Relativistic Hydrodynamics

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Errata/Corrige

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1 A Brief Review of General Relativity

• Page 28, Eq. (1.115). Change

$$v^{x} = \frac{v^{x'} + V}{1 + v^{x'}V}, \qquad v^{y} = \frac{Wv^{y'}}{1 + v^{x'}V}, \qquad v^{z} = \frac{Wv^{z'}}{1 + v^{x'}V}.$$

$$\rightarrow \qquad v^{x} = \frac{v^{x'} + V}{1 + v^{x'}V}, \qquad v^{y} = \frac{v^{y'}}{W(1 + v^{x'}V)}, \qquad v^{z} = \frac{v^{z'}}{W(1 + v^{x'}V)}.$$

• Page 32, Eq. (1.140). Change

$$\mathscr{L}_{\phi V}T = \phi \mathscr{L}_V T$$
,

 \rightarrow

$$\mathscr{L}_{\phi V} T = \phi \mathscr{L}_V T - V \mathscr{L}_T \phi \,,$$

• Page 32, Eq. (1.141). Change

$$\mathscr{L}_{\mathbf{V}}\phi = V^{\nu}\partial_{\nu}\phi_{\nu} = \frac{d\phi}{d\lambda}\,,$$

 \rightarrow

$$\mathscr{L}_{\boldsymbol{V}}\phi = V^{\nu}\partial_{\nu}\phi = \frac{d\phi}{d\lambda}\,,$$

- Page 33, four lines before Eq. (1.147). Change ...not all bases are such that e_μ · e_ν ≠ η_{μν} →
 - ...not all bases are such that $oldsymbol{e}_{\mu}\cdotoldsymbol{e}_{
 u}=\eta_{\mu
 u}$
- Page 38, Eq. (1.174). Change

$$\mathscr{L}_{\boldsymbol{\eta}}\boldsymbol{\xi} = \mathscr{L}_{\boldsymbol{\xi}}\boldsymbol{\eta} = 0\,,$$

$$\mathscr{L}_{\boldsymbol{\eta}}\,\boldsymbol{\xi} = -\mathscr{L}_{\boldsymbol{\xi}}\,\boldsymbol{\eta} = 0\,,$$

• Page 43, Eq. (1.196). Change

$$\frac{d^2(x^{\mu}+\xi^{\mu})}{d\lambda^2}+\Gamma^{\mu}_{\alpha\beta}(x+\xi)\frac{d(x^{\alpha}+\xi^{\mu})}{d\lambda}\frac{d(x^{\beta}+\xi^{\mu})}{d\lambda}=0\,.$$

 \rightarrow

$$\frac{d^2(x^\mu+\xi^\mu)}{d\lambda^2}+\Gamma^\mu_{\alpha\beta}(x^\mu+\xi^\mu)\frac{d(x^\alpha+\xi^\alpha)}{d\lambda}\frac{d(x^\beta+\xi^\beta)}{d\lambda}=0\,.$$

• Page 44, Eq. (1.202). Change

$$R_{\alpha[\beta\gamma\delta]} = 2(R_{\alpha\beta\gamma\delta} + R_{\alpha\delta\beta\gamma} + R_{\alpha\gamma\delta\beta}) = 0.$$

 \rightarrow

$$3!R_{\alpha[\beta\gamma\delta]} = 2(R_{\alpha\beta\gamma\delta} + R_{\alpha\delta\beta\gamma} + R_{\alpha\gamma\delta\beta}) = 0$$

• Page 45, Eq. (1.207). Change

$$\Gamma^r_{\theta\theta} = -\sin\theta\cos\theta, \qquad \qquad \Gamma^\theta_{r\theta} = \cot\theta.$$

 \rightarrow

$$\Gamma^{\theta}_{\phi\phi} = -\sin\theta\cos\theta$$
, $\Gamma^{\phi}_{\theta\phi} = \cot\theta$.

• Page 45, Eq. (1.208). Change

$$R^{r}_{\theta\theta r} = -\frac{1}{R^{2}_{s}}g_{\theta\theta} = -\sin^{2}\theta, \qquad \qquad R^{r}_{\theta\theta r} = \frac{1}{R^{2}_{s}}g_{rr} = 1,$$

 \rightarrow

$$R^{\theta}_{\phi\phi\theta} = -\frac{1}{R^2_s} g_{\phi\phi} = -\sin^2\theta, \qquad \qquad R^{\phi}_{\theta\phi\theta} = \frac{1}{R^2_s} g_{\phi\phi} = 1.$$

• Page 45, after Eq. (1.208). Change while the Ricci scalar is simply $R = 1/R_s^2$.

while the Ricci tensor is $R_{ij} = g_{ij}/R_s^2$ and the Ricci scalar is simply given by $R = 2/R_s^2$.

• Page 48, Eq. (1.220). Change

$$R_{\mu\nu} = 8\pi \left(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu} + \frac{1}{4\pi}\Lambda g_{\mu\nu} \right) \,,$$

 \rightarrow

$$R_{\mu\nu} = 8\pi \left(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu} + \frac{1}{8\pi}\Lambda g_{\mu\nu} \right) \,,$$

• Page 49, one line after Eq. (1.223). Change ...the coordinate time runs slower than the proper time.

 \rightarrow

...the proper time runs slower than the coordinate time.

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• Page 51, Eq. (1.229). Change

$$\frac{d}{d\tau} \left[\left(1 - \frac{2M}{r} \right)^{-1} \frac{dr}{d\tau} \right] = r \left[\left(\frac{d\theta}{d\tau} \right)^2 + \sin^2 \theta \left(\frac{d\phi}{d\tau} \right)^2 \right] \,,$$

$$\frac{d}{d\tau} \left[\left(1 - \frac{2M}{r} \right)^{-1} \frac{dr}{d\tau} \right] = r \left[\left(\frac{d\theta}{d\tau} \right)^2 + \sin^2 \theta \left(\frac{d\phi}{d\tau} \right)^2 \right] \\ - \frac{M}{r^2} \left[\left(\frac{dt}{d\tau} \right)^2 + \left(1 - \frac{2M}{r} \right)^{-2} \left(\frac{dr}{d\tau} \right)^2 \right],$$

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• Page 77, Eq. (2.30). Change

$$S(t) \coloneqq -k_{\scriptscriptstyle \mathrm{B}} V H(t) = -k_{\scriptscriptstyle \mathrm{B}} V \int f(t, \vec{\boldsymbol{x}}, \vec{\boldsymbol{u}}) \, \ln(f(t, \vec{\boldsymbol{x}}, \vec{\boldsymbol{u}})) \, d^3 u \,,$$

 \rightarrow

$$S(t) \coloneqq -k_{\scriptscriptstyle \mathrm{B}} V H(t) = -k_{\scriptscriptstyle \mathrm{B}} V \int f(t, \vec{\boldsymbol{u}}) \, \ln(f(t, \vec{\boldsymbol{u}})) \, d^3 \boldsymbol{u} \, d^3 \boldsymbol{u$$

• Page 79, after Eq. (2.39). Change "...through the evolution of the momentum flux ρv_j (i.e., the rate of change of linear momentum per unit time and unit area), ..."

 \rightarrow "...through the flux of the *momentum density tensor* $\rho v_i v_j + P_{ij}$ (*i.e.*, the rate of change linear momentum), ..."

• Page 80, after Eq. (2.42). Change "...unlike the *kinetic energy*, $\frac{1}{2}\rho v^i v_i$,..." \rightarrow

"...unlike the kinetic energy density, $\frac{1}{2}\rho v^i v_i$,..."

• Page 80. Change Eq. (2.51)

$$\langle ec{m{u}}^2
angle = rac{3k_{\scriptscriptstyle
m B}T}{m} - \langle ec{m{u}}
angle^2 = rac{3k_{\scriptscriptstyle
m B}T}{m} - ec{m{v}}^2 \,,$$

 \rightarrow

$$\langle \vec{\boldsymbol{u}}^2 \rangle = \frac{3k_{\scriptscriptstyle \rm B}T}{m} + \langle \vec{\boldsymbol{u}} \rangle^2 = \frac{3k_{\scriptscriptstyle \rm B}T}{m} + \vec{\boldsymbol{v}}^2 \,, \label{eq:alpha}$$

• Page 81, Eq. (2.47). Change

$$\epsilon = \frac{3}{2} \frac{k_{\scriptscriptstyle \mathrm{B}} T}{m} \,, \qquad \qquad p = \frac{2}{3} \frac{\epsilon}{nm} = n k_{\scriptscriptstyle \mathrm{B}} T \,,$$

$$\epsilon = \frac{3}{2} \frac{k_{\scriptscriptstyle \mathrm{B}} T}{m} \,, \qquad \qquad p = \frac{2}{3} n \, m \, \epsilon = n k_{\scriptscriptstyle \mathrm{B}} T \,,$$

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 - Page 83, Eq. (2.54). Change

$$f_0(u) = 4\pi n u^2 \left(\frac{m}{2\pi k_{\rm B} T}\right)^{3/2} \exp\left(-\frac{m u^2}{2k_{\rm B} T}\right)$$

 \rightarrow

$$4\pi u^2 f_0(u) = 4\pi n u^2 \left(\frac{m}{2\pi k_{\rm B} T}\right)^{3/2} \exp\left(-\frac{m u^2}{2k_{\rm B} T}\right)$$

- Page 84, Fig. 2.5. Change the labels on the axes: $v \rightarrow u$.
- Page 84, Eq. (2.59). Change

$$T = \frac{mn}{3k_{\rm B}} \int (\vec{\boldsymbol{u}} - \vec{\boldsymbol{v}})^2 f_0 \, d^3 \boldsymbol{u} = \frac{m}{3k_{\rm B}} \langle (\vec{\boldsymbol{u}} - \vec{\boldsymbol{v}})^2 \rangle \,.$$

 \rightarrow

$$T = \frac{m}{3nk_{\rm B}} \int (\vec{\boldsymbol{u}} - \vec{\boldsymbol{v}})^2 f_0 \ d^3 \boldsymbol{u} = \frac{m}{3k_{\rm B}} \langle (\vec{\boldsymbol{u}} - \vec{\boldsymbol{v}})^2 \rangle \,.$$

 Page 84, last paragraph of Sect. 2.2.4. Change "(Problem 1 is dedicated to showing...)" →

"(Problem 4 is dedicated to showing...)"

• Page 86, after Eq. (2.70). Change

"...represents the flux of energy per unit surface and unit time, *i.e.*, the *Newtonian energy flux density vector*."

$$\rightarrow$$

"...represents the rate of change of energy per unit time and unit area, *i.e.*, the *Newtonian energy density flux.*"

• Page 90, after Eq. (2.82). Change "...and recalling that $p'_x = 0$ in the local Lorentz rest frame..." \rightarrow

"...and recalling that $p_{x'} = 0$ in the local Lorentz rest frame..."

• Page 90, last line. Change

"The *relativistic Maxwell–Boltzmann* equation can then be obtained..." \rightarrow

"The relativistic Boltzmann equation can then be obtained ... "

• Page 91, Eq. (2.88). Change

$$K \coloneqq \sqrt{(p_1)^{\alpha}(p_2)_{\alpha} - m^4 c^4}.$$

 \rightarrow

$$K \coloneqq \sqrt{(p_1)^{\alpha}(p_2)_{\alpha} - m^2 c^2} \,.$$

• Page 91, after Eq. (2.88). Change

"Note that the collisionless Maxwell-Boltzmann equation, namely (2.86) with..." \rightarrow

"Note that the collisionless Boltzmann equation, namely (2.86) with ... "

• Page 95, second line of Sec. 2.3.4. Change

"...we multiply the relativistic Maxwell–Boltzmann equation (2.86) by..." \rightarrow "...we multiply the relativistic Boltzmann equation (2.86) by..."

- Page 95, after Eq. (2.110). Change

"...can be transformed into a volume integral in momentum space ... " \rightarrow

"...can be transformed into a surface integral in momentum space..."

• Page 97, second line of Sec. 2.3.6. Change

"...is a solution of the relativistic Maxwell-Boltzmann equation (2.86)..." \rightarrow

 $c_s^2\coloneqq$

"...is a solution of the relativistic Boltzmann equation (2.86)..."

• Page 107, Eq. (2.168). Change

$$c_s^2 \coloneqq c^2 \left(\frac{\partial p}{\partial e}\right)$$
,

 $\left(\frac{\partial p}{\partial e}\right)$

• Page 108, Eq. (2.171). Change

$$c_s^2 = \frac{c^2}{h} (c_s^2)_{\rm N} \,.$$

 $c_s^2 = \frac{1}{h} (c_s^2)_{\mathrm{N}} \,.$

• Page 108, Eq. (2.175). Change

$$\begin{split} \mathscr{G} &> \frac{3}{2}c_s^2 \,, \qquad \qquad \left(\mathscr{G} < \frac{3}{2}c_s^2 \right) \,. \\ \\ \mathscr{G} &> \frac{3}{2}\frac{c_s^2}{c^2} \,, \qquad \qquad \left(\mathscr{G} < \frac{3}{2}\frac{c_s^2}{c^2} \right) \,. \end{split}$$

 \rightarrow

$$c_s^2 = \frac{1}{h} \left(\frac{dp}{d\rho}\right)_s = \left(\frac{d\ln h}{d\ln \rho}\right)_s, \qquad (2.1)$$
$$= \frac{1}{h} \left[\left(\frac{\partial p}{\partial \rho}\right)_{\epsilon} + \frac{d\epsilon}{d\rho} \left(\frac{\partial p}{\partial \epsilon}\right)_{\rho} \right] = \frac{1}{h} \left[\left(\frac{\partial p}{\partial \rho}\right)_{\epsilon} + \frac{p}{\rho^2} \left(\frac{\partial p}{\partial \epsilon}\right)_{\rho} \right]. \qquad (2.2)$$

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$$c_s^2 = \frac{c^2}{h} \left(\frac{dp}{d\rho}\right)_s = c^2 \left(\frac{d\ln h}{d\ln \rho}\right)_s,$$

$$c_s^2 \left[\left(\frac{\partial n}{\partial r}\right)_s - \frac{dc}{dr}\left(\frac{\partial n}{\partial r}\right)_s\right],$$
(2.3)

$$= \frac{c^2}{h} \left[\left(\frac{\partial p}{\partial \rho} \right)_{\epsilon} + \frac{d\epsilon}{d\rho} \left(\frac{\partial p}{\partial \epsilon} \right)_{\rho} \right] = \frac{c^2}{h} \left[\left(\frac{\partial p}{\partial \rho} \right)_{\epsilon} + \frac{p}{\rho^2} \left(\frac{\partial p}{\partial \epsilon} \right)_{\rho} \right].$$
(2.4)

• Page 116, Eq. (2.232). Change

$$c_s^2 = \frac{p(5\rho h - 8p)}{3\rho h(\rho h - p)} \, .$$

 \rightarrow

$$c_s^2 = c^2 \frac{p(5\rho h - 8p)}{3\rho h(\rho h - p)},$$

• Page 117, Eq. (2.234). Change

$$c_s^2 = \frac{\gamma \epsilon (\gamma - 1)}{c^2 + \gamma \epsilon} = \left(\frac{h - c^2}{h}\right)(\gamma - 1) = \frac{\gamma p}{\rho h}.$$

-

$$c_s^2 = \frac{c^2 \gamma(\gamma - 1)\epsilon}{c^2 + \gamma \epsilon} = \left[\frac{c^2(h - c^2)}{h}\right](\gamma - 1) = \frac{c^2 \gamma p}{\rho h}.$$

• Page 118, footnote 34. Change

"A fluid obeying the ideal-fluid equation of state with $\epsilon = 0$ would also have a zero temperature and could provide a reasonable model for a cold and old neutron star." \rightarrow

"A fluid obeying a general polytropic equation of state can have, at least mathematically, $\epsilon = 0$, although such a choice would be difficult to justify from a physical point of view. However, if the polytropic transformation is *isentropic*, then the specific internal energy is fully determined and is proportional to the rest-mass density [cf. Eq. (2.248) and discussion around it]. A polytropic and isentropic equation of state is often used to obtain a reasonable approximation for a cold and old neutron star.".

• Page 119, after Eq. (2.247)

Put differently, a polytropic equation of state is equivalent to an ideal-fluid equation of state *only* under those *isentropic transformations* for which the *adiabatic index* of the fluid γ is the same as the *adiabatic index of the polytrope* Γ ." \rightarrow

"Put differently, if a fluid obeys the ideal-fluid equation of state and is isentropic, then its equation of state can also be written in a polytropic form [*cf.*, Eq. (2.242)], with polytropic exponent $\Gamma = \gamma$; in this case, the polytropic exponent is also the adiabatic index. On the other hand, if a fluid obeys the polytropic equation of state and is isentropic, then it is at least formally possible to express the pressure as $p = \rho \epsilon (\Gamma - 1)$ [*cf.*, Eq. (2.228)]. However, this does not necessarily mean that such a fluid obeys an ideal-fluid equation of state. For this to be the case, Γ must be the ratio of the specific heats c_p/c_v and the specific internal energy must be a function of the temperature only."

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• Page 119, Eq. (2.249). Change

$$c_s^2 = \frac{\Gamma p}{\rho h} = \frac{\Gamma(\Gamma-1)p}{\rho(\Gamma-1) + \Gamma p} = \left(\frac{1}{\Gamma K \rho^{\Gamma-1}} + \frac{1}{\Gamma-1}\right)^{-1}.$$

$$c_s^2 = c^2 \frac{\Gamma p}{\rho h} = c^2 \frac{\Gamma(\Gamma-1)p}{\rho(\Gamma-1) + \Gamma p} = c^2 \left(\frac{1}{\Gamma K \rho^{\Gamma-1}} + \frac{1}{\Gamma-1}\right)^{-1} \,.$$

3 Relativistic Perfect Fluids

• Page 139, before Eq. (3.28). Change

flux of rest-mass current density in the $\hat{\mu}$ -direction,
flux of rest-mass density current in the $\hat{\mu}$ -direction,
:

• Page 139, before Eq. (3.29). Change

 $T^{\hat{\mu}\hat{\nu}}$: flux of $\hat{\mu}$ -momentum in the $\hat{\nu}$ -direction,

 $T^{\hat{i}\hat{0}}$: flux of \hat{i} -momentum in $\hat{0}$ -th direction (\hat{i} -momentum density),

 $T^{\hat{\mu}\hat{\nu}}$: flux of $\hat{\mu}$ -momentum density in the $\hat{\nu}$ -direction, $T^{\hat{i}\hat{0}}$: flux of \hat{i} -momentum density in $\hat{0}$ -th direction ,

• Page 139, (3.29). Change the second line as follows:

$$T^{\hat{0}\hat{i}} = T^{\hat{0}\hat{i}} = 0$$

 \rightarrow

 \rightarrow

$$T^{\hat{0}\hat{i}} = T^{\hat{i}\hat{0}} = 0,$$

• Page 146, Eq. (3.69). Change

$$\mathscr{L}_{\boldsymbol{u}}(hu_{\boldsymbol{\mu}}) = -\frac{1}{\rho} \nabla_{\boldsymbol{\mu}} p - \nabla_{\boldsymbol{\mu}} h \,.$$

 \rightarrow

$$\mathscr{L}_{\boldsymbol{u}}(hu_{\mu}) = -\frac{1}{\rho} \nabla_{\mu} p = -\nabla_{\mu} h \,.$$

• Page 146, Eq. (3.71). Change

$$\mathscr{L}_{\boldsymbol{u}}(hu_{\mu}\xi^{\mu}) = -\frac{\xi^{\mu}\nabla_{\mu}p}{\rho} - \xi^{\mu}\nabla_{\mu}h = -\frac{1}{\rho}\mathscr{L}_{\boldsymbol{\xi}}p - \mathscr{L}_{\boldsymbol{\xi}}h\,,$$

$$\mathscr{L}_{\boldsymbol{u}}(hu_{\boldsymbol{\mu}}\boldsymbol{\xi}^{\boldsymbol{\mu}}) = -\frac{\boldsymbol{\xi}^{\boldsymbol{\mu}}\nabla_{\boldsymbol{\mu}}p}{\rho} = -\frac{1}{\rho}\mathscr{L}_{\boldsymbol{\xi}}p = -\mathscr{L}_{\boldsymbol{\xi}}h\,,$$

• Page 146, before Eq. (3.72). Change

"and thus use the condition (3.65) with $\mathscr{L}_{u}p = 0$ and $\mathscr{L}_{u}h = 0$, to finally obtain" \rightarrow

"and thus use the condition (3.65) with $\mathscr{L}_{\boldsymbol{\xi}} p = 0$ to finally obtain"

Page 146, after Eq. (3.72). Change
Note the similarity between expression (3.72) and the corresponding equation (1.185)
along geodesic trajectories, *i.e.*, L_u(u_μξ^μ).

 Note the similarity between expression (3.72) and the corresponding equation (1.185)

Note the similarity between expression (3.72) and the corresponding equation (1.185) along geodesic trajectories, *i.e.*, $\mathscr{L}_{u}(u_{\mu}\xi^{\mu}) = 0$.

• Page 155, caption of Fig. 3.4. Change ... Show with blue solid lines

 \rightarrow

- ... Shown with blue solid lines
- Page 177, Eq. (3.253). Change

$$P_{R}^{\alpha\beta} \coloneqq \int I_{\nu} N^{\alpha} N^{\beta} d\nu d\Omega \,.$$

 \rightarrow

$$P^{\alpha\beta}_{_R}\coloneqq h^{\alpha}_{_{\gamma}} h^{\beta}_{_{\delta}} T^{\gamma\delta}_{_R} = h^{\alpha}_{_{\gamma}} h^{\beta}_{_{\delta}} \int I_{\nu} N^{\gamma} N^{\delta} d\nu d\Omega \,.$$

• Page 179, footnote 26. Change

"Multifluids of this type as sometimes also referred to as..." \rightarrow

"Multifluids of this type are sometimes also referred to as..."

4 Linear and Nonlinear Hydrodynamic Waves

• Page 200, Eq. (4.54). Change

$$\det(\mathcal{A}^t - \lambda \mathcal{A}^x) = 0$$

 \rightarrow

$$\det(\mathcal{A}^x - \lambda \mathcal{A}^t) = 0\,,$$

- Page 213, Fig. 4.5. The tangents to the fluidlines on either side of the rarefaction tail should be exactly the same and not as shown.
- Page 216, below Eq. (4.114). Change " It is also convenient to rewrite the continuity equation (4.112) and the conservation of energy (4.113)" \rightarrow

" It is also convenient to rewrite the continuity equation (4.112) and the conservation of momentum (4.113)"

• Page 217, first paragraph. Change "The classical Hugoniot adiabat is readily obtained from (4.118) after recalling that in the Newtonian limit $h_{\rm \scriptscriptstyle N} = 1 + \epsilon + p/\rho \approx 1"$ \rightarrow

"The classical Hugoniot adiabat is readily obtained from (4.118) after recalling that in the Newtonian limit $h = 1 + \epsilon + p/\rho \approx 1$ "

• Page 216. Change Eq. (4.117) as follows

$$(h_b W_b v_b)^2 - (h_a W_a v_a)^2 = -\left(\frac{h_a}{\rho_a} + \frac{h_b}{\rho_b}\right) \llbracket p \rrbracket .$$
$$(h_b W_b v_b)^2 - (h_a W_a v_a)^2 = \left(\frac{h_a}{\rho_a} + \frac{h_b}{\rho_b}\right) \llbracket p \rrbracket .$$

 \rightarrow

$$(n_b W_b v_b) - (n_a W_a v_a) = \left(\frac{1}{\rho_a} + \frac{1}{\rho_a}\right)$$

• Page 221, Eq. (4.138). Change

$$W_{ab}^2 = \frac{(3e_a + e_b)(3e_b + e_a)}{16e_1e_2} = \frac{4}{9}W_a^2W_b^2,$$

$$W_{ab}^2 = \frac{(3e_a + e_b)(3e_b + e_a)}{16e_a e_b} = \frac{4}{9}W_a^2 W_b^2 ,$$

• Page 222, after Eq. (4.141), the expression for the shock velocity should be modified as follows:

$$V_{s}^{\pm} = \rho_{b}W_{b}v_{b}/(\rho_{b}W_{b} \pm \rho_{a})$$

$$\rightarrow$$

$$V_{s}^{\pm} = \rho_{b}W_{b}v_{b}/(\rho_{b}W_{b} \mp \rho_{a})$$

• Page 225, Fig. 4.11, panel on bottom right. The labels in the spacetime diagram should be corrected as follows:

$$\mathscr{R}_{\leftarrow} \to \mathscr{R}_{\to} \text{ and } \mathscr{S}_{\to} \to \mathscr{S}_{\leftarrow}.$$

• Page 229, last sentence in the first paragraph should be modified as follows:

These values mark the transition from one wave pattern to another one, and that are directly computed from the initial conditions (Rezzolla and Zanotti, 2001). \rightarrow

These values mark the transition from one wave pattern to another one, and are directly computed from the initial conditions (Rezzolla and Zanotti, 2001).

5 Reaction Fronts: Detonations and Deflagrations

• Page 284, problem 2. Change

"Derive the inequalities (5.4)–(5.6) across a reaction front [Hint: start from the laws of conservation of momentum and energy (4.114)–(4.113)]. \rightarrow

"Derive the inequalities (5.4)–(5.6) across a reaction front [Hint: start from the laws of conservation of momentum and energy (4.113)–(4.114)].

6 Relativistic Non-Perfect Fluids

• Page 297, Eq. (6.73). Change: $\chi \rightarrow \chi_t$

7 Formulations of the Einstein–Euler **Equations**

• Page 337, Eq. (7.100). Change

$$\begin{split} \tilde{\Gamma}^{i}_{jk} &= \Gamma^{i}_{jk} - \frac{1}{3} (\delta^{i}_{j} \Gamma^{m}_{km} + \delta^{i}_{k} \Gamma^{m}_{jm} - \gamma_{jk} \gamma^{il} \Gamma^{m}_{lm}) \\ &= \Gamma^{i}_{jk} + 2 (\delta^{i}_{j} \partial_{k} \ln \phi + \delta^{i}_{k} \partial_{j} \ln \phi - \gamma_{jk} \gamma^{il} \partial_{l} \ln \phi) \,, \\ &\rightarrow \\ \tilde{\Gamma}^{i}_{jk} &= \Gamma^{i}_{jk} - \frac{1}{2} (\delta^{i}_{j} \Gamma^{m}_{km} + \delta^{i}_{k} \Gamma^{m}_{jm} - \gamma_{jk} \gamma^{il} \Gamma^{m}_{lm}) \\ &= \Gamma^{i}_{jk} + \delta^{i}_{j} \partial_{k} \ln \phi + \delta^{i}_{k} \partial_{j} \ln \phi - \gamma_{jk} \gamma^{il} \partial_{l} \ln \phi \,, \end{split}$$

$$\tilde{\Gamma}^{i}_{jk} = \Gamma^{i}_{jk} - \frac{1}{3} (\delta^{i}_{j} \Gamma^{m}_{km} + \delta^{i}_{k} \Gamma^{m}_{jm} - \gamma_{jk} \gamma^{il} \Gamma^{m}_{lm}) = \Gamma^{i}_{jk} + \delta^{i}_{j} \partial_{k} \ln \phi + \delta^{i}_{k} \partial_{j} \ln \phi - \gamma_{jk} \gamma^{il} \partial_{l} \ln \phi$$

• Page 339, Eq. (7.108). Change

$${}^{(3)}R + K^2 = K^{ij}K_{ij} + 4\pi E = \tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2 + 4\pi E$$

 \rightarrow

 \rightarrow

$${}^{(3)}R + K^2 = K^{ij}K_{ij} + 4\pi E = \tilde{A}_{ij}\tilde{A}^{ij} + \frac{1}{3}K^2 + 16\pi E$$

• Page 339, last two lines:

(note that $\tilde{\gamma}_{ij}$ and \tilde{A}_{ij} , have only five independent components each since they are traceless)

(note that $\tilde{\gamma}_{ij}$ and \tilde{A}_{ij} , have only five independent components each since they have traces that are equal to 3 or 0, respectively)

• Page 345, first term in Eq (7.131). Change

$$D_j \Theta_{ij} = 0 \,,$$

 \rightarrow

$$D^j \Theta_{ij} = 0 \,,$$

• Page 345, Eq (7.132). Change

$$\Sigma_{ij} \coloneqq \Theta_{ij} - \frac{1}{3} \gamma_{ij} \Theta_{kl} \Theta^{kl} = \frac{1}{2} \gamma^{1/3} \mathscr{L}_t \bar{\gamma}_{ij} \,,$$

$$\Sigma_{ij} \coloneqq \Theta_{ij} - \frac{1}{3} \gamma_{ij} \Theta_{kl} \gamma^{kl} = \frac{1}{2} \gamma^{1/3} \mathscr{L}_t \tilde{\gamma}_{ij} \,,$$

• Page 345, after Eq (7.132). Change where $\bar{\gamma}_{ij} \coloneqq \gamma^{-1/3} \gamma_{ij}$ is the conformal metric. \rightarrow

where $\tilde{\gamma}_{ij} \coloneqq \gamma^{-1/3} \gamma_{ij}$ is the conformal metric.

• Page 345, first term in Eq (7.133). Change

 $D_j \Sigma_{ij} = 0 \,,$

 \rightarrow

$$D^j \Sigma_{ij} = 0 \,,$$

• Page 354, first line. Change

"constraint decouples from the Hamiltonian constraint and it is possible to solve the latter to obtain the three vectors \bar{V}^{i} "

 \rightarrow

"constraint decouples from the Hamiltonian constraint and it is possible to solve the former to obtain the three vectors \bar{V}^{i} "

• Page 354, fourth line. Change

"The calculation of initial data via the solution of the constrains simplifies considerably if"

 \rightarrow

"The calculation of initial data via the solution of the constraints simplifies considerably if"

• Page 356, first sentence before Eq. (7.180). Change

"we further introduce the conformal metric $\bar{\gamma}$ [*cf.*, Eq. (7.152)], such that" \rightarrow

"we further introduce the conformal metric $\tilde{\gamma}$ [*cf.*, Eq. (7.152), although we here use a tilde rather than a bar to be closer to the notation of Bonazzola *et al.* (2004)], such that"

• Page 356, Eq. (7.180). Change

$$f \coloneqq \det(f_{ij}) = \bar{\gamma} \coloneqq \det(\bar{\gamma}_{ij}),$$

 \rightarrow

$$f \coloneqq \det(f_{ij}) = \tilde{\gamma} \coloneqq \det(\tilde{\gamma}_{ij}),$$

• Page 356, Eq. (7.181). Change

$$\psi = (\gamma/\bar{\gamma})^{1/12} = (\gamma/f)^{1/12}$$
.

 \rightarrow

$$\psi = (\gamma/\tilde{\gamma})^{1/12} = (\gamma/f)^{1/12}$$
.

• Page 384, second equation in Exercise 7. Change

$$D_i D_j \phi = -\frac{1}{2\phi} \tilde{D}_i \tilde{D}_j + \frac{1}{2\phi^2} \partial_i \phi \, \partial_j \phi \, .$$

$$D_i D_j \phi = \tilde{D}_i \tilde{D}_j \phi + \frac{2}{\phi} \partial_i \phi \, \partial_j \phi - \frac{1}{\phi} \gamma_{ij} \partial^k \phi \, \partial_k \phi \,.$$

8 Numerical Relativistic Hydrodynamics: Finite-Difference Methods

• Page 393, Eq. (8.16). Change

$$\epsilon_j^{(h)} = \tilde{C}h^{\tilde{p}_j} + \mathcal{O}(h^{\tilde{p}_j+1}),$$

 \rightarrow

 \rightarrow

$$\epsilon_j^{(h)} = Ch^{p_j} + \mathcal{O}(h^{p_j+1}),$$

• Page 393, Eq. (8.30). Change

$$\begin{split} \tilde{p} &\coloneqq \frac{\log R(h,k)}{\log(h/k)} \,, \\ \tilde{p} &\coloneqq \frac{\log |R(h,k)|}{\log(h/k)} \,, \end{split}$$

• Page 395, the 7th line before Eq. (8.34). Change

"...its application across a time interval Δt introduces an associate truncation error $\epsilon_j(h)$." \rightarrow

"...its application across a time interval Δt introduces an associated truncation error $\epsilon^{(h)}$." • Page 406, Eq. (8.87). Change

$$\tilde{u}(x,t) = e^{-\varepsilon k^2 t} e^{ik \left[x - \left(v + \beta k^2\right)t\right]},$$

$$\tilde{u}(x,t) = e^{-\varepsilon k^2 t} e^{ik\left[x - \left(\lambda + \beta k^2\right)t\right]},$$

9 Numerical Relativistic Hydrodynamics: HRSC Methods

• Page 439. The following note should be added: Note that $\rho_{*,L}$ is different from $\rho_{*,R}$, thus reproducing the contact discontinuity and justify that the Riemann solver is complete.

11 Relativistic Hydrodynamics of Non-Selfgravitating Fluids

• Page 518, Eq. (11.84). Change

$$\frac{d\mathcal{W}}{\mathcal{W}} = \frac{M}{r^2 \mathcal{W}} dr + \frac{u}{\mathcal{W}} du.$$
$$\frac{d\mathcal{W}}{\mathcal{W}} = \frac{M}{r^2 \mathcal{W}^2} dr + \frac{u}{\mathcal{W}^2} du.$$

12 Relativistic Hydrodynamics of Selfgravitating Fluids

- Page 596, 12th line after Eq. (12.13) " $M = 2.01 \pm 0.4 M_{\odot}$ " \rightarrow " $M = 2.01 \pm 0.04 M_{\odot}$ "
- Page 597, caption of Fig. 12.1 " $M = 2.01 \pm 0.4 M_{\odot}$ " \rightarrow " $M = 2.01 \pm 0.04 M_{\odot}$ "
- Page 601, second term in Eq. (12.31). Change

$$\begin{split} H_0^2 &= \frac{1}{R_{\rm i}^2} \left[1 - \frac{\varepsilon (1+w_{\rm R})^2}{w_{\rm i}} \right] \,. \end{split}$$

$$H_0^2 = \frac{1}{R_{\rm i}^2} \left[1 - \frac{\varepsilon (1+w_{\rm i})^2}{w_{\rm i}} \right]$$

- Pagr 601, second paragraph, change:
- ... the energy density $\rho(r)$ and the pressure p(r) ... \rightarrow
- ... the energy density $\boldsymbol{e}(r)$ and the pressure $\boldsymbol{p}(r)$...
- Page 606, second but last line:

 \rightarrow

"K = 100" \rightarrow "K = 164"