto \frac{1}{k} \]
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Euclidean Lorentzian

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Asymptotically Safe Lorentzian Gravity
FP for Euclidean and Lorentzian signature
- characteristics are similar
- also similar to covariant formulation
- time circle collapses toward UV
- signature does NOT matter in UV
- formulation prepares ground for comparison to other theories
Thank you for your attention!

Questions?